Fluid electron, gyrokinetic ion simulations of global modes in tokamaks and stellarators

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Motivation

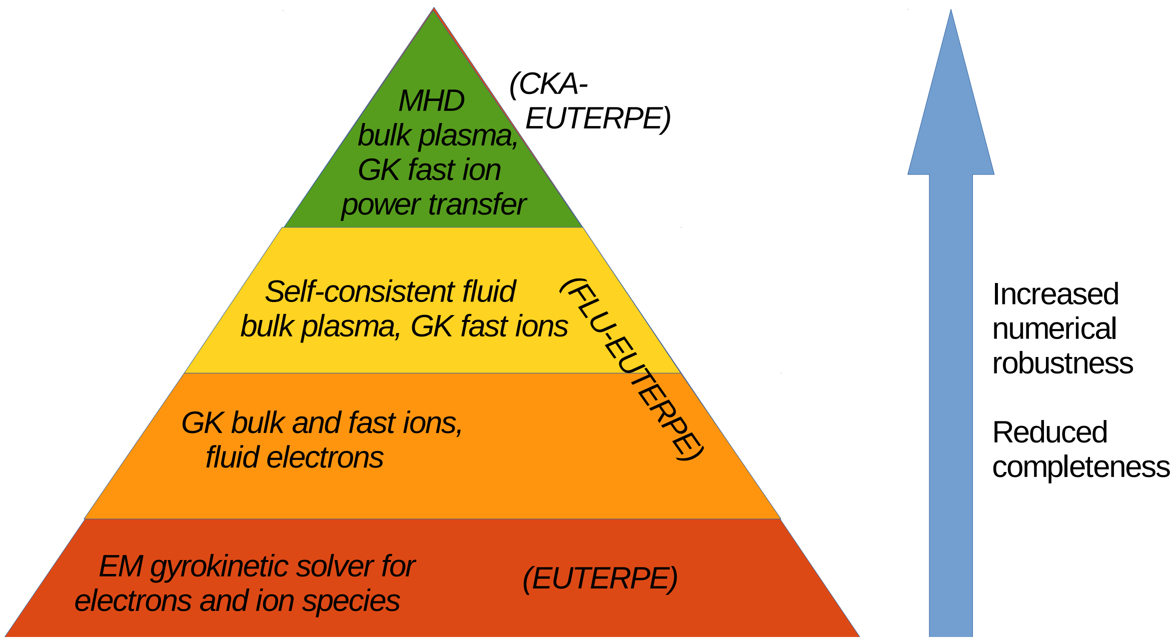
- **Global modes are important in fusion devices**
  - **e.g. TAE, sawtooth cycle**

- **Simulations are computationally demanding, more so for stellarators**

- **Gyrokinetic PIC makes full torus simulations practical, but has drawbacks**
  - **e.g. cancellation problem**

Reducing computational requirements makes physics studies possible

EUTERPE Code Family



Hierarchy of numerical models for addressing different problems.

- **Charge and current calculated on grid using markers.**
- **4th order Runge-Kutta scheme to solve gyrokinetic equations of motion in phase space.**

$$\frac{\partial f_{1s}}{\partial t} + \dot{\vec{R}} \cdot \frac{\partial f_{1s}}{\partial \vec{R}} + \dot{v}_{\|} \frac{\partial f_{1s}}{\partial v_{\|}} = -\dot{\vec{R}}^{(1)} \cdot \frac{\partial F_{0s}}{\partial \vec{R}} - \dot{v}_{\|}^{(1)} \frac{\partial F_{0s}}{\partial v_{\|}}$$

$$-\nabla \cdot \left[\left(\sum_{s=i,f} \frac{q_s^2 n_s}{T_s} \rho_s^2 \right) \nabla_{\perp} \phi \right] = \sum_{s=i,e,f} q_s \bar{n}_s \qquad (1)$$

$$\left(\sum_{s=i,e,f} \frac{\hat{\beta}_s}{\rho_s^2} - \nabla_{\perp}^2 \right) A_{\|} = \mu_0 \sum_{s=i,e,f} \bar{j}_{\|s}$$

- **Global, non-linear, collisional, δf, but neglects $\delta B_{\|}$**

FLU-EUTERPE

- **Fluid** model for **electrons** combined with **gyrokinetic** model for **bulk and fast ions**.

$$\frac{\partial n_{1e}}{\partial t} = f(u_{\parallel 1e}, \phi, A_\parallel, P_{1e})$$

- **Electron continuity equation** connected to GK quantities by **quasineutrality equation** and **Ampère's law**:

$$-\nabla_\perp \frac{m_i n_0}{eB^2} \nabla_\perp \phi = n_{1i} - n_{1e}, \qquad j_{\parallel 1i} = e n_0 u_{\parallel 1e} - \frac{1}{\mu_0} \nabla_\perp^2 A_\parallel$$

- **Closures** needed for $E_\parallel$ (Ohm's law) and pressure:

$$E_\parallel = -\nabla_\parallel \phi - \frac{\partial A_\parallel}{\partial t} = \eta (j_{\parallel i} + j_{\parallel e}), \quad \frac{\partial P_{1e}}{\partial t} = -\vec{v}_E \cdot \nabla P_{0e} = -\frac{\vec{b} \times \nabla \phi}{B} \cdot \nabla n_0 T_0$$

$v_{\|}$ formulation

Equations of motion, $v_{\|}$-formulation:

$$\dot{\vec{R}} = v_{\|}\vec{\tilde{b}}^{*} + \frac{1}{q_s \tilde{B}_{\|}^{*}}\vec{b} \times \left[\mu \nabla B + q_s \left(\nabla \langle\phi\rangle + \frac{\partial \langle A_{\|}\rangle}{\partial t}\vec{b}\right)\right]$$

and

$$\dot{v}_{\|} = -\frac{1}{m_s}\vec{\tilde{b}}^{*} \cdot \mu \nabla B - \frac{q_s}{m_s}\left(\vec{\tilde{b}}^{*} \cdot \nabla \langle\phi\rangle + \frac{\partial \langle A_{\|}\rangle}{\partial t}\right)$$

where

$$\vec{\tilde{B}}^{*} = \vec{B} + \frac{m_s}{q_s} v_{\|s}(\nabla \times \vec{b}) + \nabla \times \langle A_{\|}\rangle \vec{b}.$$

Simplified closure eliminates $\frac{\partial A_{\|}}{\partial t}$ in $v_{\|}$-formulation:

$$E_{\|} = -\nabla_{\|}\phi - \frac{\partial A_{\|}}{\partial t} = 0$$

Fluid Bulk Plasma model

- Solve the gyrokinetic equation only for fast ions.

- Deriving continuity equations for both electrons and bulk ions, calculate total charge density $\rho = q_i n_i + q_e n_e$:

$$\frac{\partial \rho_1}{\partial t} + \vec{B} \cdot \nabla \left(\frac{j_{\|1}}{B} \right) + \left(\nabla \times A_\| \vec{b} \right) \cdot \nabla \left(\frac{j_{\|0}}{B} \right) + \rho_0 \vec{v}_* \cdot \frac{\nabla B}{B} - \frac{\nabla \times \vec{B}}{B^2} \cdot \nabla P_1 = 0$$

with

$$\vec{v}_* = 2 \frac{\vec{b} \times \nabla P_{1e}}{n_0 m_e e B}.$$

- With appropriate quasineutrality equation and Ampère's law:

$$-\nabla_\perp \frac{m_i n_0}{B^2} \nabla_\perp \phi = \rho_1, \qquad \mu_0 j_{\|1} = \nabla_\perp^2 A_\|.$$

CKA-EUTERPE

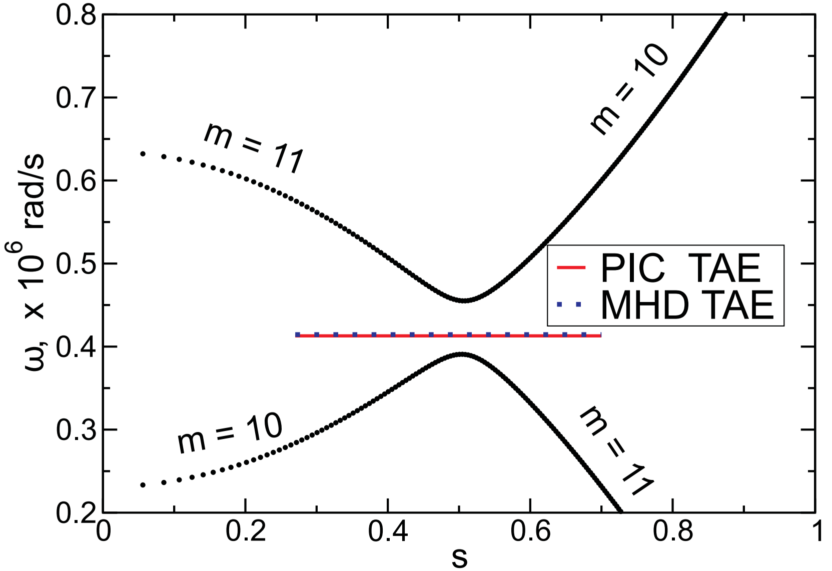
- Solve reduced ideal-MHD vorticity equations to find perturbed fields (CKA):

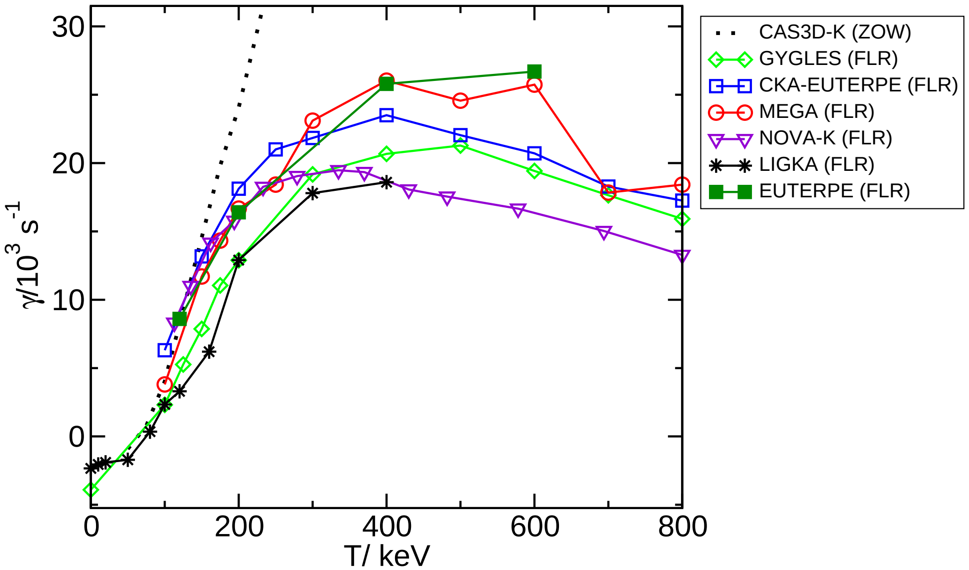
$$\omega^2 \nabla \cdot \left(\frac{1}{v_A^2} \nabla_\perp \phi \right) + \nabla \cdot \left[\vec{b} \nabla_\perp^2 (\vec{b} \cdot \nabla) \phi \right] + \nabla \cdot \left[\vec{b} \nabla \cdot \left(\frac{\mu_0 j_\parallel}{B} \vec{b} \times \nabla \phi \right) \right] - \\ - \nabla \cdot \left(\frac{2\mu_0}{B^2} \left[(\vec{b} \times \nabla \phi) \cdot \nabla p \right] (\vec{b} \times \kappa) \right) = 0$$

- Solve gyrokinetic equation for fast ion species, to calculate power transfer with mode (EUTERPE)

ITPA TAE Benchmark

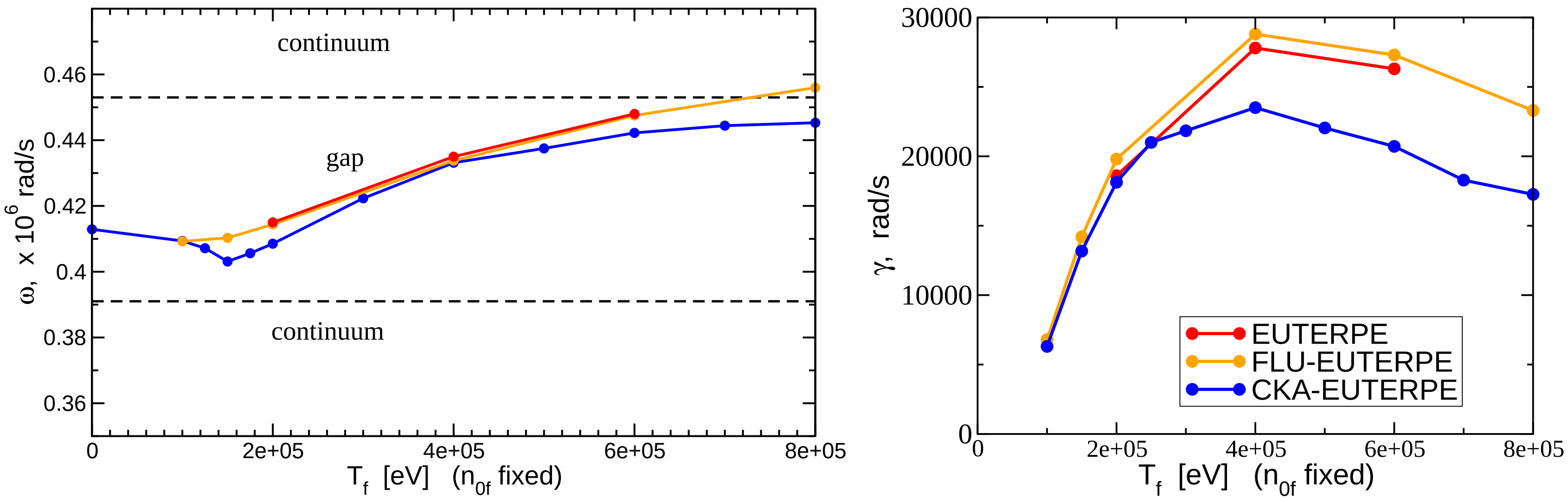
- **TAE in large aspect ratio tokamak, low shear, simple spectrum**
- **Can compare EUTERPE, FLU-EUTERPE and CKA-EUTERPE**
- **Permits benchmark of all models with other codes worldwide**





TAE Benchmark Results

- Can compare **EUTERPE**, **FLU-EUTERPE** and **CKA-EUTERPE**



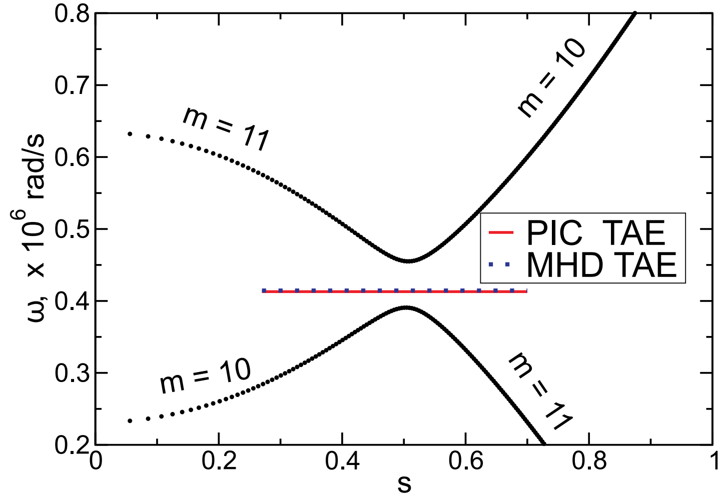
- Some divergence of **perturbative hybrid** model, but **significant speed-up**.
  - Roughly order of magnitude separation between models in run time.

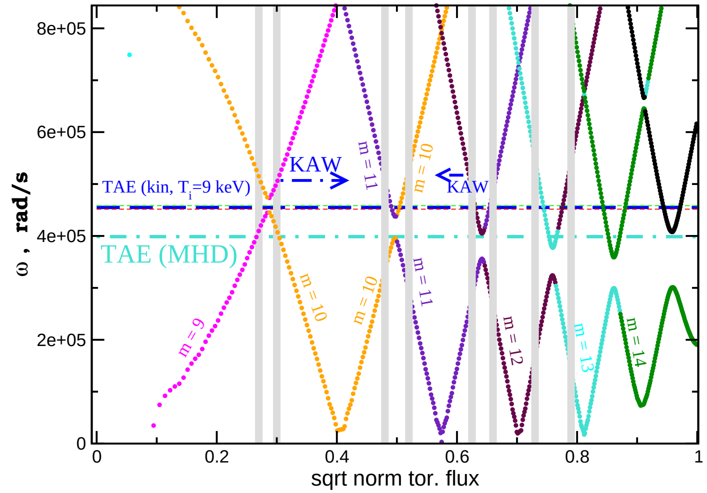
WENDELSTEIN 7-X

Continuum effects

IPP

- **Stronger shear**
  - **more complex spectrum.**
- **Mode structure modification by fast particles captured by fluid-electron model but not perturbative hybrid.**
- **Limits of closure?**





A. Mishchenko et al., Phys. Plasmas 21, 052114 (2014)

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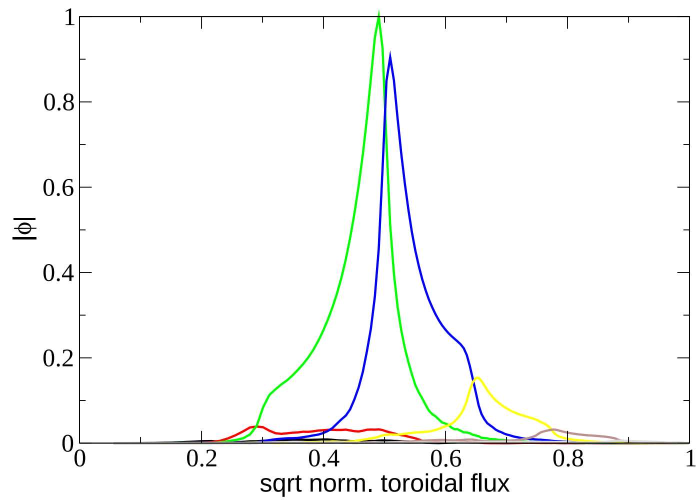
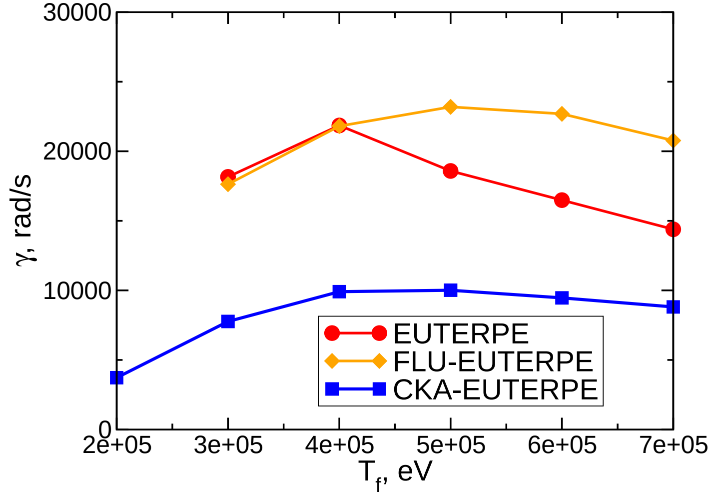
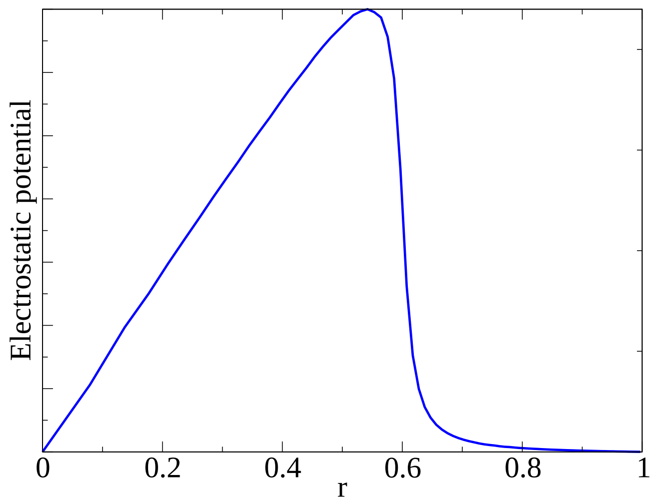


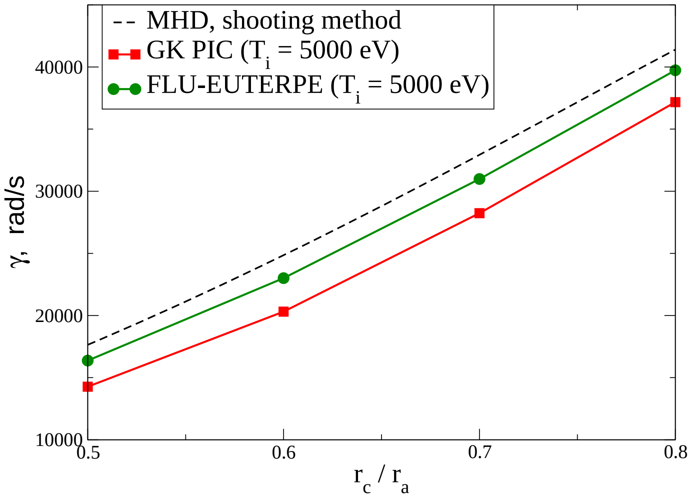


A. Mishchenko et al., Phys. Plasmas 21, 052114 (2014)

Internal kink mode in a screw pinch

- Full gyrokinetics possible in screw pinch, but close to limits.
  - Important for resistive layer physics.
- Benchmark for fluid-electron model in ‘MHD regime’.





A. Mishchenko and A. Zocco, Phys. Plasmas 19, 122104 (2012)

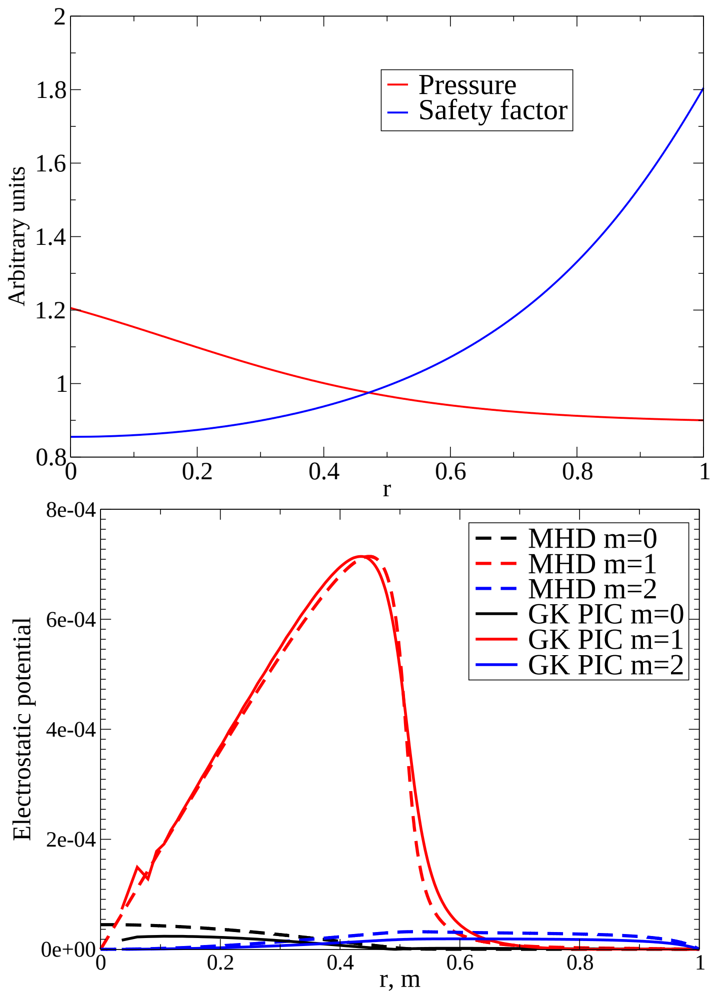
Kink in tokamak

- Finite ∇P now needed to destabilise mode.

- In fluid limit, direct comparison possible with MHD code CKA (A. Könies):

$$\gamma_{FLU} = 1.29 \times 10^6 s^{-1}$$

$$\gamma_{CKA} = 1.27 \times 10^6 s^{-1}$$

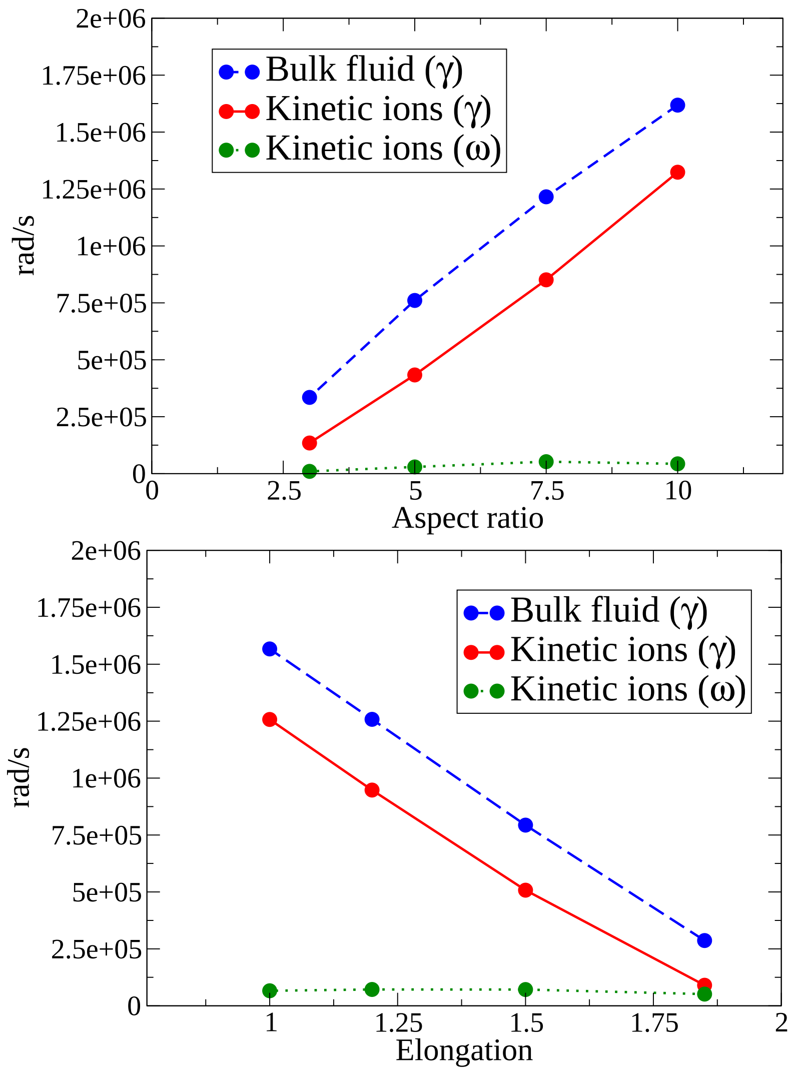


Lose resistive layer physics but tokamak simulations become practical.

Tokamak shaping

- Fully global code - consider significance of geometry
  - Strong stabilisation with JET/ITER-like aspect ratio and elongation

- Further stabilisation with inclusion of bulk ion gyrokinetic effects

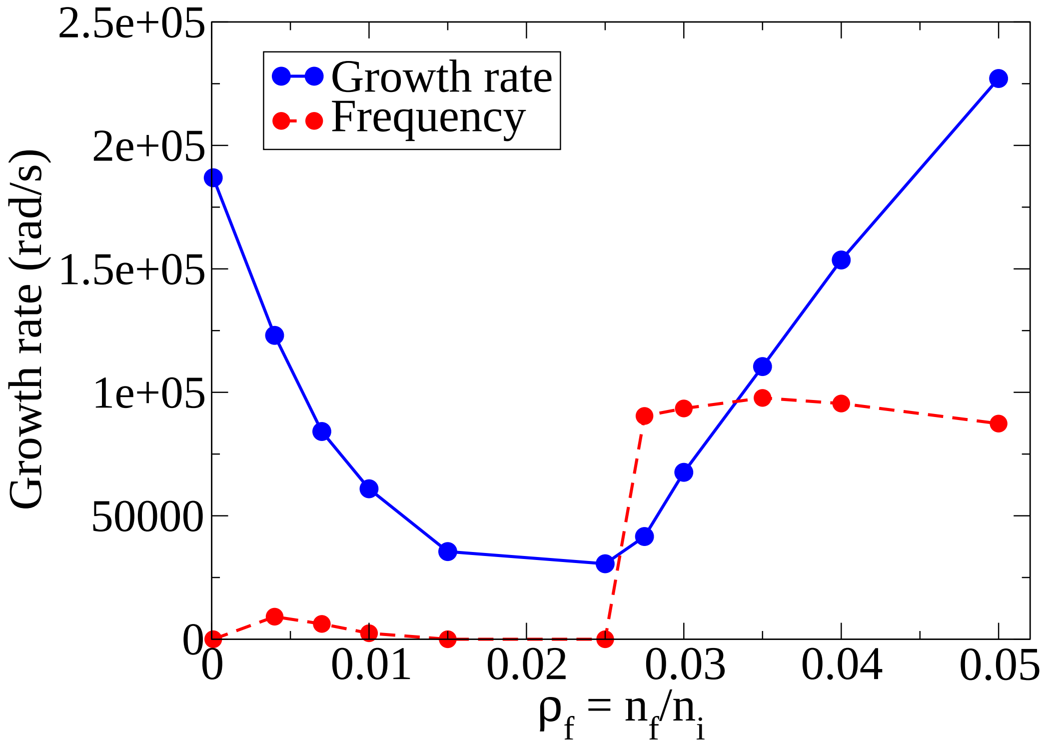


Fast particle effects - fishbone

- **Pure Energetic Particle Modes (EPMs) such as the fishbone emerge when kink is destabilised by energy transfer from fast particles.**

- **Initial stabilisation of the mode (perturbative effect) overcome by unstable fishbone EPM branch.**

- **Frequency jump with onset of fast particle destabilisation.**



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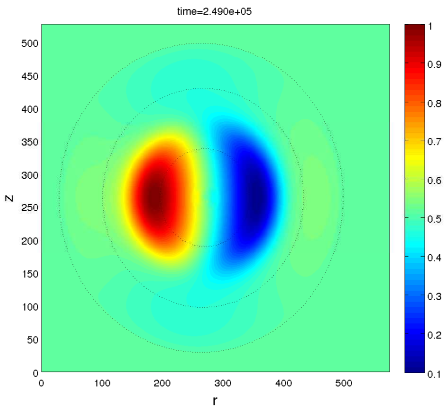
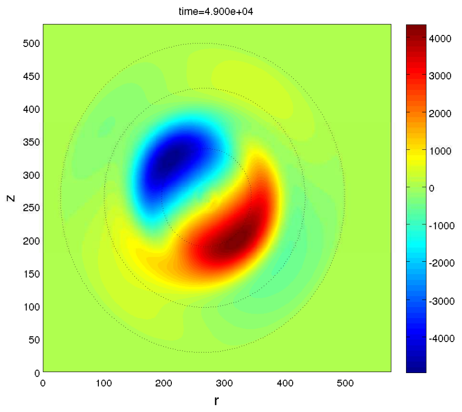
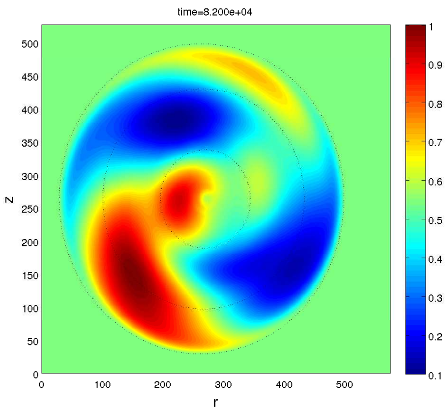
Fast particle effects - fishbone



$\rho_f = 0.0$



$\rho_f = 0.01$

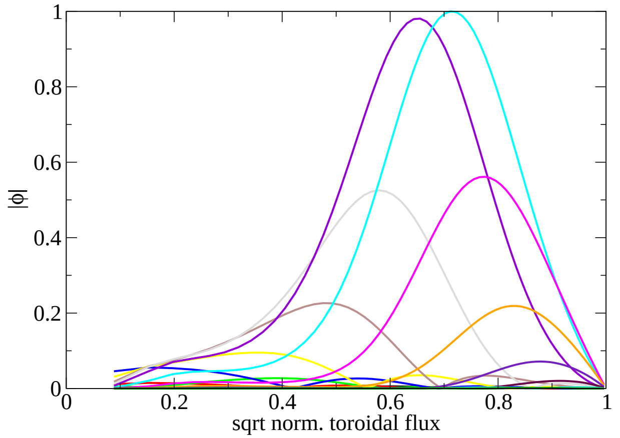


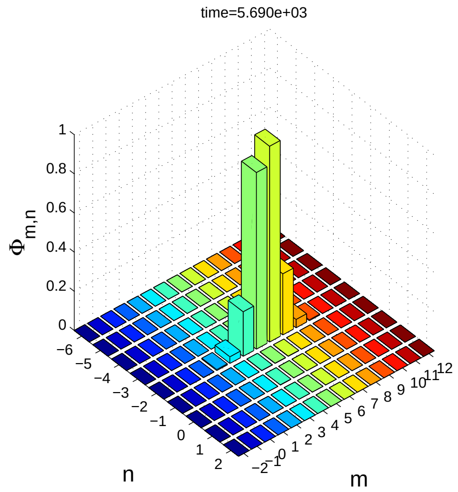
$\rho_f = 0.04$

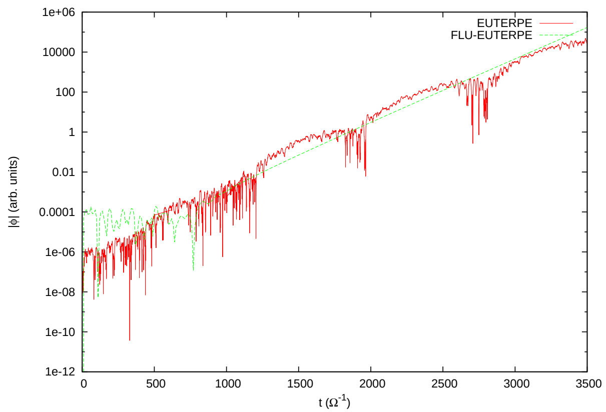
M. Cole et al., Phys. Plasmas 21, 072123 (2014)

Ballooning mode in W7-AS

Stellarator simulations with FLU-EUTERPE and EUTERPE.



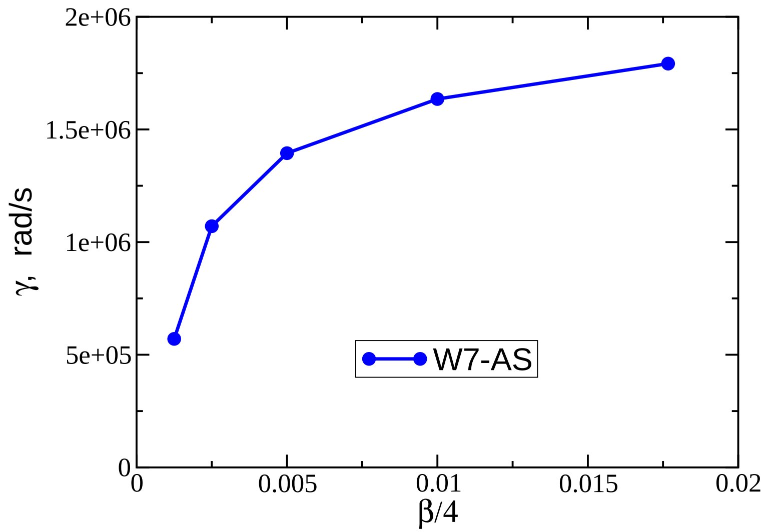


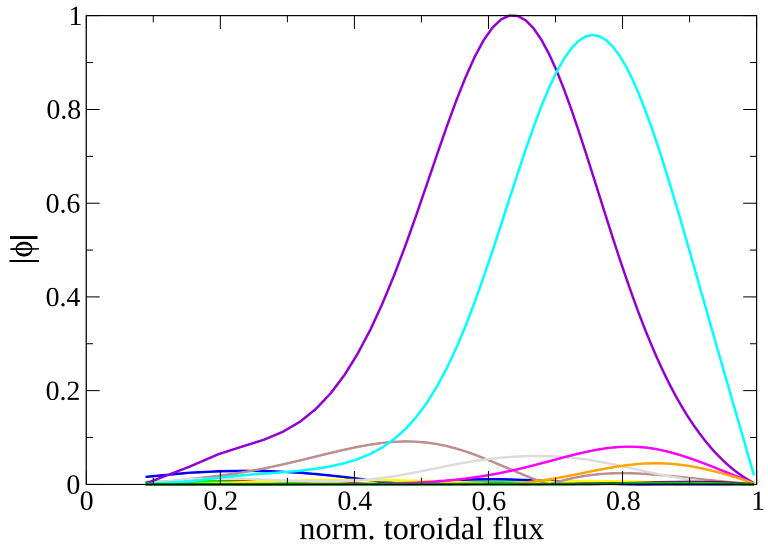


- **Bulk temperature gradient drives unstable modes at high β.**

Ballooning mode in W7-AS

Stellarator simulations with FLU-EUTERPE and EUTERPE.





- Scan in β: mode disappears below critical point.

Summary

- Simulating global modes still presents challenges.
  - Stellarator simulations especially difficult.

- Theoretical developments required to mitigate difficulties
  - pullback scheme for full gyrokinetics
  - electron-fluid, gyrokinetic-ion model
  - perturbative hybrid model

- New models open new avenues, e.g. stellarator phenomena, fishbones, saw-tooth cycle, rapid comparison with experiments.