AMSI-ANU Workshop on microlocal analysis and its applications in spectral theory, dynamical systems, inverse problems, and PDE

Murramarang Resort, Australia, March 18-23.

Dean Baskin
Texas A&M

Title: *Radiation fields on asymptotically Minkowski spacetimes.*

Abstract: Radiation fields are rescaled limits of solutions of wave equations near "null infinity" and capture the radiation pattern seen by a distant observer. They are intimately connected with the Fourier and Radon transforms and with scattering theory. We find an asymptotic expansion for the radiation field on asymptotically Minkowski spacetimes (as well as on product cones) and show that the exponents seen in the expansion are the resonances of the Laplacian on an associated asymptotically hyperbolic manifold. This talk is based on joint work with Andras Vasy, Jared Wunsch and Jeremy Marzuola.

Tanya Christiansen
University of Missouri

Title: *Resonance rigidity for Schrödinger operators in even dimensions.*

Abstract: We discuss questions related to the rigidity of the set of resonances of a Schrödinger operator on $\mathbb{R}^d$ with bounded, compactly supported real-valued potential in the case the dimension is even. For example, we show that any such nontrivial potential must have infinitely many resonances. We also show that the poles of the resolvent for a smooth potential determine the heat coefficients, and that this shows the compactness of certain sets of isoresonant potentials. One step in the proof is a computation of the leading term of the logarithmic derivative of the determinant of the scattering matrix in high energy limit, under only the assumption that the real-valued potential V is bounded with compact support.

Nguyen Viet Dang
Université de Lyon

Title: *Pollicott-Ruelle resonances and the asymptotic spectrum of Witten Laplacians.*

Abstract: This is a report on joint work with G. Rivire. On a compact Riemannian manifold and given a Morse function on it, the Witten Laplacian was introduced (by E. Witten) in the early 80s to give an analytic proof of the Morse inequalities. In the present talk, we will explain how the spectrum of the Witten Laplacian converges at the semiclassical limit to the Pollicott-Ruelle resonance spectrum of the generator of the gradient flow acting on suitable anisotropic Sobolev spaces of currents. This resonance spectrum is related to the long time behaviour of the gradient flow. We will discuss the relation with the Witten complex.

Frédéric Faure
Institut Fourier Université de Grenoble

Title: *Some properties of hyperbolic dynamics from micro-local analysis.*

Abstract: In hyperbolic dynamics (Anosov dynamics) each trajectory is strongly unstable and its behavior is unpredictable. A smooth probability distribution evolves also in a complicated way since it acquires higher and higher oscillations. Nevertheless using micro-local analysis, this evolution is predictable in the sense of distributions. It is similar to a quantum scattering problem in cotangent space as treated by Helffer and Sjöstrand using escape functions in (86'). In this talk we will use wave-packet transform (or FBI transform) and explain how to derive some spectral properties of the dynamics, as the existence of the intrinsic discrete spectrum of Ruelle resonances, a fractal Weyl law, estimates on the wave front set of the resonances, and band structure in the case of geodesic flow. Collaboration with Masato Tsujii.

Clotilde Fermanian
Université Paris Est

Title: *Wigner measures and effective mass theorems.*

Abstract: The dynamics of an electron in a crystal in the presence of impurities is described by a wave function that solves a Schrdinger equation and depends on a small parameter which is the ratio between the mean spacing of the lattice and the characteristic length scale of variation of the external potential. Effective Mass Theory consists in showing that, under suitable assumptions on the initial data, the wave function can be approximated when this parameter vanishes by the solution of a simpler equation, the effective mass equation. In this talk, we shall use Wigner measure theory to derive these effective mass equations for rather general initial data, under generic assumptions. We shall explain how, when one relaxes these conditions, a similar analysis can be performed, leading to operator-valued effective mass equations of Heisenberg type.

Jeffrey Galkowski
Stanford U

Title: *Concentration of Eigenfunctions: Sup-norms and Averages*

Abstract: In this talk we relate concentration of Laplace eigenfunctions in position and momentum to sup-norms and submanifold averages. In particular, we present a unified picture for sup-norms and submanifold averages which characterizes the concentration of those eigenfunctions with maximal growth. We then exploit this characterization to derive geometric conditions under which such growth cannot occur.

Colin Guillarmou
Laboratoire de Mathématiques d'Orsay,

Title: *Horocyclic invariance of Ruelle resonant states in dimension 3.*

Abstract: (joint with Faure) We show that for smooth contact Anosov flows the Ruelle resonances in the "first band" are invariant by the unstable horocyclic flow. We will discuss also possible extensions and related questions.

Lysianne Hari
Laboratoire de Mathématiques de Besançon

Title: *Propagation of coherent states.*

Abstract: We study the propagation of a coherent state for systems of coupled nonlinear Schrdinger equations in the semi-classical limit. We will consider different cases, leading to different physical phenomena. First, couplings will be induced by a cubic nonlinearity and some stability of the solution will be studied: an initial coherent state polarized along an eigenvector of the potential remains - at leading order - in the same eigenspace ("adiabaticity"). Then we will add a linear coupling, thanks to a matrix-valued potential presenting an "avoided crossing" at one given point: the gap between its eigenvalues reduces as the semi-classical parameter becomes smaller. We will show that when an initial coherent state polarized along an eigenvector of the potential propagates through the avoided crossing point, there are transitions between the modes at leading order, whose probabilities are given by the linear Landau-Zener formula. We will finally handle a "smooth" exact crossing, and see that one can proceed as in the linear case to study the propagation in our setting. A comparison with linear results will be made in each case.

Peter Hintz
UC Berkeley

Title: *Global stability problems in General Relativity.*

Abstract: I will discuss the problem of proving the stability of (families of) exact spacetimes (M, g) such as Minkowski space or the family of Kerr-de Sitter (KdS) black holes as solutions of Einstein's vacuum equation: spacetimes evolving from initial data close to those of (M, g) stay globally close to (M, g), and are indeed asymptotic to (M, g) or another nearby member of the KdS family. I will focus on geometric aspects of this problem: how to compactify M for the purpose of analyzing the underlying nonlinear wave equation; how to choose a gauge to break the diffeomorphism invariance of Einstein's equation; and the role of constraint damping. (Joint with Andras Vasy)

Katya Krupchyk
UC Irvine

Title: *Inverse boundary problems for elliptic PDE in low regularity setting*

Abstract: In this talk, we shall discuss recent progress in the global uniqueness issues for inverse boundary problems for second order elliptic equations, such as the conductivity, magnetic Schrödinger, and advection equations, with low regularity coefficients. Generally speaking, in an inverse boundary problem, one wishes to determine the coefficients of a PDE inside a domain from the knowledge of its solutions along the boundary of the domain. While ubiquitous in practice, the mathematical analysis of such problems is quite challenging, and the consideration of

the low regularity setting, motivated by applications, brings additional substantial difficulties. In this talk, we shall discuss the case of full, as well as partial, measurements, both for domains in the Euclidean space, as well as in the more general setting of transversally anisotropic compact Riemannian manifolds with boundary. Some of the important ingredients in our approach are semiclassical Carleman estimates with limiting Carleman weights with an optimal gain of derivatives, precise smoothing estimates, as well as a construction of Gaussian beam quasimodes in a low regularity setting. This is joint work with Gunther Uhlmann.

Matthieu Léautaud
Ecole Polytechnique

Title: *Control from an internal hypersurface.*

Abstract: In this talk, we consider the observability and controllablity problems for the wave and heat equations. As opposed to classical results, the region where observation takes place (resp. where a control acts on the equation) is an internal hypersurface. We prove sufficient geometric conditions on the hypersurface and the geodesic flow for observability estimates to hold. In turn, these results also provide lower bounds for the Cauchy data of Laplace eigenfunctions on hypersurfaces. This is joint work with Jeffrey Galkowski.

Melissa Tacy
University of Otago

Title: *L^p estimates for joint eigenfunctions.*

Abstract: Consider the problem of the L^p growth of joint eigenfunctions of a set of pseudodifferential operators $\Delta = P_1, \dots, P_r$ that satisfy a suitable non-degeneracy assumption. In the special case of symmetric spaces of rank r Marshall obtained L^p estimates that indicated that the L^p growth of u behaves like that of products $u(x) = u_1(x_1) \dots u_r(x_r)$ where each $u_i(x_i)$ is a Laplace eigenfunction in n/r variables. In this talk I will discuss a more general case where we only assume that the normals to the characteristic sets of each P_i are linearly independent. In this case we are able to obtain L^p results which are as good as the symmetric space results for some p and examples to show that in this general setting we cannot improve the results.

Andras Vasy
Stanford

Title: *Global analysis for linear and nonlinear waves.*

Abstract: I will describe a framework for the Fredholm analysis of non-elliptic problems, with a view towards wave propagation on Minkowski and Kerr-de Sitter spacetimes. The main ingredients are, first, the microlocal control of the regularity of waves by means of elliptic, real principal type, and radial point estimates on a suitable compactification of the spacetime; and second, the asymptotic analysis in which model operators and resonance expansions play a key role. (Joint with Peter Hintz)

Jared Wunsch
Northwestern

Title: *Diffraction of semiclassical singularities by conormal potentials*

Abstract: I will report on work in progress, joint with Oran Gannot, in which we analyze the propagation of semiclassical singularities in the presence of a potential with a conormal singularity at a hypersurface. These potentials cause splitting of rays, with reflected rays generally carrying weaker singularities than those directly propagated; the gain in regularity depends on the regularity of the potential. This extends closely related work of De Hoop-Uhlmann-Vasy in the case of wave equations with conormal metrics. I will also report on recent work of Kleinhenz concerning analogous effects for the damped wave equation on tori, in situations where there is a non-smooth damping term that does not exert geometric control.

10 MINUTE TALKS

Yaiza Canzani
UNC

Title: *Recurrent conormal directions to a submanifold for Anosov flows.*

Xi Chen
Fudan U

Title: *TBA.*

Hans Christianson
UNC

Title: *Equidistribution of Neumann data mass on triangles and applications.*

Kiril Datchev
Purdue

Title: *Exterior resolvent estimates and wave decay.*

Suresh Eswarathasan
U Sydney

Title: *Tangent points to nodal sets of random eigenfunctions.*

Jesse Gell-Redman
U Melbourne

Title: *Dirac operators and the index formula on iterated wedge spaces.*

Zihua Guo
U Cardiff

Title: *Generalised Strichartz estimates for Schrodinger equation.*

Xiaolong Han
California State University, Northridge

Title: *Nodal curves of eigenfunctions and geodesics on Riemannian surfaces.*

Andrew Hassell
Australian National University

Title: *Quantum ergodicity at small scales.*

Michael Hitrik
UCLA

Title: *Toeplitz operators and positive canonical transformations.*

Maxime Ingremeau
Strasbourg

Title: *a lower bound on the Bogomolny-Schmit constant.*

Etienne Le Masson
University of Bristol

Title: *L^p norms of joint eigenfunctions on the sphere.*

Katrina Morgan
UNC

Title: *Local Energy, Resolvents, and Wave Decay.*

Gabriel Riviere
Lille

Title: *Equidistribution of toral eigenfunctions along hypersurfaces.*

Julie Rowlett
Chalmers University

Title: *The Poisson relation and billiards in polygons.*

Jacob Shapiro
Purdue

Title: *Semiclassical resolvent estimates in low regularity.*

Adam Sikora
Macquarie

Title: *Meromorphic extension of a resolvent in one dimension.*

Leo Tzou
U Sydney

Title: *Tomography and rigidity in non-compact spaces*

Mathai Varghese
U Adelaide

Title: *Spectral Gap-labelling conjecture with nonzero magnetic field.*

Fang Wang
Shanghai Jiao Tong University

Title: *Some positivity results for fractional GJMS operators or order between 2 and 4.*