HAGIS Code

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… on behalf of Simon Pinches
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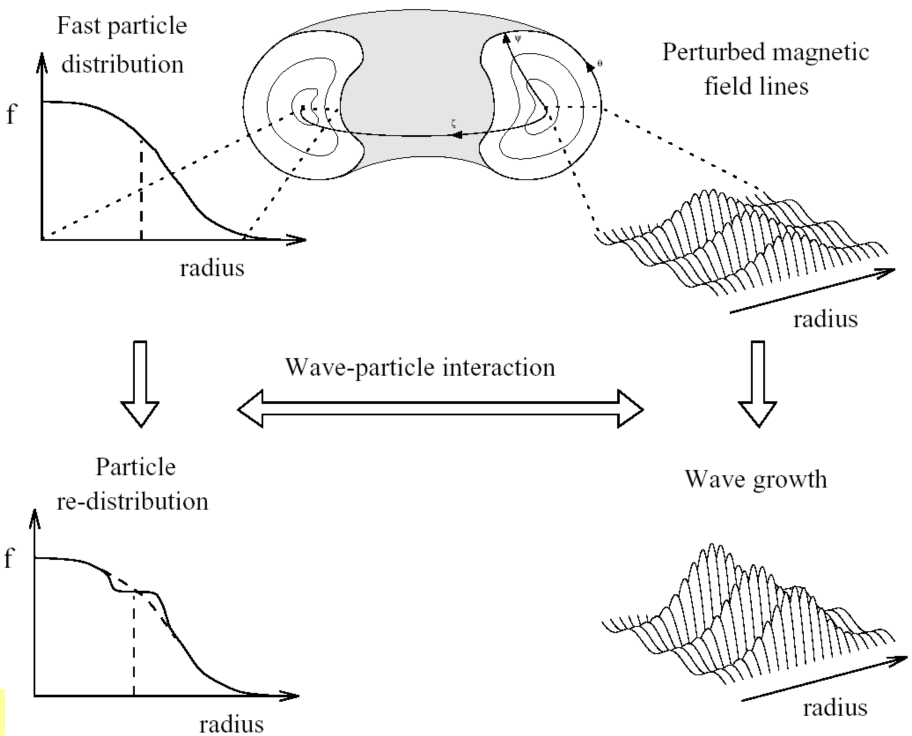
HAGIS Description[1]

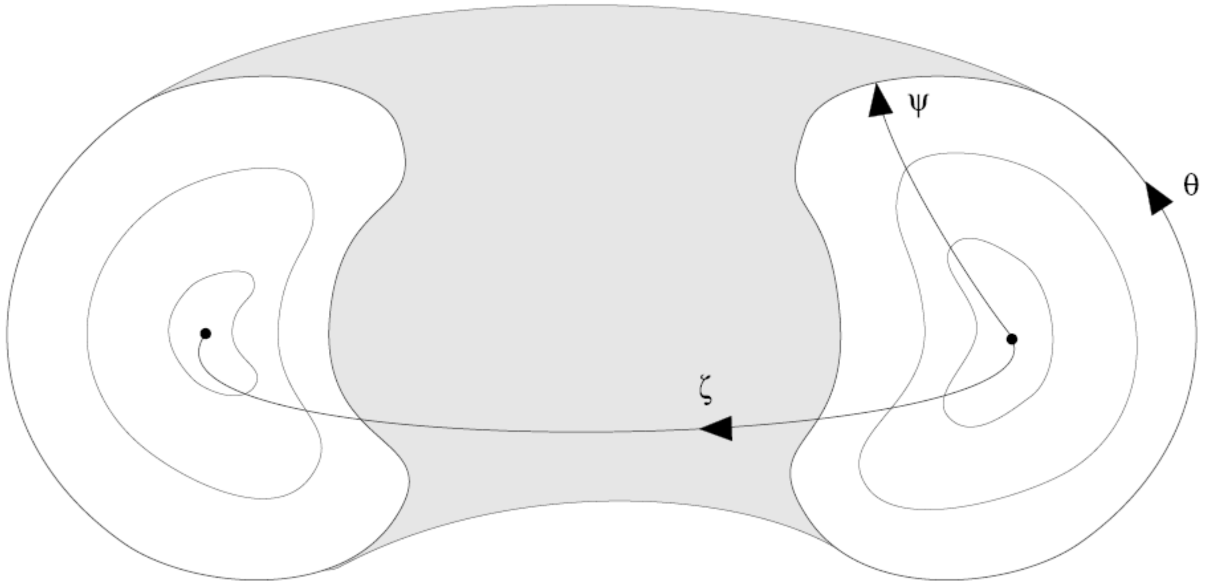
- Self-consistently models the nonlinear interaction between spectrum of linear eigenfunctions and fast particle distribution function
  - Weakly damped global Alfvén Eigenmodes (AE) could be driven unstable by fusion-born α-particles

- Straight field-line equilibrium
  - Boozer coordinates
- Hamiltonian description of particle motion
- Fast ion distribution function
  - δf method
- Evolution of waves
  - Wave eigenfunctions computed by CASTOR/MISHKA/CAS-3D and held invariant throughout system evolution



[1]SD Pinches, LC Appel et al., Comput. Phys. Commun. 111 131 (1998)

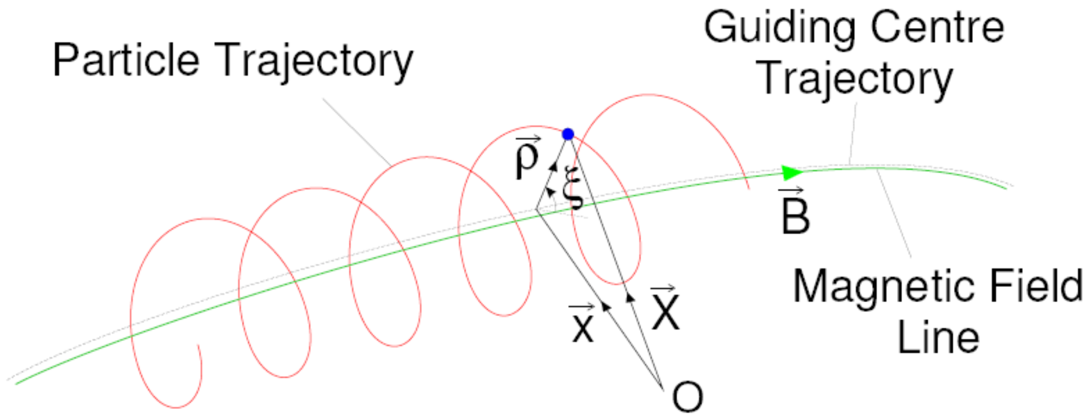
Boozer Coordinates



The coordinates, ψ_p, θ, ζ are chosen to produce straight field lines in the $\theta - \zeta$ plane and lead to a field representation of the form:

$$
\begin{aligned}
\mathbf{B} &= \delta(\psi_p,\theta)\boldsymbol{\nabla}\psi_p + I(\psi_p)\boldsymbol{\nabla}\theta + g(\psi_p)\boldsymbol{\nabla}\zeta, \\
\mathbf{B} &= \boldsymbol{\nabla}\psi \wedge \boldsymbol{\nabla}\theta - \boldsymbol{\nabla}\psi_p \wedge \boldsymbol{\nabla}\zeta, \\
\Rightarrow \quad \mathbf{A} &= \psi\boldsymbol{\nabla}\theta - \psi_p\boldsymbol{\nabla}\zeta.
\end{aligned}
$$

Evolution of Energetic Particles



The exact particle Lagrangian,

$$\mathcal{L}_{exact} = \sum_{ep} \frac{1}{2} m V^2 + e\mathbf{V} \cdot \mathbf{A} - e\phi,$$

is gyro-averaged and written in the form,

$$\mathcal{L}_{ep} = \sum_{j=1}^{n_p} P_{\theta j} \dot{\theta}_j + P_{\zeta j} \dot{\zeta}_j - \mathcal{H}_j,$$

with

$$\mathcal{H}_j = \frac{1}{2} m_j v_{\parallel j}^2 + \mu_j B_j + e_j \phi_j,$$

leading to $4 \times n_p$ first order differential equations.

Equations of Motion

$$\dot{\theta} = \frac{1}{D}\left[\rho_{\|}B^2(1-\rho_c g' - g\tilde{\alpha}') + g\left\{(\rho_{\|}^2 B + \mu)B' + \tilde{\Phi}'\right\}\right],$$

$$\dot{\zeta} = \frac{1}{D}\left[\rho_{\|}B^2(\rho_c I' + q + I\tilde{\alpha}') - I\left\{(\rho_{\|}^2 B + \mu)B' + \tilde{\Phi}'\right\}\right],$$

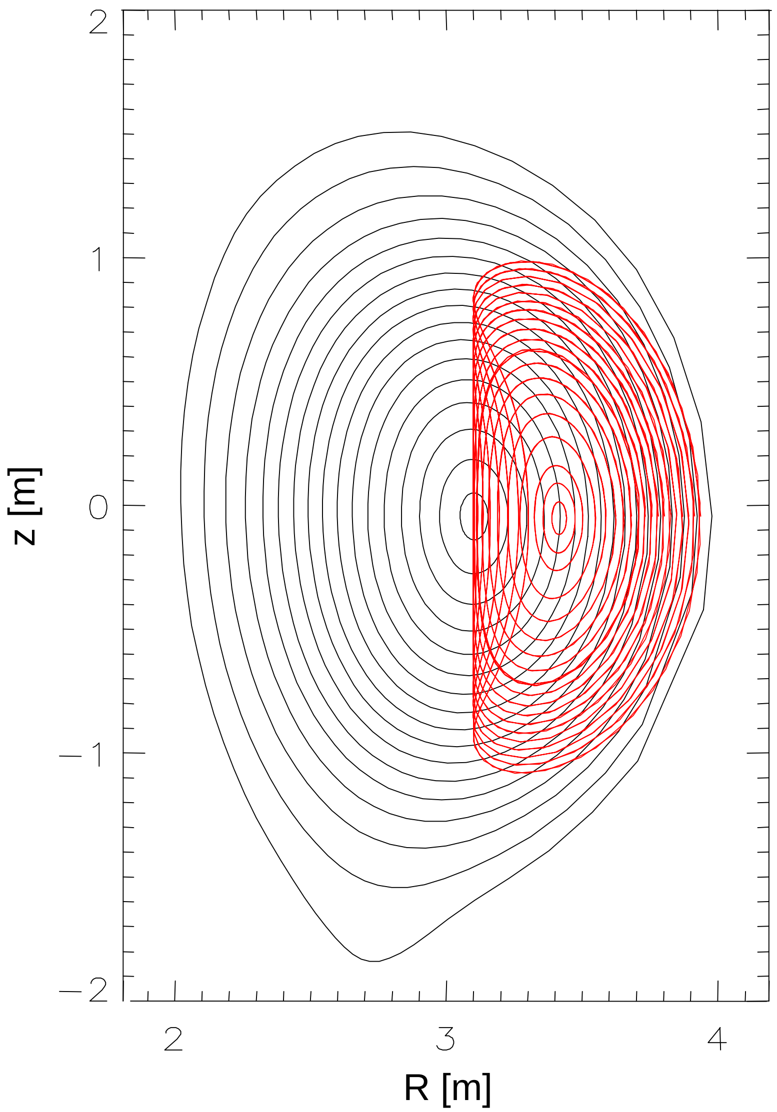
$$\dot{\psi}_p = \frac{1}{D}\left[\rho_{\|}B^2\left(g\frac{\partial\tilde{\alpha}}{\partial\theta} - I\frac{\partial\tilde{\alpha}}{\partial\zeta}\right) - \left(g\frac{\partial\tilde{\Phi}}{\partial\theta} - I\frac{\partial\tilde{\Phi}}{\partial\zeta}\right) - g(\rho_{\|}^2 B + \mu)\frac{\partial B}{\partial\theta}\right],$$

$$\dot{\rho}_{\|} = \frac{1}{D}\left[\left(I\frac{\partial\tilde{\alpha}}{\partial\zeta} - g\frac{\partial\tilde{\alpha}}{\partial\theta}\right)\left\{(\rho_{\|}^2 B + \mu)B' + \tilde{\Phi}'\right\} - (q + \rho_c I' + I\tilde{\alpha}')\frac{\partial\tilde{\Phi}}{\partial\zeta}\right.$$
$$\left. + (\rho_c g' - 1 + g\tilde{\alpha}')\left\{(\rho_{\|}^2 B + \mu)\frac{\partial B}{\partial\theta} + \frac{\partial\tilde{\Phi}}{\partial\theta}\right\}\right] - \frac{\partial\tilde{\alpha}}{\partial t},$$

- Derived from total system Hamiltonian

Fast Particle Orbits

- ICRH ions in JET deep shear reversal
  - On axis heating: $\Lambda = \mu B_0/E = 1$
  - E = 500 keV



Calculation of AE Eigenfunctions

Wave Lagrangian:

$$\mathcal{L}_w = \sum \left[\frac{1}{2}mv^2 + e\left(\mathbf{A}\cdot\mathbf{v} - \phi\right)\right] + \frac{1}{2\mu_0}\int_V \left(\frac{1}{c^2}E^2 - B^2\right) dx^3$$

Expanding in perturbed field powers:

- $\mathcal{L}^{(0)}$ describes the equilibria and is solved by the HELENA code.
- $\mathcal{L}^{(1)}$ describes first order force balance.
- $\mathcal{L}^{(2)}$ describes fixed amplitude AE and is solved by the CASTOR code.

Wave Evolution

Linear eigenmode structure is assumed to remain fixed throughout the simulation.

Each wave is allowed two degrees of freedom, amplitude and phase-shift; A_k and α_k.

$$\tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi) e^{i(n_k\zeta - m\theta - \omega_k t - \alpha_k(t))}$$

The wave Lagrangian can then be written as

$$L_w = \sum_{k=1}^{n_w} \frac{E_k}{\omega_k} A_k^2 \dot{\alpha}_k,$$

where

$$E_k = \frac{1}{2\mu_0} \int_V \frac{\left| \nabla_\perp \tilde{\Phi}_k \right|^2}{v_A^2} d^3x,$$

and n_w is the number of eigenmodes in the system.

Wave Equations

- Linear eigenstructure assumed invariant
- Introduce slowly varying amplitude and phase:

$$\tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi) e^{i(n_k\zeta - m\theta - \omega_k t - \alpha_k(t))}$$

- Gives wave equations as:

$$\dot{\mathcal{X}}_k = \frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k_{\|m} v_{\|j} - \omega_k) S_{jkm} + \mathcal{X}_k \gamma_d,$$

$$\dot{\mathcal{Y}}_k = -\frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k_{\|m} v_{\|j} - \omega_k) C_{jkm} + \mathcal{Y}_k \gamma_d,$$

Additional mode damping rate, γ_d

- where

$$\mathcal{X}_k \equiv A_k \cos(\alpha_k),$$
$$\mathcal{Y}_k \equiv A_k \sin(\alpha_k),$$

$$C_{jkm} \equiv \Re e[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_{jkm}}]$$
$$S_{jkm} \equiv \Im m[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_{jkm}}]$$
$$\Theta_{jkm} \equiv n_k \zeta_j - m\theta_j - \omega_k t$$

Delta f Method

Numerical noise can be dramatically reduced by making use of a powerful technique known as the δf method.

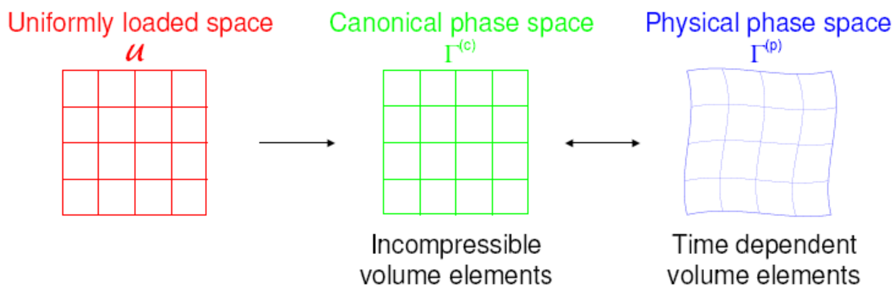
$$f = \underbrace{f_0(\mathcal{E}, P_\zeta; \mu)}_{\text{analytic}} + \underbrace{\delta f(\Gamma^{(p)}, t)}_{\text{markers}}$$

$$\frac{df}{dt} = 0 \Rightarrow \dot{\delta f} = -\dot{P}_\zeta \frac{\partial f_0}{\partial P_\zeta} - \dot{\mathcal{E}} \frac{\partial f_0}{\partial \mathcal{E}} - \nu_{\mathrm{eff}}\, \delta f$$

$$\int f\, g\, d\Gamma^{(p)} \longleftrightarrow \int f_0\, g\, d\Gamma^{(p)} + \sum_{j=1}^{n_p} \delta n_j\, g_j,$$

where

$$\delta n_j(t) \equiv \delta f_j(t)\, \Delta\Gamma_j^{(p)}(t)$$



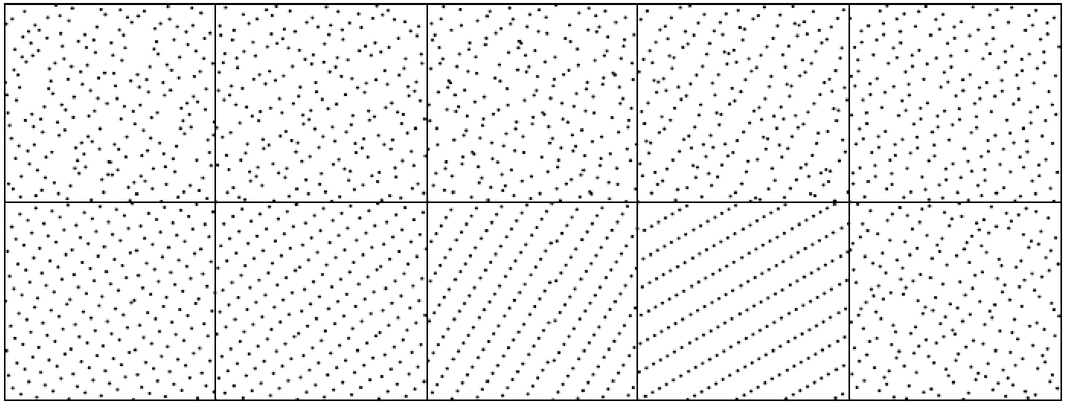
Quiet Start Method

Markers are uniformly loaded using Hammersley's sequence⋆,

$$x_i = \{i/N, \phi_2(i), \phi_3(i), \phi_5(i), \phi_7(i)\}.$$

If the integer i is written in base r,

$$\begin{aligned} i &= a_0 + a_1 r + a_2 r^2 + \cdots \\ \text{then} \quad \phi_r(i) &= a_0 r^{-1} + a_1 r^{-2} + a_2 r^{-3} + \cdots \end{aligned}$$



Projections of uniformly loaded 5-D hypercube

This achieves a discrepancy $\propto 1/N$, where a random distribution has a discrepancy $\propto 1/\sqrt{N}$.

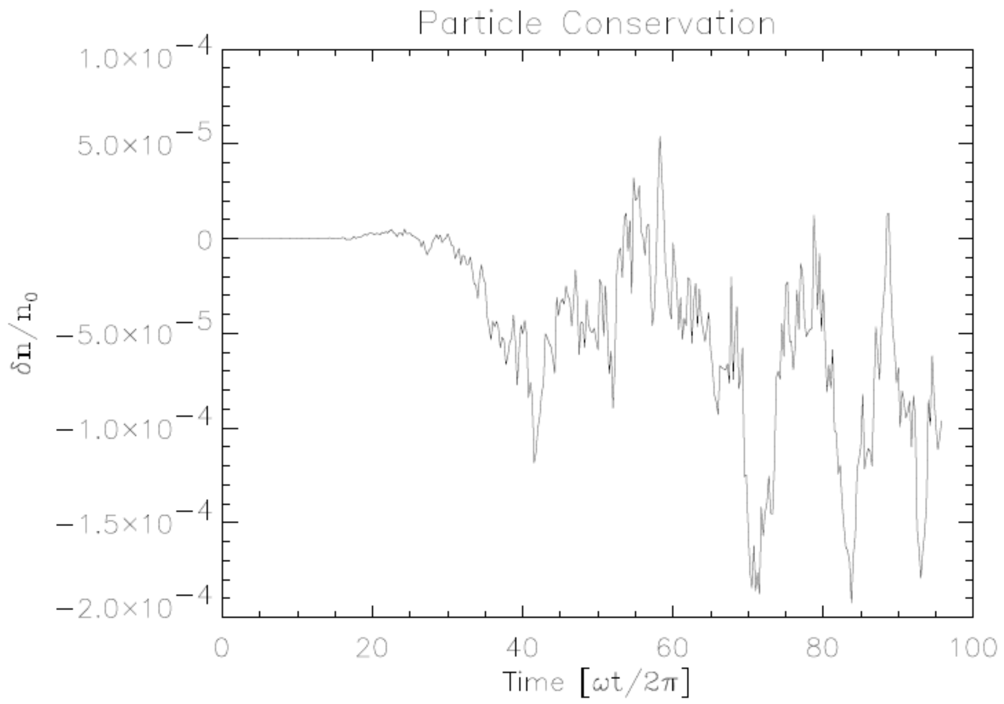
Conservation of Particles

The total number of particles in the system is very well conserved over long time periods.

Particle fluctuation,

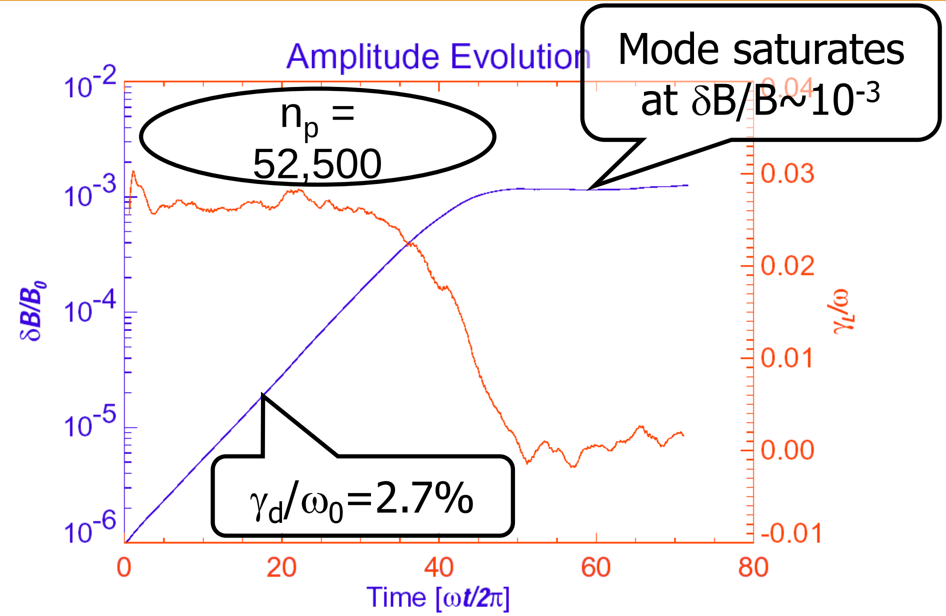
$$\delta n = \sum_j \delta f_j(t)\, \Delta\Gamma_j^{(p)}(t),$$

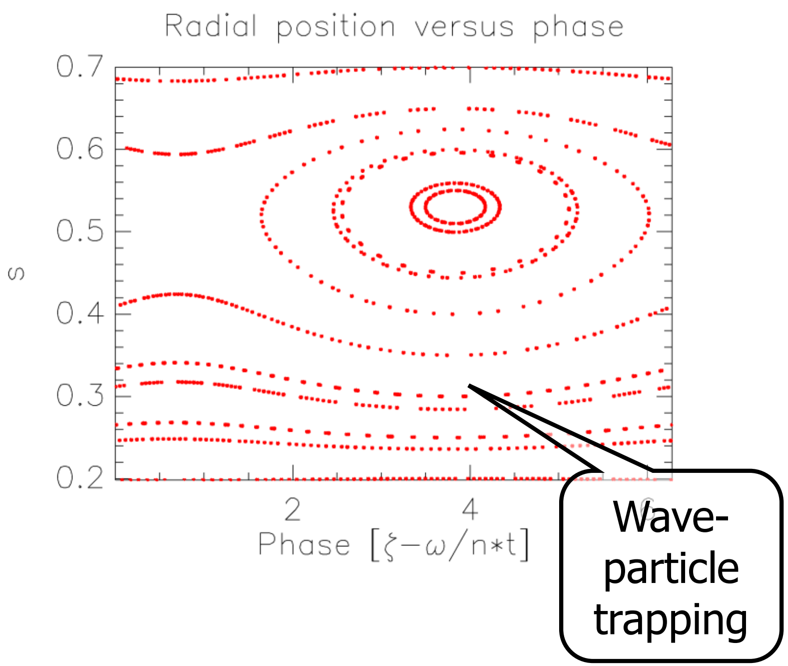
for $n = 5$ KTAE in JET:

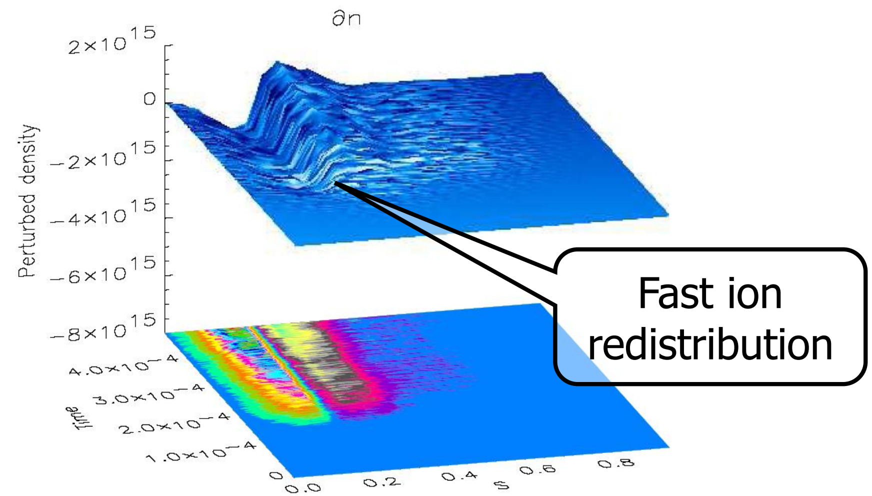


Applications

- AE linear growthrates and nonlinear saturation amplitudes
- Determination of
  - fast particle re-distribution
  - Fast particle losses



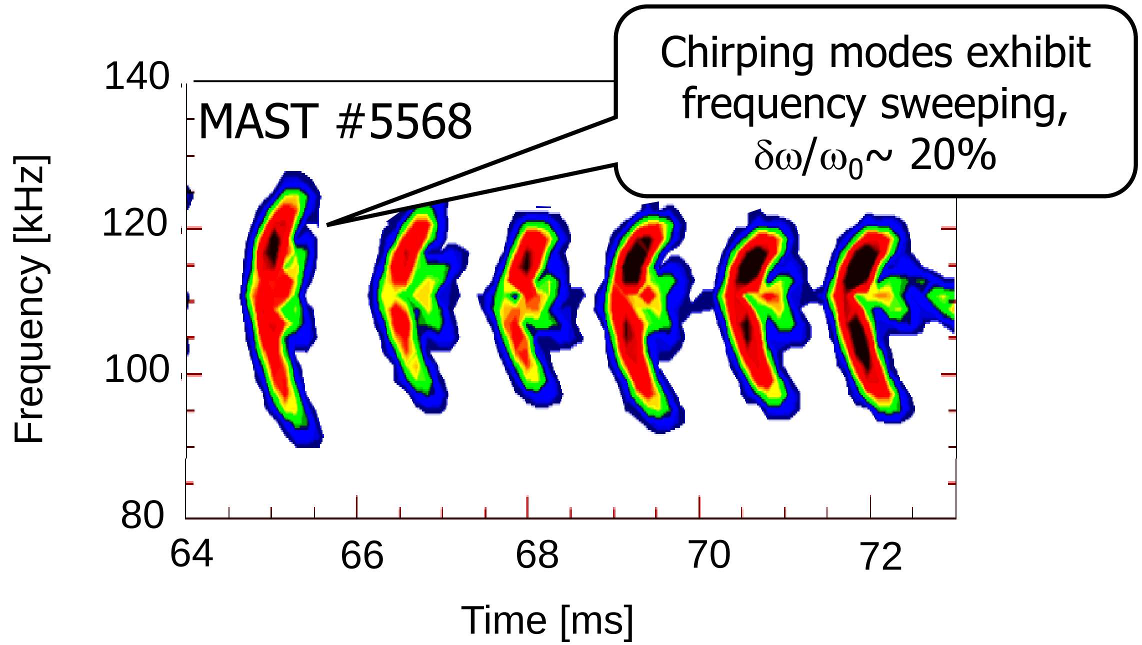


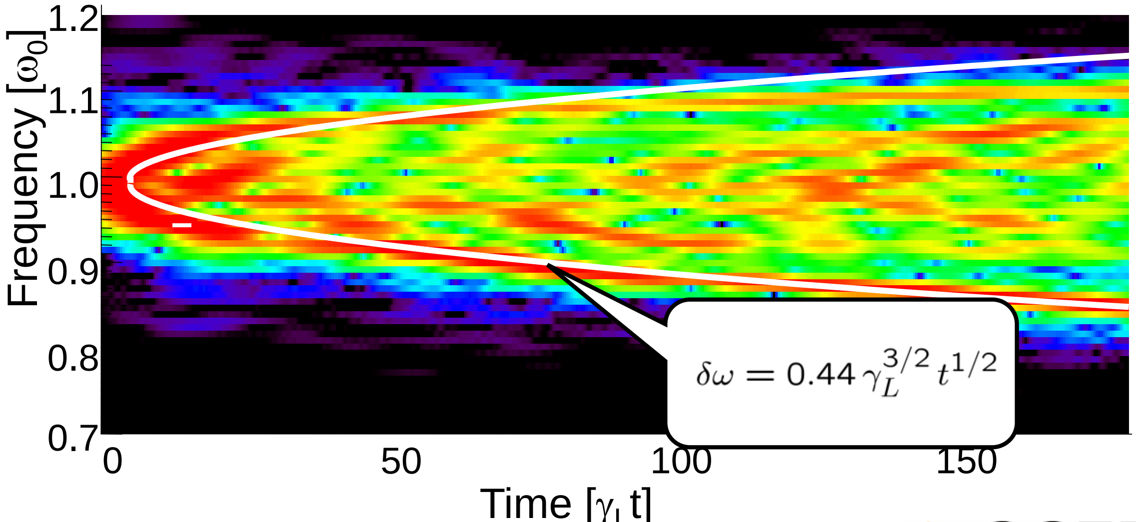


TAE Amplitude Determination

- Frequency sweeping observed when damping rate ~ linear growth rate and holes/clumps form in particle phase space
- Modelling with HAGIS allows determination of experimental internal mode amplitudes from observed frequency sweeping
- Particle trapping frequency determined using HAGIS

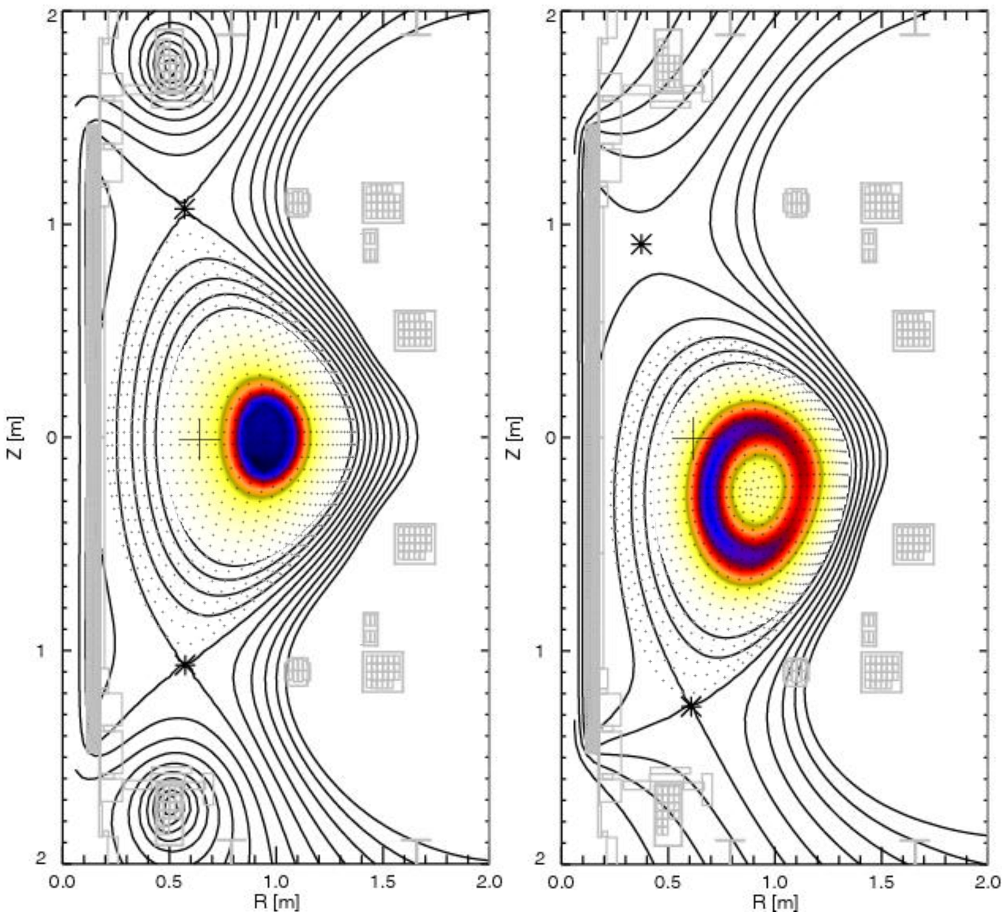
$$\frac{\delta B}{B} = \frac{1}{(1.156\times 10^6)^2}\left(\frac{32\,\delta f^2}{\delta t}\right)^{2/3} = 4\times 10^{-4}$$





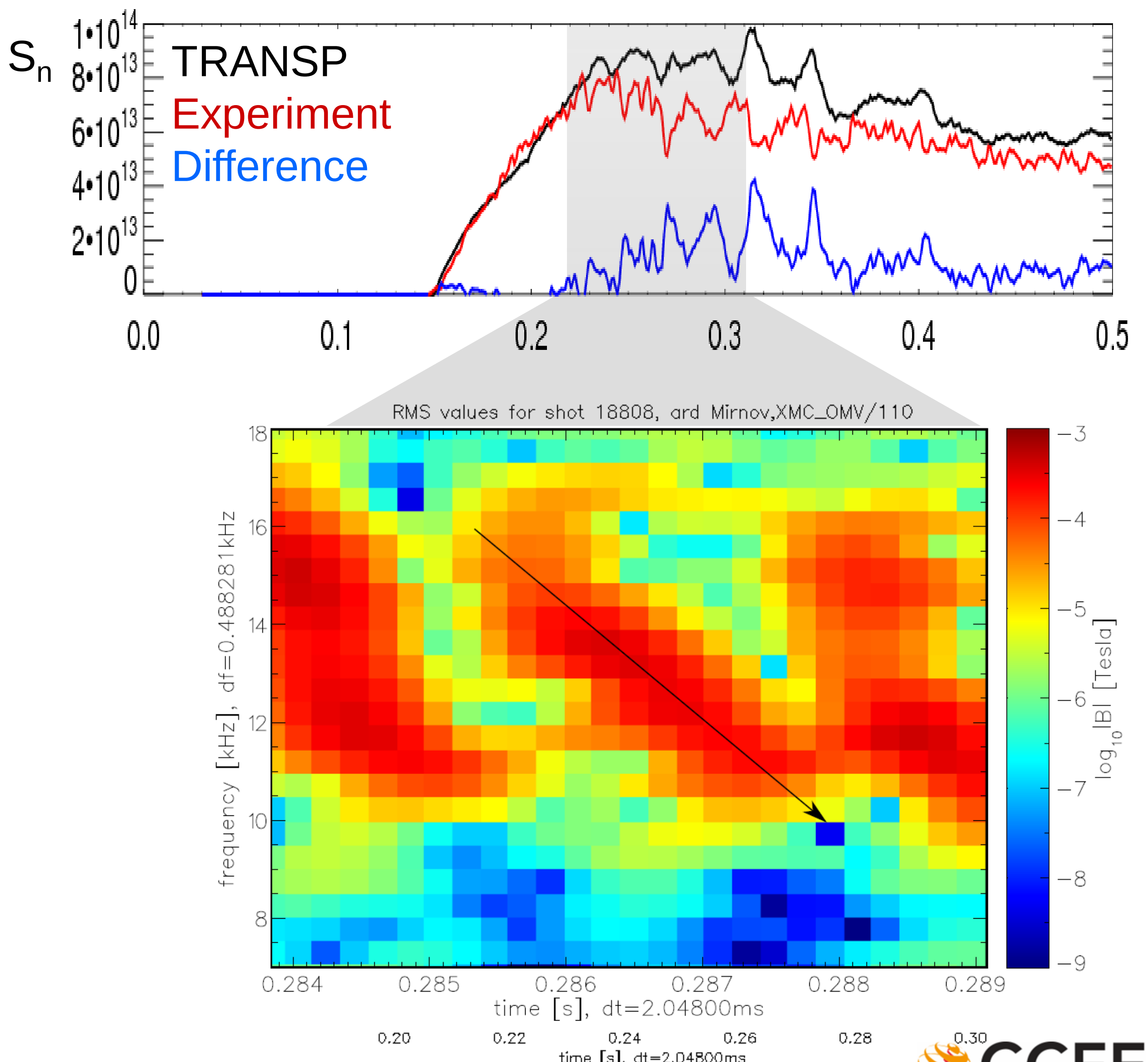
Fast Ions in MAST

- High performance MAST plasmas achieved with off-axis NBI
  - Off-axis NBI current drive used to tailor current profile
- Fast ion distribution very different with on and off axis injection
  - Changes to current profile with off-axis beams studied in MAST



Transport Code Modelling

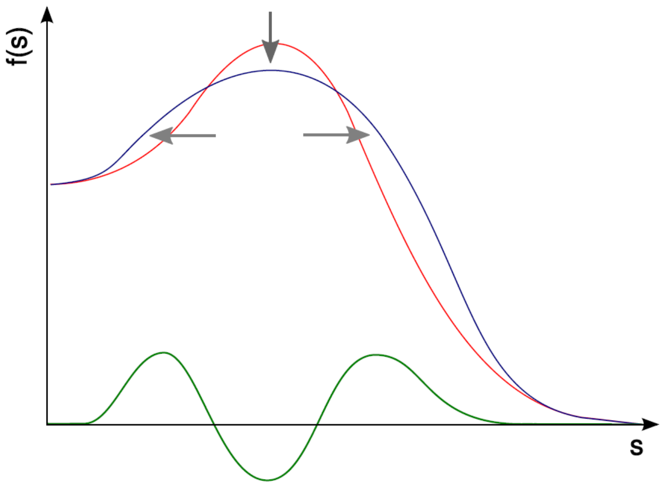
- Discrepancy in neutron rate coincident with observation of fast particle driven modes
  - Fishbones
- Can be explained by introducing an anomalous fast ion diffusion, D~0.5 m²/s

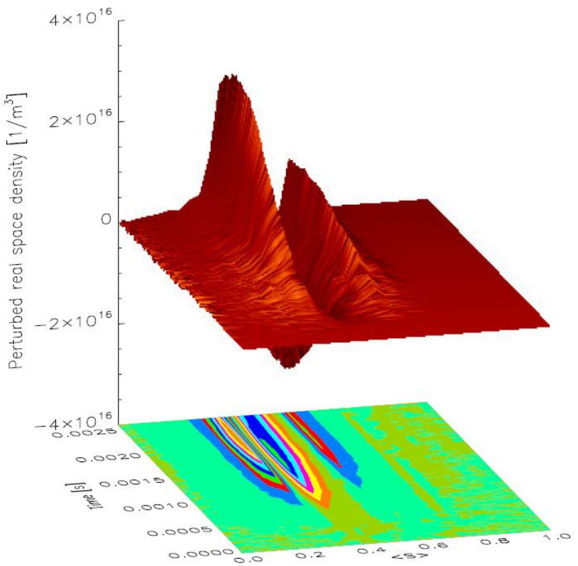


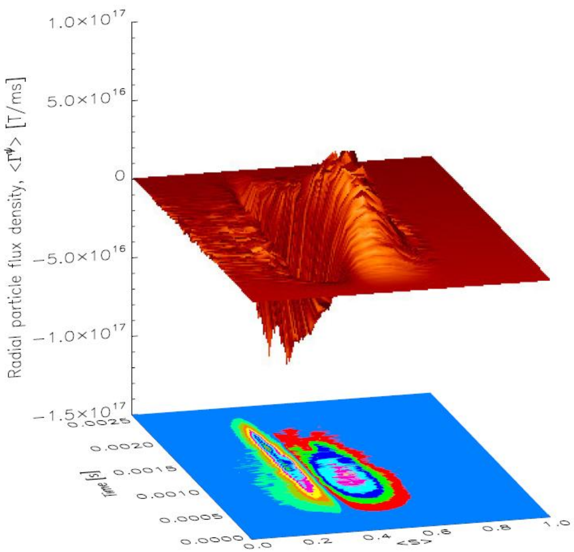
Fast Particle Simulations

- Fishbones simulated in HAGIS to study effect on fast ion population and calculate an effective fast ion diffusion

$$\Gamma_{\psi_p}(\psi_p, t) = -D_r \frac{\partial n(\psi_p, t)}{\partial \psi_p}$$

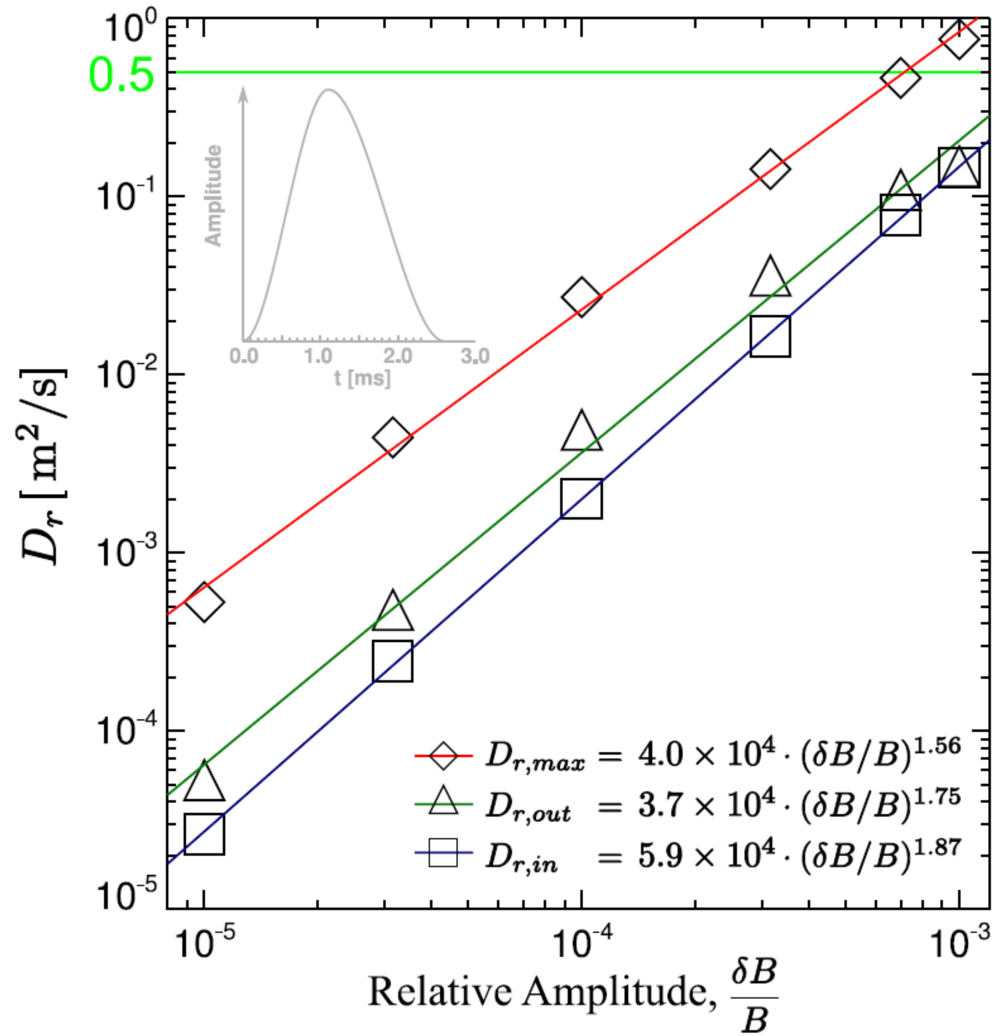




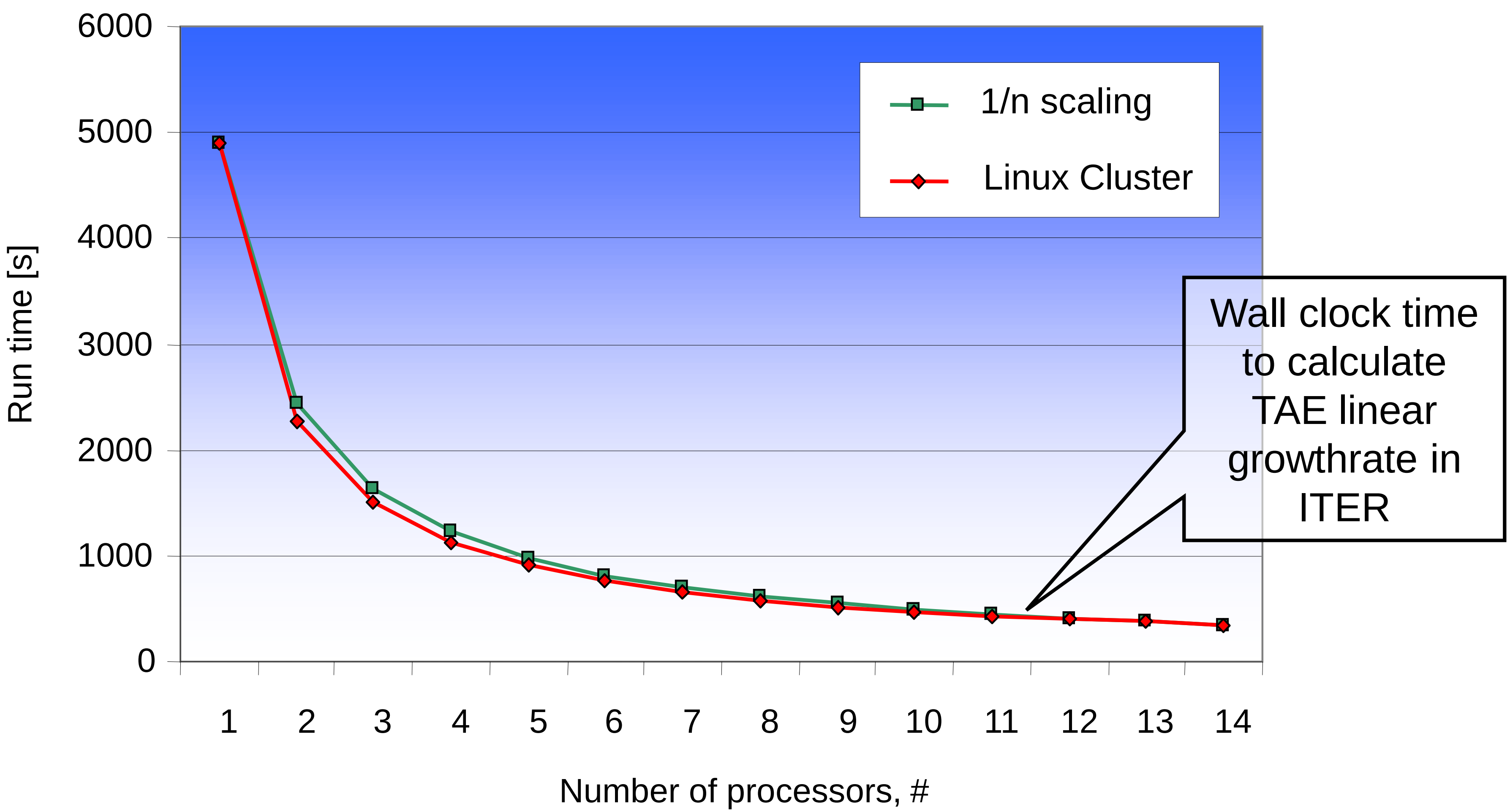


Comparison with Experiment

- Levels of anomalous fast ion diffusion found in simulations consistent with those required in transport codes to explain observations:
  - Neutron rate
  - Stored energy
  - Current profile



HAGIS code performance



- HAGIS code parallelises very well
  - relatively low level of inter-processor communication traffic