HAGIS Code

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HAGIS Description

- Self-consistently models the nonlinear interaction between spectrum of linear eigenfunctions and fast particle distribution function
  - Weakly damped global Alfvén Eigenmodes (AE) could be driven unstable by fusion-born $\alpha$-particles

- Straight field-line equilibrium
  - Boozer coordinates

- Hamiltonian description of particle motion

- Fast ion distribution function
  - $\delta f$ method

- Evolution of waves
  - Wave eigenfunctions computed by CASTOR/MISHKA/CAS-3D and held invariant throughout system evolution

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Boozer Coordinates

The coordinates, \( \psi_p, \theta, \zeta \) are chosen to produce straight field lines in the \( \theta - \zeta \) plane and lead to a field representation of the form:

\[
\mathbf{B} = \delta(\psi_p, \theta) \nabla \psi_p + I(\psi_p) \nabla \theta + g(\psi_p) \nabla \zeta,
\]

\[
\mathbf{B} = \nabla \psi \wedge \nabla \theta - \nabla \psi_p \wedge \nabla \zeta,
\]

\[
\Rightarrow \mathbf{A} = \psi \nabla \theta - \psi_p \nabla \zeta.
\]
Evolution of Energetic Particles

The exact particle Lagrangian,

\[ \mathcal{L}_{\text{exact}} = \sum_{ep} \frac{1}{2} m V^2 + e V \cdot A - e \phi, \]

is gyro-averaged and written in the form,

\[ \mathcal{L}_{ep} = \sum_{j=1}^{n_p} P_{\theta j} \dot{\theta}_j + P_{\zeta j} \dot{\zeta}_j - \mathcal{H}_j, \]

with

\[ \mathcal{H}_j = \frac{1}{2} m_j v_j^2 + \mu_j B_j + e_j \phi_j, \]

leading to \(4 \times n_p\) first order differential equations.
Equations of Motion

\[ \dot{\theta} = \frac{1}{D} \left[ \rho_{\parallel} B^2 (1 - \rho c g' - g\tilde{\alpha}') + g \left\{ (\rho_{\parallel}^2 B + \mu) B' + \tilde{\Phi}' \right\} \right], \]

\[ \dot{\zeta} = \frac{1}{D} \left[ \rho_{\parallel} B^2 (\rho c I' + q + I\tilde{\alpha}') - I \left\{ (\rho_{\parallel}^2 B + \mu) B' + \tilde{\Phi}' \right\} \right], \]

\[ \dot{\psi}_p = \frac{1}{D} \left[ \rho_{\parallel} B^2 \left( g \frac{\partial \tilde{\alpha}}{\partial \theta} - I \frac{\partial \tilde{\alpha}}{\partial \zeta} \right) - \left( g \frac{\partial \tilde{\Phi}}{\partial \theta} - I \frac{\partial \tilde{\Phi}}{\partial \zeta} \right) - g(\rho_{\parallel}^2 B + \mu) \frac{\partial B}{\partial \theta} \right], \]

\[ \dot{\rho}_{\parallel} = \frac{1}{D} \left[ \left( I \frac{\partial \tilde{\alpha}}{\partial \zeta} - g \frac{\partial \tilde{\alpha}}{\partial \theta} \right) \left\{ (\rho_{\parallel}^2 B + \mu) B' + \tilde{\Phi}' \right\} - (q + \rho c I' + I\tilde{\alpha}') \frac{\partial \tilde{\Phi}}{\partial \zeta} \right. \]

\[ + (\rho c g' - 1 + g\tilde{\alpha}') \left\{ (\rho_{\parallel}^2 B + \mu) \frac{\partial B}{\partial \theta} + \frac{\partial \tilde{\Phi}}{\partial \theta} \right\} \left. - \frac{\partial \tilde{\alpha}}{\partial t} \right], \]

- Derived from total system Hamiltonian
Fast Particle Orbits

- ICRH ions in JET deep shear reversal
  - On axis heating:
    \[ \Lambda = \frac{\mu B_0}{E} = 1 \]
  - \( E = 500 \text{ keV} \)
Calculation of AE Eigenfunctions

Wave Lagrangian:

$$\mathcal{L}_w = \sum \left[ \frac{1}{2} m v^2 + e (A \cdot \mathbf{v} - \phi) \right]$$

$$+ \frac{1}{2 \mu_0} \int_V \left( \frac{1}{c^2} E^2 - B^2 \right) dx^3$$

Expanding in perturbed field powers:

- $\mathcal{L}^{(0)}$ describes the equilibria and is solved by the HELENA code.

- $\mathcal{L}^{(1)}$ describes first order force balance.

- $\mathcal{L}^{(2)}$ describes fixed amplitude AE and is solved by the CASTOR code.
Linear eigenmode structure is assumed to remain fixed throughout the simulation.

Each wave is allowed two degrees of freedom, amplitude and phase-shift; $A_k$ and $\alpha_k$.

$$\tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi)e^{i(n_k \zeta - m \theta - \omega_k t - \alpha_k(t))}$$

The wave Lagrangian can then be written as

$$L_w = \sum_{k=1}^{n_w} \frac{E_k}{\omega_k} A_k^2 \dot{\alpha}_k,$$

where

$$E_k = \frac{1}{2\mu_0} \int_V \frac{\left| \nabla \cdot \tilde{\Phi}_k \right|^2}{v_A^2} d^3x,$$

and $n_w$ is the number of eigenmodes in the system.
Wave Equations

- Linear eigenstructure assumed invariant
- Introduce slowly varying amplitude and phase:
  \[ \tilde{\Phi}_k = A_k(t) \sum_m \tilde{\phi}_{km}(\psi) e^{i(n_k \zeta - m \theta - \omega_k t - \alpha_k(t))} \]
- Gives wave equations as:
  \[ \dot{\chi}_k = \frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k||m|| - \omega_k) S_{jkm} + \chi_k \gamma_d, \]
  \[ \dot{\gamma}_k = -\frac{1}{2E_k} \sum_{j=1}^{n_p} \delta f_j \Delta \Gamma_j^{(p)} \sum_m (k||m|| - \omega_k) C_{jkm} + \gamma_k \gamma_d, \]
- where
  \[ \chi_k \equiv A_k \cos(\alpha_k), \quad C_{jkm} \equiv \Re[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_jkm}] \]
  \[ \gamma_k \equiv A_k \sin(\alpha_k), \quad S_{jkm} \equiv \Im[\tilde{\phi}_{km}(\psi_j) e^{i\Theta_jkm}] \]
  \[ \Theta_{jkm} \equiv n_k \zeta_j - m \theta_j - \omega_k t \]
Delta f Method

Numerical noise can be dramatically reduced by making use of a powerful technique known as the $\delta f$ method.

\[
f = f_0(\mathcal{E}, P_\zeta; \mu) + \delta f(\Gamma^{(p)}, t)\]

\[
\frac{df}{dt} = 0 \Rightarrow \delta f = -\dot{P}_\zeta \frac{\partial f_0}{\partial P_\zeta} - \dot{\mathcal{E}} \frac{\partial f_0}{\partial \mathcal{E}} + v_{eff} \delta f
\]

\[
\int f \ g \ d\Gamma^{(p)} \leftrightarrow \int f_0 \ g \ d\Gamma^{(p)} + \sum_{j=1}^{n_p} \delta n_j g_j,
\]

where

\[
\delta n_j(t) \equiv \delta f_j(t) \Delta \Gamma_j^{(p)}(t)
\]
Markers are uniformly loaded using Hammersley’s sequence*:

\[ x_i = \{i/N, \phi_2(i), \phi_3(i), \phi_5(i), \phi_7(i)\}. \]

If the integer \( i \) is written in base \( r \),

\[ i = a_0 + a_1 r + a_2 r^2 + \cdots \]

then \( \phi_r(i) = a_0 r^{-1} + a_1 r^{-2} + a_2 r^{-3} + \cdots \)

Projections of uniformly loaded 5-D hypercube

This achieves a discrepancy \( \propto 1/N \), where a random distribution has a discrepancy \( \propto 1/\sqrt{N} \).
Conservation of Particles

The total number of particles in the system is very well conserved over long time periods.

Particle fluctuation,

$$\delta n = \sum_j \delta f_j(t) \Delta \Gamma_j^{(p)}(t),$$

for $n = 5$ KTAE in JET:
Applications

- AE linear growthrates and nonlinear saturation amplitudes
- Determination of
  - fast particle re-distribution
  - Fast particle losses

[Graphs and diagrams showing wave-particle trapping and fast ion redistribution]

\[ n_p = 52,500 \]

\[ \gamma_d/\omega_0 = 2.7\% \]

Mode saturates at \( \delta B/B \sim 10^{-3} \)
### TAE Amplitude Determination

- Frequency sweeping observed when damping rate ~ linear growth rate and holes/clumps form in particle phase space.
- Modelling with HAGIS allows determination of experimental internal mode amplitudes from observed frequency sweeping.
- Particle trapping frequency determined using HAGIS.

\[
\frac{\delta B}{B} = \frac{1}{(1.156 \times 10^6)^2} \left( \frac{32 \delta f^2}{\delta t} \right)^{2/3} \\
= 4 \times 10^{-4}
\]

Chirping modes exhibit frequency sweeping, \(\delta \omega/\omega_0 \sim 20\%\).
Fast Ions in MAST

- High performance MAST plasmas achieved with off-axis NBI
  - Off-axis NBI current drive used to tailor current profile
- Fast ion distribution very different with on and off axis injection
  - Changes to current profile with off-axis beams studied in MAST
Discrepancy in neutron rate coincident with observation of fast particle driven modes
  - Fishbones

Can be explained by introducing an anomalous fast ion diffusion, $D \sim 0.5 \text{ m}^2/\text{s}$
Fast Particle Simulations

- Fishbones simulated in HAGIS to study effect on fast ion population and calculate an effective fast ion diffusion

\[ \Gamma_{\psi_p}(\psi_p, t) = -D_r \frac{\partial n(\psi_p, t)}{\partial \psi_p} \]
Comparison with Experiment

- Levels of anomalous fast ion diffusion found in simulations consistent with those required in transport codes to explain observations:
  - Neutron rate
  - Stored energy
  - Current profile
**HAGIS code performance**

- HAGIS code parallelises very well
  - relatively low level of inter-processor communication traffic

![Graph showing Run time vs. Number of processors]

- 1/n scaling
- Linux Cluster

Wall clock time to calculate TAE linear growthrate in ITER