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ABSTRACTS

Kato square root problem for parabolic operators

Pascal Auscher

CNRS, France
pascal.auscher@u-psud.fr

The Kato square root problem for elliptic operators is by now a well-known result in harmonic analysis and partial differential equations. Better understanding of the proof and development of new arguments allow to prove extensions in various directions of the elliptic theory of second and higher order operators which are called maximal accretive, so that they posses a square root . Surprisingly, parabolic operators may also be maximal accretive and be associated with a sesquilinear form. This is obvious for the heat operator $\partial_t - \Delta_x$ but this is also true when the Laplacian is replaced by a second order elliptic operator with measurable coefficients $divA(x,t)\nabla$. The Kato square root problem is to identify the domain of their square roots as the form domain. We show this is the case. We employ the first order approach that Alan popularized. The difficulty here lies in the measurable dependence of the coefficients with respect to the time variable.

This talk is based on joint work with M. Egert and K. Nyström.

This talk could never have happen without the joint works and inspiring discussions with Alan.

McIntoshery in Geometry

Lashi Bandara

University of Potsdam, Germany
lashi.bandara@uni-potsdam.de

The ideas and methods developed by McIntosh are well known to have had a significant impact across multiple areas of mathematics. This talk will focus on the application of "McIntoshery" to geometry which I will present in two parts. The first part concerns the "spectral flow" of paths of unbounded self- adjoint operators, which is more or less the net number of eigenvalues crossing zero along the path. Understanding spectral flow is not only interesting mathematically, but given its origins, also to physics. Two geometrically interesting examples are paths of Atiyah-Singer Dirac operators obtained via:

1. a time-varying family of Riemannian metrics on boundaryless manifold, and

2. through perturbations of local boundary conditions when the manifold has boundary.

Both these questions have now been answered under a set of very weak geometric assumptions, the first in a work involving both McIntosh and Rosén and the second with Rosén. These questions were motivated by Wojciechowski during his visit to ANU in the mid 2000's, when he became aware of the first-order formulation of the Kato square root problem for the Hodge-Dirac operator which was able to deal with moving domains, a situation similar that which arises for spectral flows. The second part is ongoing work with Bär and it is motivated by the desire to formulate and understand index formulae for the Rarita-Schwinger operator on 3/2-spinors on manifolds with boundary. This necessitates being able to consider boundary value problems and the state-of-the-art to do this is Bär-Ballmann framework. However, a standing assumption of this theory is that induced operator on the boundary is symmetric, which although satisfied by many of the natural operators in geometry including the Atiyah-Singer Dirac operator, is not satisfied by Rarita-Schwinger. It turns out, however, that using the ellipticity of the operator, one can extract a bi-sectorial operator on the boundary. Through combining pseudo-differential methods along with results of of McIntosh, we obtain a holomorphic bounded functional calculus for the boundary operator. This allows us to characterise the maximal domain as well as understand higher regularity, which is the first step in extending the Bär-Ballmann framework.

The Brascamp–Lieb inequalities

Michael Cowling

University of New South Wales, Australia,
m.cowling@unsw.edu.au

About 20 years ago, the mathematical physicists Herm Brascamp and Eliott Lieb discovered a family of inequalities, which now bear their names. These inequalities generalise several of the classical inequalities of analysis, including Hölder's inequality, Young's convolution inequality, and the Loomis–Whitney inequality. The Brascamp–Lieb inequalities depend on linear maps L_j with a common domain V and different ranges V_j and a family of indices θ_j in $[0, 1]$. The inequalities hold for some but not all of the possible indices. For applications in partial differential equations it is important to understand for which L_j and θ_j the inequalities hold, and whether the constants that appear in the inequalities depend continuously on the parameters. in this talk, i review the inequalities, and discuss some recent progress on these questions.

Operational calculus from hypergroups

Ian Doust

University of New South Wales, Australia,
i.doust@unsw.edu.au

Let A denote the square root of the negative of the Laplacian on a complete n-dimensional Riemannian manifold. One can use the spectral theorem to compute functions $f(A)$ of this operator. In particular, one gets that the operators $U(t) = \cos tA$ are uniformly norm bounded, and have 'finite propogation speed' in the sense that $\mathrm{supp} U(t)\delta_{x_2} \subseteq \overline{B_{|t|}(x_2)}$. Cheeger, Gromov, Taylor and others have used properties of the 'cosine family' $\{U(t)\}_{t\in\mathbb{R}}$ to obtain, for example, L^p bounds via the formula

$$f(A) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \hat{f}(t)\, \cos(tA)\, dt$$

for certain functions f. In this talk I will discuss a general framework based on the harmonic analysis associated with various hypergroup structures on $(0, \infty)$ which allows us to provide families of such representations which can give boundedness results in cases where we start with a cosine family which is not so well-behaved, This is joint work with Gordon Blower (Lancaster).

Some recent progress on estimates of singular integrals

Xuan Duong

Macquarie University
xuan.duong@mq.edu.au

We will present some recent progress on estimates of singular integrals with rough kernels on certain non-doubling spaces (including the non-doubling manifolds with ends).

Quantitative Rectifiability and absolute continuity of harmonic measure

Steve Hofmann

University of Missouri, USA,
hofmanns@missouri.edu

We discuss recent progress in an ongoing program to characterize geometrically those open sets in Euclidean space, such that harmonic measure for the open set Ω is absolutely continuous, in a quantitative, scale invariant sense, with respect to surface measure on $\partial\Omega$. In part, this involves understanding the relationship between quantitative rectifiability of the boundary of Ω, and the boundary behavior of harmonic functions.

An Optimization Approach to Multidimensional Wavelets

Jeff Hogan

University of Newcastle, Australia,
jeff.hogan@newcastle.edu.au

The first compactly supported continuous orthogonal MRA wavelets on the line were produced by I. Daubechies in 1988. Until that time it was thought that such functions did not exist. Daubechies' construction relied heavily on techniques of complex analysis (such as spectral factorization), many of which are unavailable in the higher-dimensional setting. Wavelets have become invaluable tools in the analysis and processing of single-variable, single-channel signals, offering a "zoom-in" feature not available in standard Fourier analysis while maintaining the speed of the fast Fourier transform. Continuous, compactly supported orthogonal MRA wavelets in $\mathbb{R}^n$ $(n \geq 2)$, on the other hand have proven to be elusive. I will report on unfinished joint work with David Franklin (Newcastle) and Matthew Tam (Göttingen) in which we search for (non-separable) multidimensional wavelets with the help of iterated projections, reflections and PALM (Proximal Alternating Linear Minimization). Potential extension of these constructions to the Clifford-valued case leaves open the possibility of fast algorithms for the processing of multi-variable, multi-channel signals such a colour images.

Bounded variation approximation of martingales and solutions in L^p

Tuomas Hytönen

University of Helsinki, Finlan
tuomas.hytonen@helsinki.fi

I report on joint work with Andeas Rosén (né Axelsson), a former student of Alan. For a given function and $\epsilon > 0$, we look for an ϵ-approximation, in a suitable L^p sense, of locally bounded variation. The novelty is dealing with a finite Lebesgue exponent $p < \infty$, while previous related results are concerned with $p = \infty$ (possible with BMO in place of L^∞). In this way we extend, from infinite to finite p, some classical work of Varopoulos from late 1970's, when the initial function is a dyadic martingale, and more recent work of Hofmann, Kenig, Mayboroda and Pipher, when it is a solution of an elliptic equation. Very recently, these L^p-approximability results have been connected to uniform rectifiability by Steve Hofmann, Olli Tapiola, and Simon Bortz.

Analyticity and gradient estimates in L^p of the Stokes semigroup in bounded Lipschitz domains

Sylvie Monniaux

Université d'Aix-Marseille, France,
sylvie.monniaux@univ-amu.fr

In 2012, Z. Shen proved that in the case of a bounded Lipschitz domain Ω, the semigroup generated by the Stokes operator with Dirichlet boundary conditions, originally defined in $L^2(\Omega, \mathbb{R}^d)$, extends to an analytic semigroup in $L^p(\Omega, \mathbb{R}^d)$ for p in an open interval containing $\left[\frac{2d}{d+1}, \frac{2d}{d-1}\right]$. This result answered a longstanding question. Recently, P. Tolksdorf showed gradient estimates for the Stokes operator with Dirichlet boundary conditions in the critical space $L^3(\Omega, \mathbb{R}^3)$ in Lipschitz domains.

In this talk, I will show how to prove gradient estimates of the Stokes semigroup with "natural boundary conditions", using methods developed together with Alan M^cIntosh. In particular, we reduce the problem to a first order system and prove bounded holomorphic functional calculus for the Stokes operator in a larger interval than the one obtained in the case of Dirichlet boundary conditions.

References

[1] Alan M^{c}Intosh and Sylvie Monniaux. Hodge-Dirac, Hodge-Laplacian and Hodge-Stokes operators in L^p spaces on Lipschitz domains. Submitted, 2016.
[2] Zhongwei Shen. Resolvent Estimates in L^p for the Stokes Operator in Lipschitz Domains, Arch. Rational Mech. Anal. 205 (2012) 395–424.
[3] Patrick Tolksdorf. On the L^p theory of the Navier-Stokes equations on three-dimensional bounded Lipschitz domains, arXiv:1703.01091 (2017).

Quadratic estimates for Hodge–Dirac operators with singular potentials

Andrew Morris

University of Birmingham, United Kingdom,
a.morris.2@bham.ac.uk

We will highlight the power and elegance of some first-order methods pioneered by Alan McIntosh, surveying his work on finite propagation speed and embeddings for Hardy spaces. Inspired by his vision and mentorship, we establish new quadratic estimates for singular additive perturbations of first-order elliptic systems, such as the perturbed Hodge–Dirac operator $D = d + d^* + V$, incorporating a singular potential V, on $\mathbb{R}^n$. The homogeneous estimates require a substantial reworking of existing techniques and this restricts the admissible singularities. The inhomogeneous estimates, however, follow from a more straight-forward perturbation argument that permits greater diversity in the potentials. The solution of the Kato square-root problem and the solvability of boundary value problems for magnetic Schrödinger operators follow as applications.

R-bisectoriality and boundedness of the H^∞ functional calculus for Hodge-Dirac operators associated with Ornstein Uhlenbeck operators and Witten Laplacians

Jan van Neerven

Delft University of Technology, Netherlands
J.M.A.M.vanNeerven@tudelft.nl

We discuss R-bisectoriality and boundedness of theH^∞-functional calculus in L^p, $1 < p < \infty$ for Hodge-Dirac operators associated with Ornstein-Uhlenbeck operators and Witten Laplacians. This is joint work with Jan Maas and Rik Versendaal.

Commutators and weighted norm inequalities: a survey.

María Cristina Pereyra

University of New Mexico, USA,
crisp@math.unm.edu

I will survey developments on weighted norm inequalities using as case study the commutator of the Hilbert transform and BMO functions. We will go from the celebrated Coifman-Rochberg-Weiss characterization to sharp quantitative weighted norm inequalities and the most recent domination by dyadic sparse positive operators results of Lerner, Ombrosi, and Rivera-Ríos. We will touch on a number of results that have been recently proven and techniques that have been perfected along the way, including Hytönen's resolution of the A_2 conjecture and dyadic averaging and domination techniques which are seeing a lot of applications beyond weighted norm inequalities.

Mono-Component Function Theory and Applications in Signal Analysis

Tao Qian

University of Macau, China,
tqian1958@gmail.com

Given a real-valued signal f one can associate it with a Hardy space function F whose real part coincides with f. It is known that such function F is composed as $F = f + iHf$, where H stands for the Hilbert transformation of the context. We introduce that in the standard cases one can develop fast orthogonal decompositions of F with the form

$$F = \sum_{k=1}^{\infty} c_k B_k, \tag{1}$$

where B_k's are also Hardy space functions (equivalent with the condition $HB_k = -iB_k$) with the additional properties

$$B_k(t) = \rho_k(t)e^{i\theta_k(t)}, \quad \rho_k \geq 0, \quad \theta_k'(t) \geq 0. \tag{2}$$

Correspondingly, the original real-valued function f can be decomposed as

$$f = \sum_{k=1}^{\infty} \rho_k(t) \cos \theta_k(t)$$

with, besides the properties of ρ_k and θ_k given in (2), also satisfies

$$H(\rho_k \cos \theta_k)(t) = \rho_k(t) \sin \theta_k(t).$$

Real-valued functions $f(t) = \rho(t) \cos \theta(t)$ that satisfy the condition

$$\rho \geq 0, \quad \theta'(t) \geq 0, \quad H(\rho \cos \theta)(t) = \rho(t) \sin \theta(t)$$

are called mono-components. If f is a mono-component, then the phase derivative $\theta'(t)$ is defined to be instantaneous frequency of f. The above described positive-instantaneous frequency expansions (1) are generalizations of the Fourier series and Fourier inversion formula. Since mono-components are crucial to understand the concept instantaneous frequency we will give an account of mono-component classes. Adaptive decompositions of signals into mono-components are called adaptive Fourier decomposition (AFD). Some scopes of the study can be extended to higher dimensions with vector or matrix valued functions as well.

Convolution operators in discrete Cesàro spaces

Werner Ricker

Katholische Universität Eichstätt-Ingolstadt, Germany,
werner.ricker@ku.de

The discrete Cesàro spaces $\text{ces}(p), 1 < p < \infty$, arise from the classical sequence spaces $\ell^p, 1 < p < \infty$, via the process of averaging. A sequence $b \in \mathbb{C}^{\mathbb{N}}$ is called a p-multiplier for $\text{ces}(p)$ if the convolution $a * b \in \text{ces}(p)$ for every $a \in \text{ces}(p)$. This talk will discuss various properties of such convolution operators acting in $\text{ces}(p)$. The situation is *very different* than for convolution operators in the spaces ℓ^p.

Universal Limitations of Generalized Spherical Designs

Stefan Steinerberger

Yale University, USA,
stefan.steinerberger@yale.edu

How many points does one have to place on a sphere so that the average of every polynomial up to degree k on the sphere coincides with the average on these points? These spherical designs have been introduced in the 70s and studied intensively ever since - a recent paper of Bondarenko, Radchenko & Viazovska (Annals 2013) concludes the theory. We give a vast generalization to general compact manifolds and weighted averages. The techniques are completely new and based on partial differential equations, the results are new even on S^2. If time allows, I will discuss a generalized Sturm Oscillation theorem that is based on similar ideas.

The Dirichlet-to-Neumann operator

Tom ter Elst

University of Auckland, New Zealand
t.terelst@auckland.ac.nz

We present some recent results on kernel bounds for the semigroup generated by the Dirichlet-to-Neumann operator when the underlying operator has Hölder continuous coefficients and the domain has a $C^{1+\kappa}$-boundary. The proof depends on Gaussian bounds for derivatives of the semigroup kernel of an elliptic operator with Dirichlet boundary conditions. As a consequence the Dirichlet-to-Neumann semigroup is holomorphic on the right half-plane on L_1. This is joint work with El Maati Ouhabaz.

Quasiconformal mappings and BMO on spaces of homogeneous type

Lesley Ward

University of South Australia
Lesley.Ward@unisa.edu.au

Martin Riemann discovered a beautiful connection between quasiconformal mappings and functions of bounded mean oscillation. Namely, the logarithm of the Jacobian determinant of a quasiconformal mapping is a BMO function. In joint work with Trang Nguyen, we have generalised Reimann's theorem from functions defined on Euclidean spaces $\mathbb{R}^n$ to those defined on spaces of homogeneous type (X, d, μ). As the analogue of quasiconformality we have used η-quasisymmetry. In this talk I will describe two different proofs, one modelled on Reimann's approach and one via a suitable Calderón–Zygmund decomposition, dyadic reverse-Hölder inequalities, and the one-third trick. I will also discuss our work in progress on generalisations to (X, d, μ) of Reimann's results about a second relationship: composition with a mapping satisfying some natural preconditions preserves BMO if and only if the mapping is quasiconformal.

Comments by Yves Meyer and Raphy Coifman.

My meeting with Alan changed my mathematical life. It happened in 1980. It was a time when my colleagues in Orsay for some political reasons refused to give graduate courses. They were objecting to a decision by the Minister of Education or something. I hate to follow the crowd, and so I decided to give a graduate course anyway, just for proving that I do not follow the crowd. So I gave the graduate course and there was a student following the course who was completely distinct from the other students and who seemed to be much older. So I spoke to this person. He came from Australia, he was Alan McIntosh. I invited him to have lunch at the end at the course every week. Alan?s personality was quite original and we soon became friends. He was speaking slowly with great care, with a beautiful smile and an unexpected sense of humor or even irony. After three weeks he explained what he was trying to find, his program.

So, his program was exactly what I was trying to do with Raphy Coifman, but Alan was greatly influenced by another mathematician, Tosio Kato. Kato, who is not alive anymore , was working in operator theory but from a very abstract view- point as compared to the real variable methods introduced by Alberto Calderón and Antoni Zygmund. Kato had a general conjecture from which Calderoń's conjecture would follow as a simple corollary. Calderoń did not know Kato's work, and Kato did not know Calderón's work. When they were in the US, Calderoń was at the University of Chicago and Kato was at Berkeley. McIntosh explained that the problem I was trying to solve could be rephrased in the terminology of Kato. McIntosh has proved that the abstract version of Kato's conjecture was wrong. Nevertheless Alan believed that Kato's conjecture on the domain of the square root of an accretive operator remained true in the setting of second order partial differential operators. He thought that some real variable methods (in the sense of Calderón and Zygmund school) were needed to solve Kato's problem in this particular case.

When I got this information I went immediately to Yale to discuss with Coifman about the possibility of solving Calderón's problem through this new formulation. Coifman was excited and wrote a kind of draft version of the solution. Then I returned to France and I managed to find the missing points. This happened in May 1981. So, without my discussion with McIntosh, who knows if the problem would have been solved ? McIntosh knew that the problem had a double meaning, that it could be rephrased inside another completely distinct theory, and with this new perspective on the problem, the problem could be solved.

Yves Meyer.

Continuing Yves words;

I recall Yves, when he arrived at Yale, all excited about this new interpretation of the Cauchy integral problem, as an analytic perturbation of d/dx.

We have been struggling with the multilinear expansion of this operator in terms of high order commutators , we had the basic paraproduct multilinear approach but could not control the convergence . Within days of playing with Alan's idea we got a real variable version of Calderon's theorem showing boundedness for the operator, in the small constant regime , which we presented in the conference honoring Antony Zygmund in march 1981.

It was Yves, who, on his return to France figured out the the result for norm < 1, which again with an idea from Alan for renormalization , gave the full result in L^2. This was an exciting beginning for a long range collaboration over four continents , within a short time Alan visited me at Yale where we pursued his ideas for developing operator functional calculi , extending these methods to high dimensions. The idea was to use Clifford algebras with generalized holomorphy, and generalized resolvents.

Shortly later, the Kato problem for small perturbation constant in higher dimensions was obtained in collaboration with Yves and Dong Gao Deng (who was visiting Yale) initiating many collaborations in China. Later Tao Qian joined Alan, many of Yves students, and mine joined his team in Australia, continuing to develop new powerful analytic tools, finally solving the full Kato problem in 2001. (Pascal Auscher; Steve Hofmann; Michael Lacey; Alan McIntosh; Philippe Tchamitchian, (2002). "The solution of the Kato square root problem for second order elliptic operators on $\mathbb{R}^n$". Annals of Mathematics. 156 (2): 633654. doi:10.2307/3597201. MR 1933726).

This story is remarkable, this is a team of mathematical friends, striving together to pursue Alan's program, working across the world collaboratively on the singular integral and operator theory needed to understand Functional Calculi, and their analysis.

Alan through his friendly disposition, and his deep insights provided both leader- ship and opportunity for this extraordinary productive teamwork.

Raphy Coifman