IDEAL STRUCTURE OF GROUPOID CROSSED PRODUCT C^* ALGEBRAS

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We generalise to groupoid crossed products a theorem of E.Gootman and J. Rosenberg [GR], which asserts that every primitive ideal of the crossed product C^*–algebra is contained in an induced primitive ideal.

More precisely, if G is a second countable locally compact groupoid acting continually on a separable continuous field A of C^*–algebras over the unit space $G^{(0)}$ of G, then every representative L of the (full) crossed product C^*–algebra $C^*(G,A)$ weakly contains the representation induced from the restriction of L to the isotropy group bundle of the action of G on Prim A. The reverse inclusion holds if the action of G on Prim A is amenable.

Just as in [GR], the key ingredient of the proof is a "local cross–section theorem" which is better phrased in the following topological setting. If G is a topological groupoid, x a point of continuity of the isotropy and K a neighbourhood of x in G, which is symmetric and conditionally compact (c.c. for short)– that is, KL is compact for each compact subset L of $G^{(0)}$, then there exists a neighbourhood V of x in $G^{(0)}$ such that the relation $y \overset{K}{\sim} z$ if yKz is non–void becomes on V an open and Hausdorff equivalence relation. This result is applied to the semi–direct product of the action of G on Prim A endowed with the regularized topology. Another tool is a G–equivariant version of a decomposition theorem for representations of C^*– algebras of E. Effros [E]. If L is a representation of $C^*(G,A)$, then there exist a transverse measure class Λ on Prim A and a covariant representation of (G,A) on a measurable field H of Hilbert spaces over Prim A, such that for almost every x the representation of A on H_x is homogeneous with kernel x, which provide by integration a representation unitarily equivalent to L.

This theorem, which compares a given representation with the representation induced from its restriction to the isotropy, does not give enough information on the regular representations when the isotropy is not trivial. It is completed by the following result which was given in [R] in the case of a discrete groupoid and in [FS] in the case of the holonomy groupoid of a foliation. If G is a second countable locally compact and Hausdorff groupoid which is minimal and has discrete isotropy with at least one isotropy group being trivial, then the reduced C^*–algebra $C^*_{red}(G)$ is simple. This is no longer true when the groupoid is not Hausdorff. G. Skandalis provides the following example. Let G be the group of transformations of the circle S^1 generated by an irrational rotation S_0 and two homeomorphisms S_1 and S_2 having respectively as set of fixed points the intervals. $[0,\pi]$ and $[\pi,2\pi]$ and let G be the groupoid of its germs. Since S_0 acts minimally, G is minimal. The isotropy at $\theta \in S^1$ is trivial unless θ is in the orbit of 0 or π, where it is $\mathbb{Z}$. The function of f taking the values 1 at the germs $(0,1)$ and $(\pi,1)$ and 0 elsewhere is in $C_c(G)$ since it can be written as $f = \chi_1 - \chi_{S_1} - \chi_{S_2} + \chi_{S_1S_2}$, where χ_S is the characteristic function of the germs of S. It is in the kernel of the regular representations R_θ when θ is not in the orbit of 0 or π but not in the kernal of the regular representation R_0. Hence $C^*_{red}(G)$ is not simple.

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