MAXIMUM PRINCIPLE FOR NON-LINEAR DEGENERATE INEQUALITIES OF PARABOLIC TYPE

J. Chabrowski and R. Výborný

In recent years the maximum principle was extended to degenerate elliptic parabolic equations, and has been studied by several authors, for example in [2], [3], [4], [5], [6], [8], [9], [10]. In this paper we consider a differential inequality

$$(1) \qquad \alpha(t,x)u_t - f(t,x,u(t,x),Du(t,x),D^2u(t,x)) \leq \alpha(t,x)v_t - f(t,x,v(t,x),Dv(t,x),D^2u(t,x))$$

in $Q = (0,T] \times \Omega$, where Ω is an open and bounded set in R^n, and $\alpha(t,x) \geqq 0$ in Q. Du denotes the gradient of u with respect to x, D^2u is the Hessian matrix of the second order derivatives (also with respect to the variable x). $f(t,x,u,p,r)$ is assumed to be defined for $(t,x) \in \phi$, $u \in R$, $p \in R^n$ and $r \in R^{n^2}$.

The main assumptions are that (i) f is weakly parabolic in sense of Besala (see [1]) (ii) f is decreasing with respect to u, (iii) f is Lipschitz with respect to p and r and (iv) there exists a positive constant h and non-negative function γ such that

$$(2) \qquad \alpha(t,x) + \gamma(t,x) \geq h$$

for all $(t,x) \in Q$. If u and w are regular, (for definition see [7]) satisfy (1) and $u - v$ has a non-negative maximum on $\bar{Q}$ then this maximum is attained at some point of the parabolic boundary of Q. Simple examples show that one cannot expect a strong maximum principle with the assumption (2). Our weak maximum principle leads immediately to the uniqueness of the Dirichlet problem for the equation

$$\alpha(t,x)u_t = f(t,x,u,Du,D^2u)$$

to the uniqueness of the Cauchy problem for equation (3) in the class of functions which decay at infinity. With various additional assumptions concerning f we obtain uniqueness of the Cauchy problem in classes of functions which grow at infinity not faster than

$$\text{a)}\quad M \exp k\,|x|^2\,, \qquad \text{b)}\quad M \exp\left(k \sum_{i=1}^{n} |x_i|\right).$$

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Department of Mathematics,
The University of Queensland,
St. Lucia,
Queensland 4067
AUSTRALIA