Entropy & Vorticity in Self-Organizing Fluid/Plasma Systems

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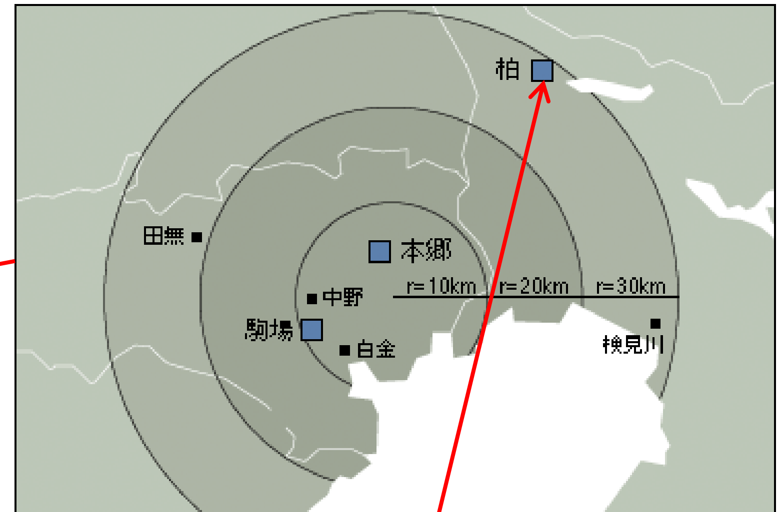
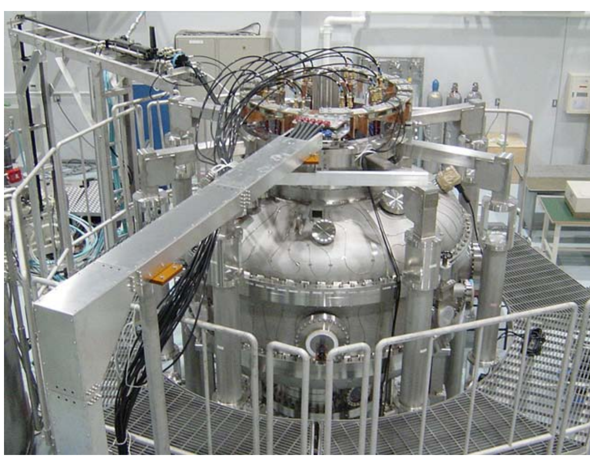
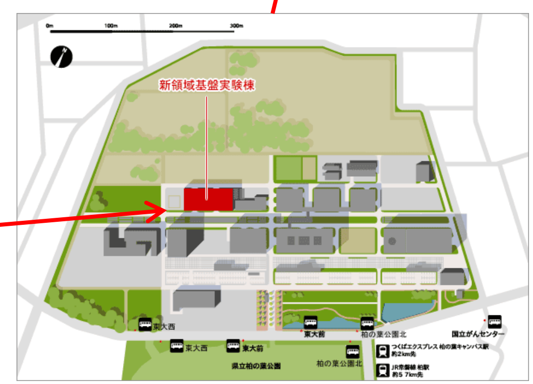


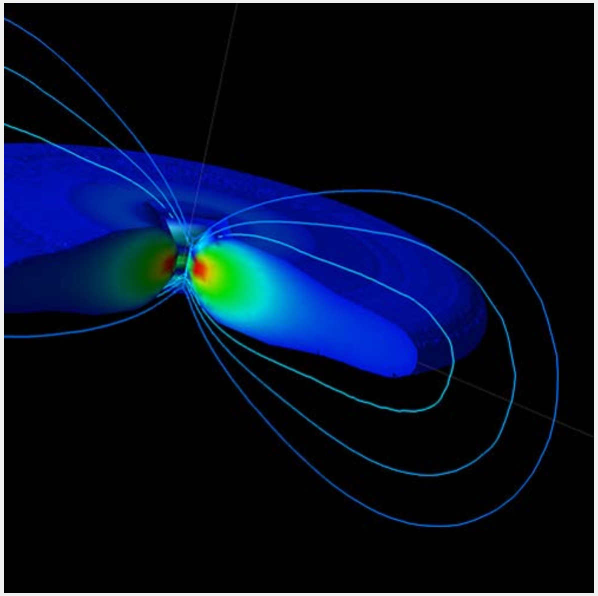




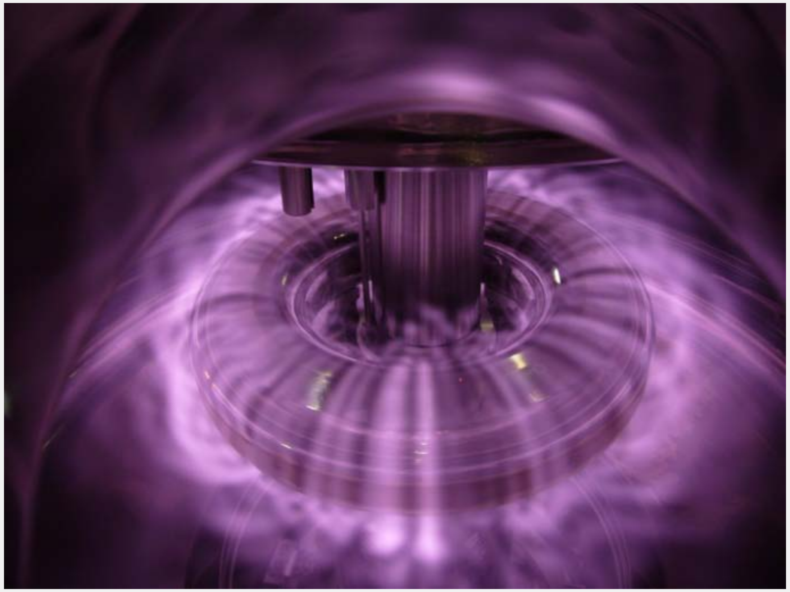
Kashiwa campus

A magnetosphere on the Earth

RT-1 project

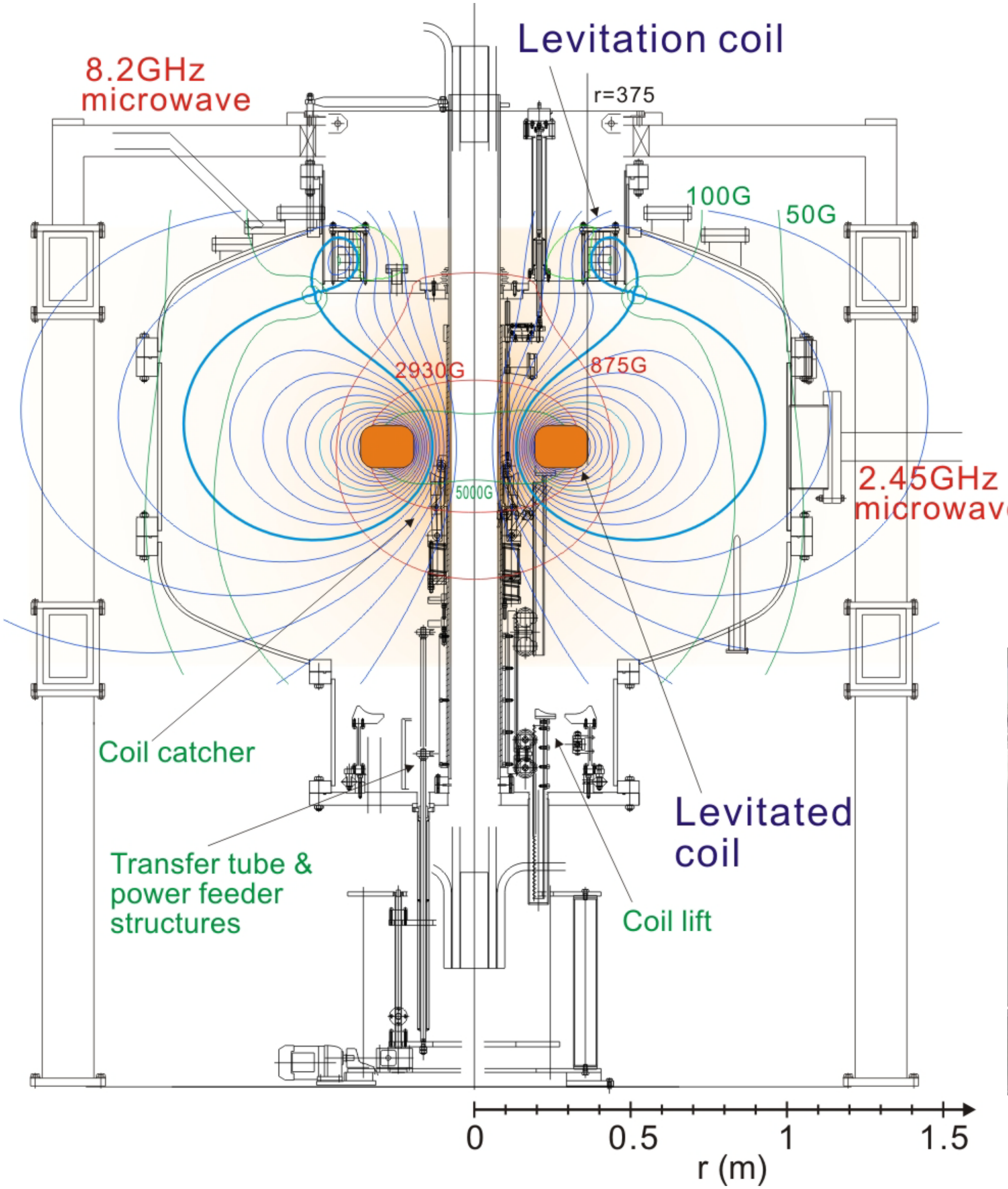


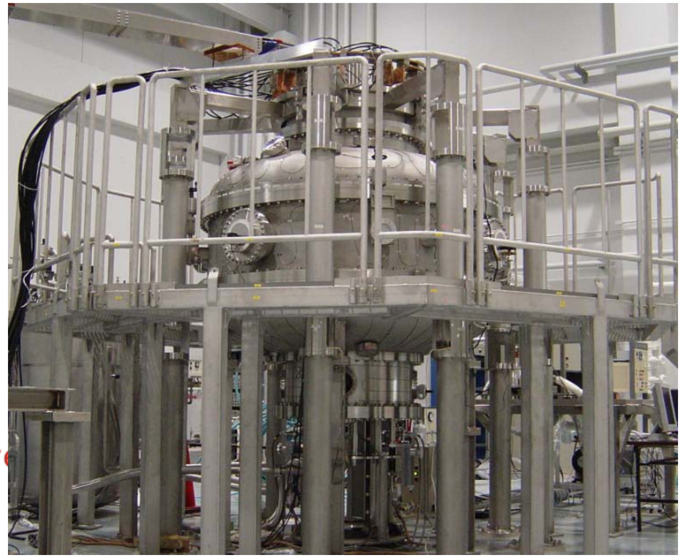
Jovian magnetosphere



RT-1 magnetospheric plasma

Levitating HTC superconducting magnet system





Recent results from RT-1

- High-β plasma

$$\beta \approx \beta_e \leq 0.7$$

$$T_e \approx 10\sim 20\text{keV}, \quad n_e = 10^{16} - 10^{17}\ \text{m}^{-3}$$

$$\tau_E \leq 0.5\,\text{sec}$$

- Long-term trap of non-neutral plasma

$$\tau_P \approx 300\ \text{sec}$$

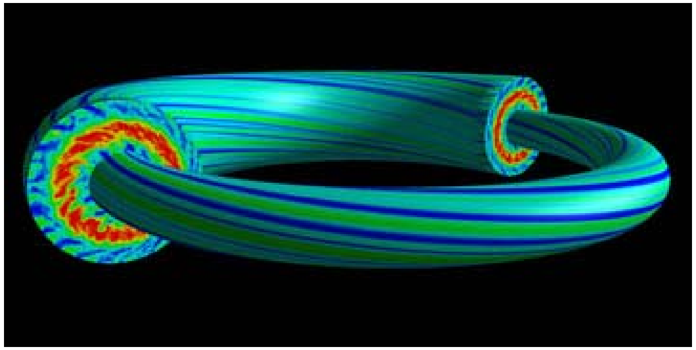
$$n_e = 10^{11} - 10^{12}\ \text{m}^{-3}$$

Plasma production by ECH

max/min entropy production in self-organization

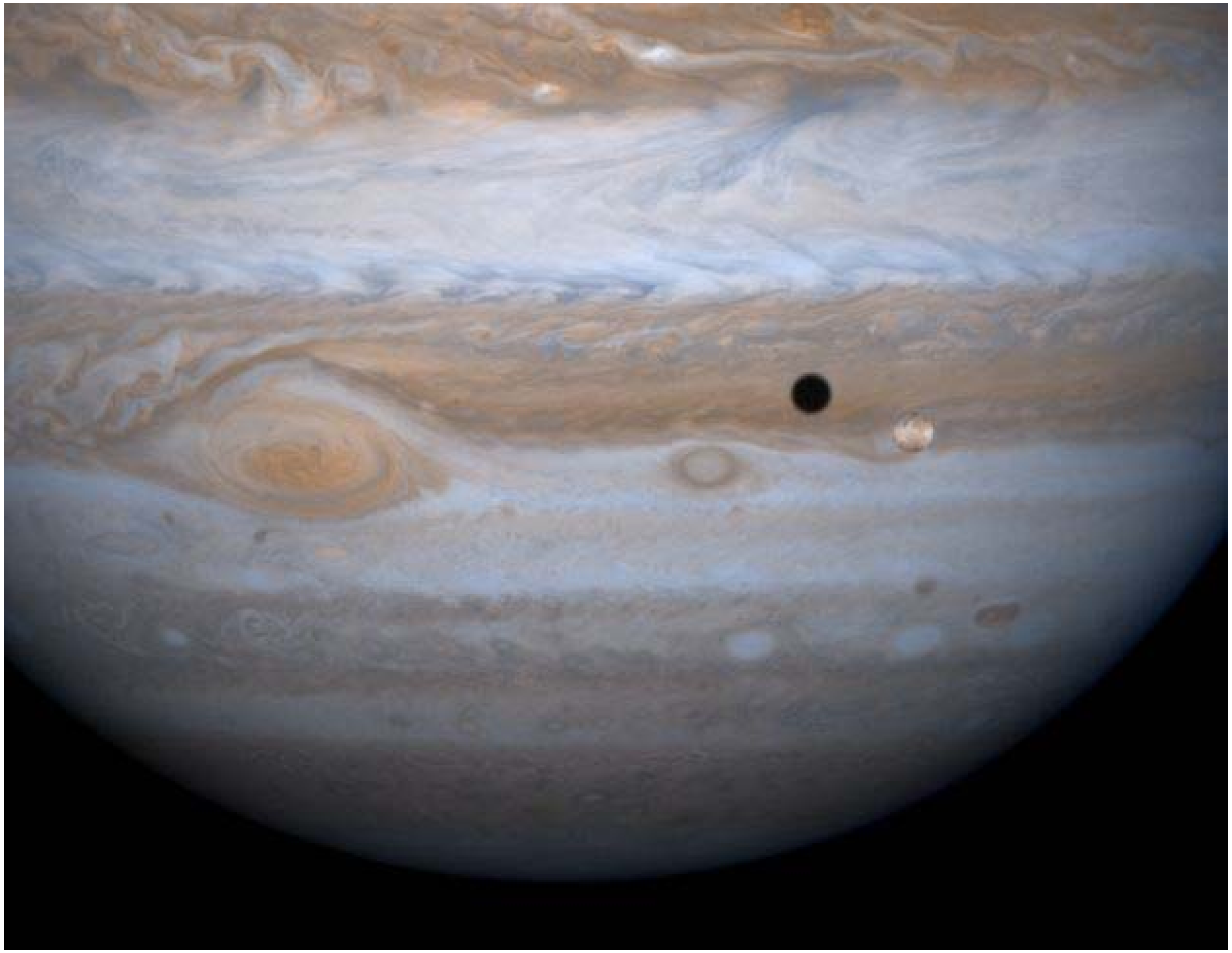
Self-organized plasma boundary layer

- Observation of *H-mode* transport barrier (large ΔT) in *tokamaks*.
- Spontaneous transition at high heat flux.
- Self-organized shear flow (zonal flow) is believed to suppress turbulent heat transport.

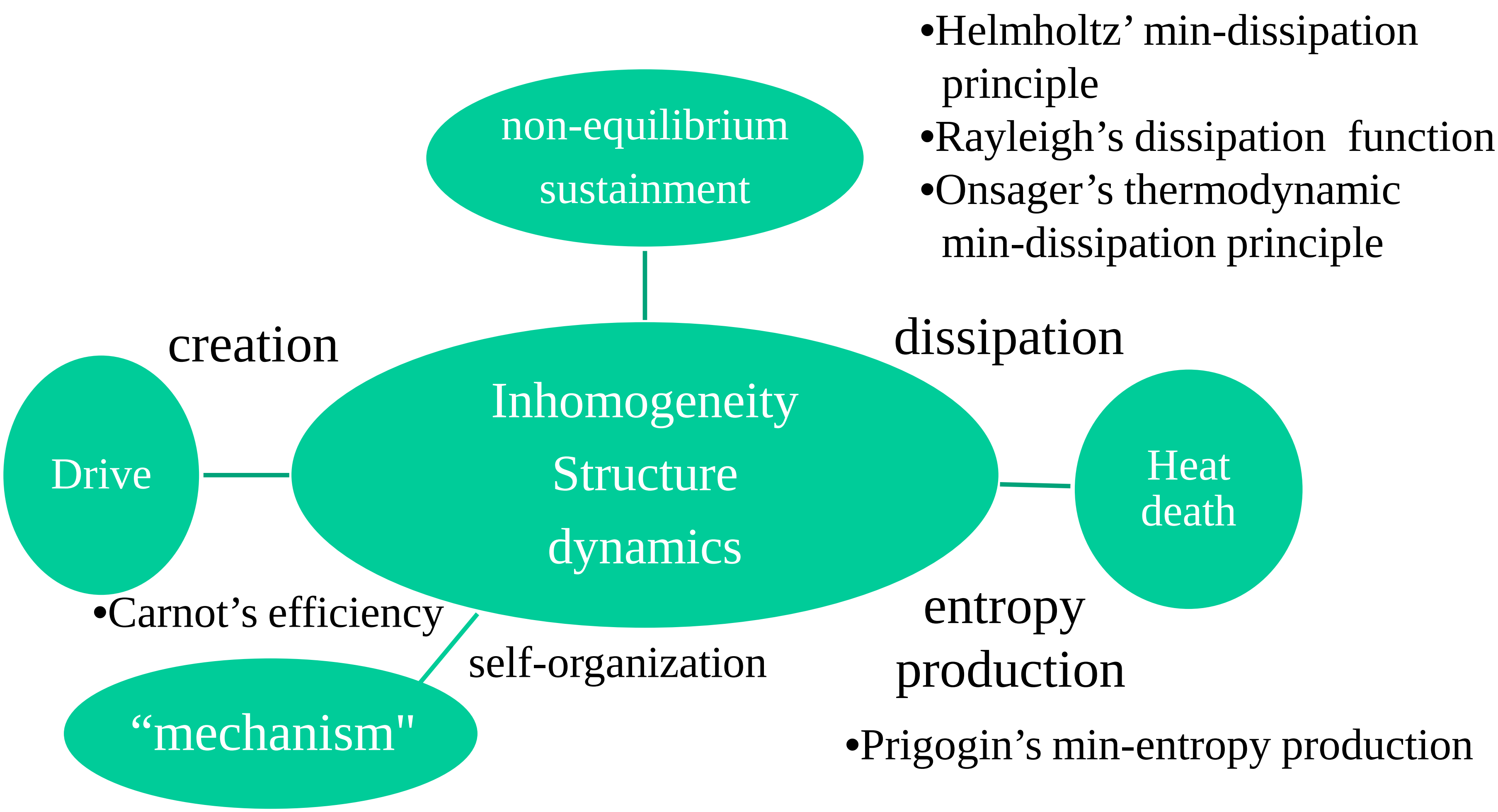


Gyro-kinetic simulation by T. Watanabe (NIFS)

Zonal flow



A brief review of the narratives on non-equilibrium systems



Core model: Dirichlet's principle in linear diffusion model

Linear diffusion equation:

$$\partial_t u = \nabla \cdot (D \nabla u)$$

Minimization of the "inhomogeneity: $W(u) := \int D |\nabla u|^2 \, dx / 2$

$$\partial_t u = -\partial_u W(u)$$

Dissipation function: $w = -\boldsymbol{F} \cdot \nabla u$

Fick's law: $\boldsymbol{F} = -D \nabla u$ → $W(u)$: Dissipation function

If $u \sim \phi$ (thermodynamic intensity) → $w \sim -\boldsymbol{F} \cdot \nabla \phi$

Entropy production rate

Generalization by combining with mechanics:

$$\partial_t u = J \partial_u H(u) - \partial_u W(u)$$

Rayleigh's dissipation function

Problems to be carefully considered

- Dissipation / Entropy production

  ex. Fourier's law contradicts:

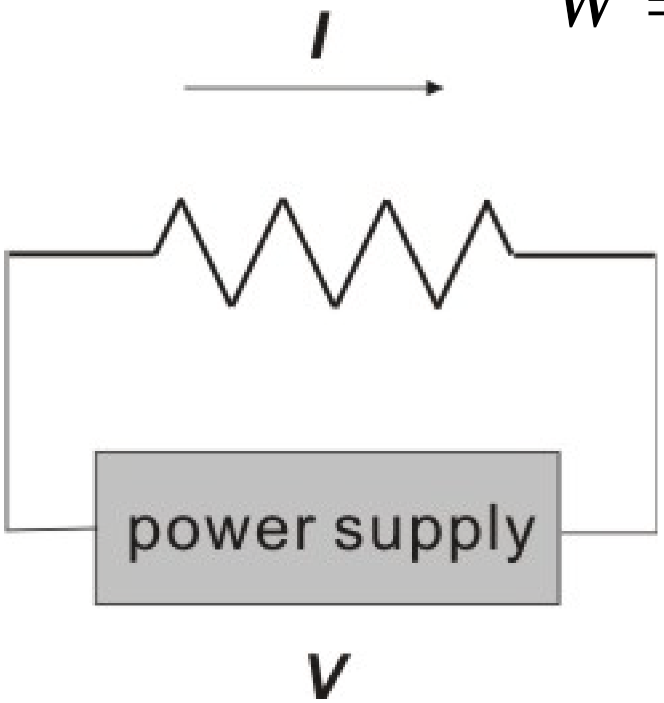
$$EPR = \int \left(D\nabla T\right)\cdot\nabla\beta dx = \int D\left|\nabla\log T\right|^2 dx$$

- Relaxation / Sustainment (Driven System)

  Connection to the drive can change the mode of operation

- Nonlinearity in Dissipation
- Scale hierarchy: Large-scale coherent structure may create different modes of entropy production (primarily in small scales)

Drive and nonlinearity

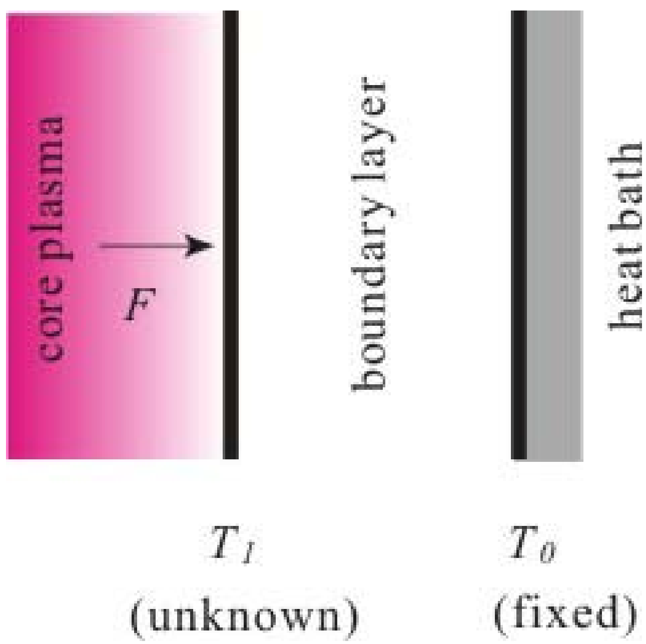


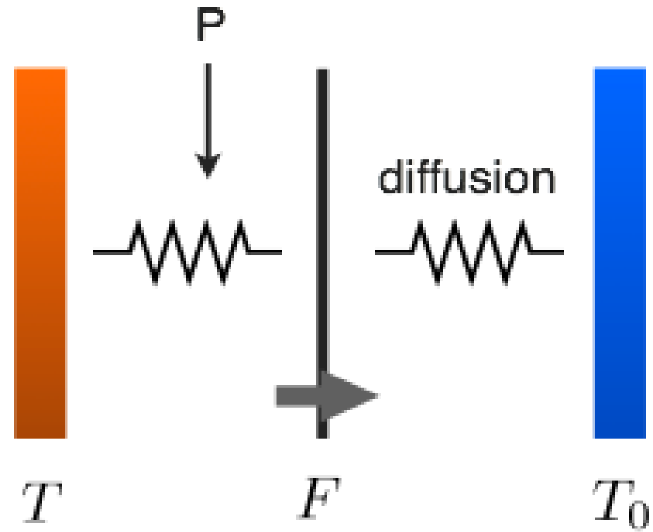
$$W = I \cdot V = \begin{cases} RI^2 & \text{Flux (current) driven} \\ R^{-1}V^2 & \text{Force (voltage) driven} \end{cases}$$

Nonlinearity: $R(W)$

If $dR/dW > 0$,
Flux driven → thermal runaway

A model of self-organized transport barrier



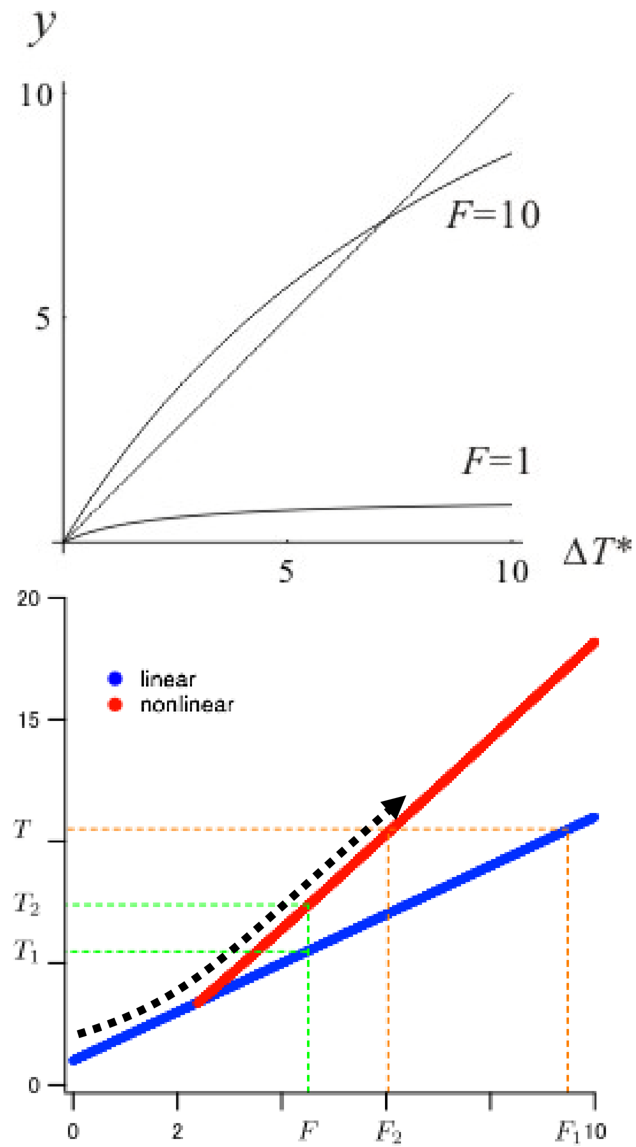


$$T = T_0 + \eta(P)F$$

$$\eta(P) = \eta_0 + \eta_1(P) = \eta_0 + aP$$

$$P = \left(1 - \frac{T_0}{T}\right)F - \left(1 - \frac{T_0}{T_0 + \eta_0 F}\right)F$$

Bifurcation of MEP branch



non-organized solution

$$\begin{cases} T = T_0 + \eta_0 F \\ T = \frac{aF^2 T_0}{T_0 + \eta_0 F} \end{cases}$$

organized solution

$$F > F_{\min} \equiv \frac{T_0}{\sqrt{T_0 a} - \eta_0}$$

Thermodynamic potential and variational principle

- General operating-point equation:

$$\left.\begin{array}{c} T = T_0 + (\eta_0 + aP)F \\ \text{or} \\ (\chi + \chi(P))\,(T - T_0) = F \end{array}\right\} \quad \Rightarrow \quad F = f(\beta)$$

$$\beta \equiv \frac{1}{T_0} - \frac{1}{T} (\geq 0)$$

- Potential function: $\Phi(F,\beta) \equiv F\beta - \Psi(\beta)$

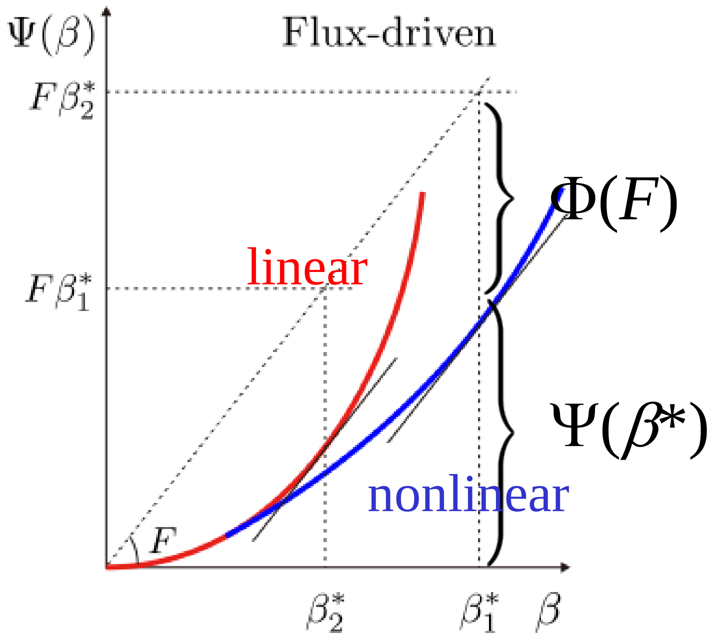
$$\Psi(\beta) \equiv \int_0^{\beta} f(\beta')d\beta'$$

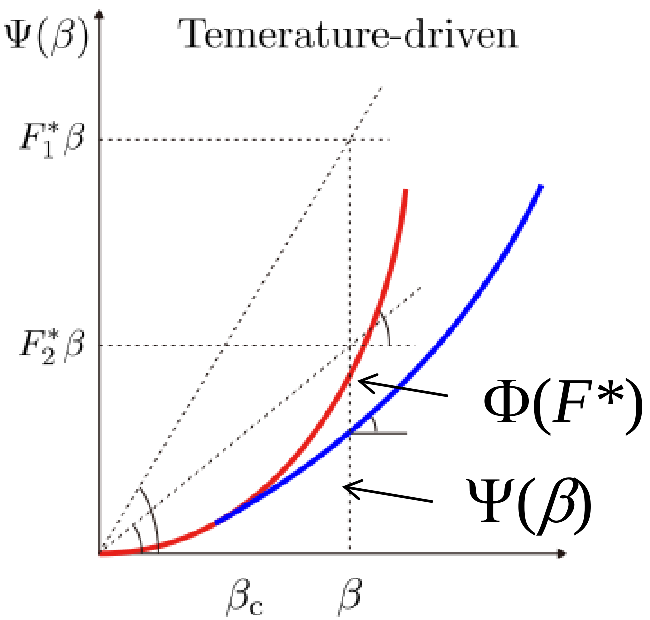
- Variational principle: $\partial\Phi(F,\beta)/\partial\beta = 0$
- In fact $\mathrm{Max}_{\beta}\,\Phi(F,\beta)$

$= \Phi(F)$: Legendre transformation of $\Psi(\beta)$

Entropy production

$$d(F\beta) = Fd\beta + \beta dF \quad \Rightarrow \quad F\beta = \Psi(\beta) + \Phi(F)$$

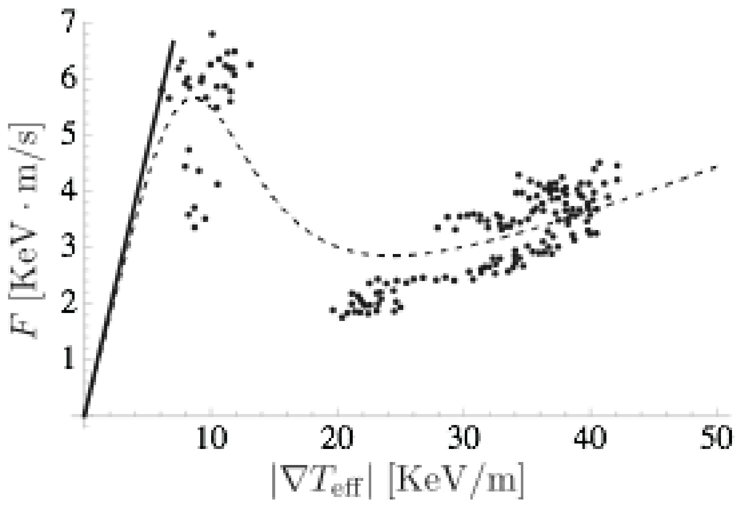


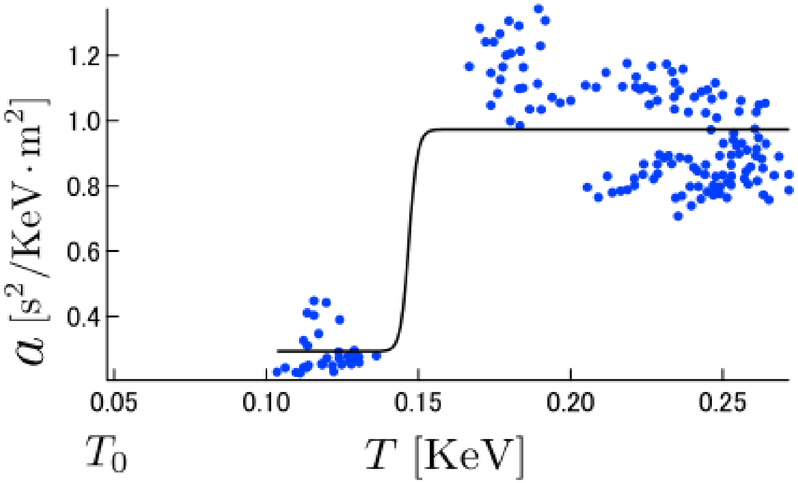


More general nonlinearity: hysteresis

- When *a* depends on *T*, hysteresis can occur:

$$\frac{da(T)}{dT} > \frac{1}{FP(T)}\left(1 - \frac{aT_0F^2}{T^2}\right) = \frac{1}{FP(T)}\left(1 - \frac{T_D}{T}\right)$$

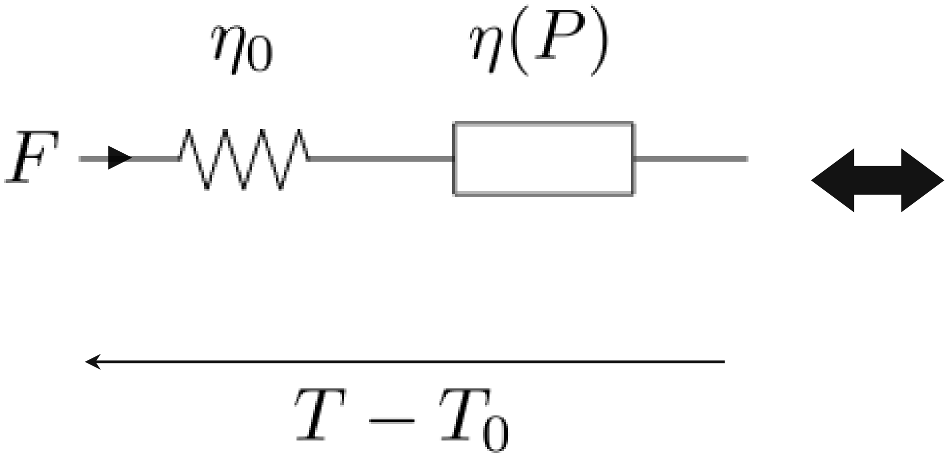




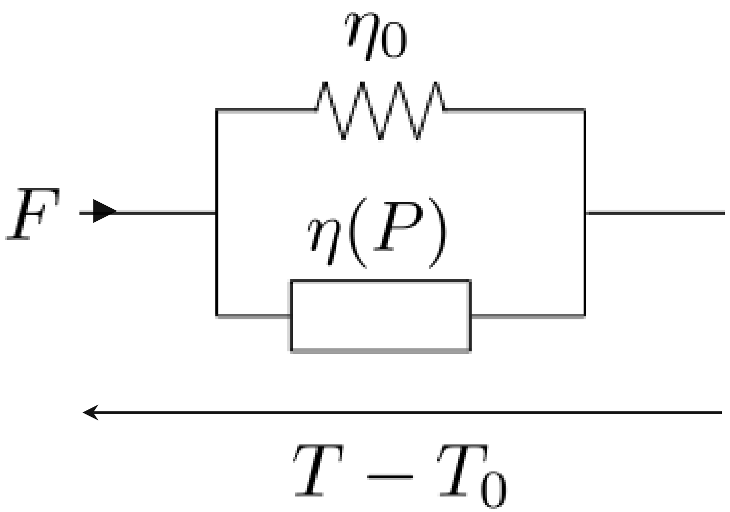
Data from A. E. Hubbard et al., Plasma Phys. Controlled Fusion **44,** A359 (2002)..

Different types (topologies) of nonlinearities

Zonal flow type



Bénard convection type



Compare with the Linear Dissipation function

Linear response: $\beta_i = \sum_{k=1}^{N} R_{ik} F_k$ or $F_i = \sum_{k=1}^{N} L_{ik} \beta_k$

$$\Psi(\beta) \equiv \frac{1}{2} \sum_{i,j=1}^{N} L_{ij} \beta_i \beta_j \qquad \Phi(F) \equiv \frac{1}{2} \sum_{i,j=1}^{N} L_{ij} F_i F_j$$

Onsager's dissipation function

$$d\Psi = \sum_{i,j} L_{ij} \beta_j d\beta_i = \sum_j \beta_j d\left(\sum_i L_{ji} \beta_i\right) = \sum_j \beta_j dF_j$$

$$= \sum_j dF_j \sum_i R_{ij} F_i = \sum_{i,j} R_{ij} F_i dF_j = d\Phi$$

$$\therefore d(F\beta) = d\Psi + d\Phi = 2d\Psi$$

min/max EPR ←→ min/max dissipation

Underlying mechanism

- Self-organization of

  ordered structure = *shear flow* = *vorticity*

- *Entropy* connects *vorticity* and *bifurcation* through *Kelvin's circulation law*:

$$\frac{d}{dt}\left(\oint_{L(t)} \boldsymbol{P}\cdot d\boldsymbol{x}\right) = \oint_{L(t)} \left[\partial_t \boldsymbol{P} + (\nabla\times\boldsymbol{P})\times\boldsymbol{V}\right]\cdot d\boldsymbol{x}$$

$$= \oint_{L(t)} -\nabla(H+h)\cdot d\boldsymbol{x} + \oint_{L(t)} T\nabla S\cdot d\boldsymbol{x}$$

0

Example: Bénard convection

$$\begin{cases} \partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\dfrac{1}{\rho_0}\nabla p_1 + \alpha T \boldsymbol{g} + \mu \Delta \boldsymbol{v} \\ \partial_t T + \boldsymbol{v} \cdot \nabla T = \kappa \Delta T \\ \nabla \cdot \boldsymbol{v} = 0 \end{cases}$$

Boussinesq approximation

$$\rho_1 - \rho_0 = -\alpha(T_1 - T_0)$$

By Maxwell's relation: $\left(\dfrac{\partial S}{\partial P}\right)_T = \dfrac{1}{n^2}\left(\dfrac{\partial n}{\partial T}\right)_P = -\dfrac{\alpha}{n_0}$

$$\partial_t \boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla\left(\frac{p_1}{\rho_0} + T_0 S_1\right) + T_1 \nabla S_0 + \mu \Delta \boldsymbol{v}$$

Self-organization of structures

- Maximum entropy production
  --- creation of “disorder”
  measured by $\left(\frac{1}{T_0} - \frac{1}{T_1} \right)$
- Generation of *vorticity*
  --- creation of “order”
  measured by $\left(1 - \frac{T_0}{T_1} \right)$

Co-existence of “order” and “disorder” ← scale hierarchy

Summary

"Entropy Production" and "Vorticity Creation" are fundamentally connected.

→The entropy term *TdS* works as a non-exact differential breaking the circulation constraint.

→$TdS = \delta Q$ creates a self-organized heat engine = *vortex:* self-organized transport barrier maximizing the entropy production rate.

references

CLASSICS

- Lord Rayleigh, Phil. Mag. **26**, 776786 (1913).
- L. Onsager, Phys. Rev. **37**, 405 (1931).
- P. Gransdorff and I. Prigogine, *Thermodynamic Theory of Structure, Stability and Fluctuations* (Wiley-Interscience, New York, 1971).
- I. Gyarmati, *Nonequilibrium Thermodynamics* (North-Holland, 1962).

MEP

- G. W. Paltridge, Q. J. R. Meteorol. Soc. **101**, 475 (1975); G. W. Paltridge, Nature (London) 279, 630 (1979).
- H. Ozawa, S. Shimokawa, H. Sakuma, Phys. Rev. E, **64**, 026303 (2001).
- R. Dewar, J. Phys. A **36**, 631 (2003).

Min/Max Duality and Boundary Layer Model

- ZY & S.M. Mahajan, Phys. Plasmas **15** , 032307 (2008).
- Y. Kawazura & ZY, Phys. Rev. E **82**, 066403 (2010).