SOME OPEN PROBLEMS IN MATHEMATICAL RELATIVITY

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Theoretical general relativity has developed to such an extent that over the next few decades we should expect rigorous mathematical arguments to replace many of the formal calculations and heuristic arguments of the past. Experience suggests that this symbiosis of mathematical and physical intuition will yield new insights for both disciplines, and it is in the hope of stimulating this process that this list was conceived. Thus, the basic criteria were that questions should admit a clear mathematical formulation, and be of interest both mathematically and physically. Of course, my own interests and tastes played a large part in the selection process, so there are many topics which are not represented here. In apology I can only say that our subject is too broad, and my knowledge too limited, for me to dare to venture any farther afield than I have already done.

The questions vary from the banal, which may already lie solved, implicitly or explicitly, in the literature, to the impossible. The references are intended to be representative only — much more is known about some of the questions than I am able to indicate here.

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Apparent Horizons

A closed oriented space-like 2-surface in a spacetime determines two future null vector fields, normal to the surface. If the future evolutions of the surface along these directions are both area-non-increasing, the surface is *future trapped*. This is a condition on the null mean curvatures $k^{\pm}$, and if one of them is zero, then the surface is an *apparent horizon*. If M is a spacelike hypersurface with extrinsic curvature (second fundamental form) $K = (K_{ij})$, the apparent horizon condition for $\Sigma \subset M$ leads to the equation

$$\mathrm{tr}_{\Sigma}k + \mathrm{tr}_{\Sigma}K = 0 ,$$

where $k = (k_{ab})$ is the second fundamental form of $\Sigma \subset M$. In particular, an apparent horizon in a time symmetric (i.e. $K_{ij} \equiv 0$) spacelike hypersurface is a minimal surface. The weak energy condition [HE] implies that a time symmetric (more generally, maximal, $\mathrm{tr}_{\Sigma}K = 0$) M has non-negative scalar curvature.

- Given an asymptotically flat (AF) initial data set (M,g,K) and a (future) trapped surface Σ (i.e. $\mathrm{tr}_{\Sigma}(K+k) \geq 0$),

  (a) show there is a (smooth, spherical) apparent horizon outside Σ.

  (b) show there is an apparent horizon inside Σ.

If M is time symmetric then it should be possible to deduce these results from known results about minimal surfaces [SU;MSY;PJ;SS;FCS].

- As the previous question indicates, we understand the behaviour of apparent horizons mainly by analogy with the behaviour of minimal surfaces. To what extent is this analogy valid? Unlike the area functional and minimal surfaces, there is no known functional for which the apparent horizon equations arise as the Euler-Lagrange equations. Does the "second variation" formula for apparent horizons mean anything? (cf. stability for minimal surfaces).

- Prove there is an AF vacuum initial data set, diffeomorphic to $\mathbb{R}^3$, which contains an apparent horizon. (P. C. attributes this to Ladyzhenskaya). There are numerical examples [MS], constructed by evolving axially-symmetric, time symmetric initial data.

- Find an AF metric on $\mathbb{R}^3$ with zero scalar curvature and admitting a minimal 2-sphere, or prove there is no such metric. This is the previous question, but requiring a time-symmetric initial data set. Can the 2-sphere be a *stable* minimal surface? One motivation for this question is the quasi-local mass definition [BR6], restricted to scalar-flat metrics. (It may be possible to do this by gluing a small mass Schwarzschild metric onto a neighbourhood of a point of a scalar-flat metric on S^3, and then perturbing the metric back to scalar-flat, but this has possible problems with the Sobolev constant).

- Does an apparent horizon persist under the Einstein evolution? This is probably false, but I don't know of an explicit counterexample. An argument giving spacetime conditions under which a trapped surface can be indefinitely extended to a spacelike 3-cylinder of trapped surfaces, of non-increasing area, is given in [IW2].

- Let $g = \varphi^4\delta$ be a conformally flat, AF metric, with non-negative scalar curvature, so $\Delta\varphi \leq 0$. Find conditions on φ which ensure g has no horizon. (The relation between the mass and size of a star in a conformally flat metric has been explored in [BMOM]).

- Let M be a 3-manifold, AF with non-negative scalar curvature, and let $\Sigma \subset M$ be a stable minimal 2-sphere. Prove the Penrose inequality [PR1]

$$\text{area}(\Sigma) \leq 16\pi\,(\text{mass})^2$$

where mass is the ADM mass of M. This would give an upper bound on the area of an apparent horizon in a time-symmetric data set. Find a generalisation of this inequality to initial data sets which are not time-symmetric. (The stability assumption here is necessary, cf. [BN1,2;Gi]. Proofs have been given under various restrictive assumptions, such as the existence of Geroch-type foliations [JP,JJ], and global conditions on the spacetime generated by M [LV2]. See also [TP], which shows an analogous inequality based on the Penrose quasi-local mass.)

Initial Data Sets

An initial data set (M,g,K) is a Riemannian manifold (M,g) with a symmetric 2-tensor $K = (K_{ab})$ satisfying the constraint equations

$$\begin{aligned} 2T_{00} &= R(g) + (\mathrm{tr}K)^2 - \|K\|^2 \\ T_{0i} &= g^{jk}K_{ij|k} - (\mathrm{tr}K)_i\,. \end{aligned}$$

Initial data sets may be constructed by first specifying the conformal data $(\gamma_{ab}, \lambda^c{}_d, \tau)$, where γ_{ab} is a Riemannian metric, $\lambda^c{}_d$ is a traceless, divergence-free, symmetric tensor, and τ is a function, and then solving the Lichnerowicz/ Choquet-Bruhat/York (LCBY) system [CBY;OMY] (vacuum for simplicity);

$$\begin{aligned} \nabla_a(LW)^a{}_b &= {}^2/_3\, \varphi^6 \nabla_b \tau \\ 8\,\Delta\varphi &= R\varphi - \varphi^{-7}(\lambda^c{}_d + LW^c{}_d)(\lambda^d{}_c + LW^d{}_c) + {}^2/_3 \varphi^5 \tau^2 \end{aligned}$$

for (φ, W^a), $\varphi > 0$, where $LW_{cd} = \nabla_c W_d + \nabla_d W_c - {}^2/_3 \gamma_{cd} \nabla_e W^e$ is the conformal Killing operator. The initial data set $(g_{ab}, K^c{}_d)$

$$\begin{aligned} g_{ab} &= \varphi^4 \gamma_{ab}\,, \\ K^c{}_d &= \varphi^{-6}(\lambda^c{}_d + LW^c{}_d) + {}^1/_3\, \delta^c{}_d\, \tau \end{aligned}$$

then satisfies the (vacuum) constraint equations with mean curvature $\tau = \mathrm{tr}K$. If τ is constant and γ_{ab} is a metric on a compact manifold, then the LCBY system has been shown to be solvable [CB1;OMY;IJ].

• Determine conditions on $(\gamma_{ab}, \lambda^c{}_d, \tau)$ which ensure the LCBY system is solvable, for M compact and τ non-constant (the constant-τ argument should generalise if $|\nabla \log\tau|$ is sufficiently small). Examples of spacetimes without constant mean curvature (CMC) Cauchy surfaces [BR5;Br;WD], and numerical work with cosmological models undergoing inflation, show that it is necessary to consider initial data with non-constant mean curvature τ.

• Describe suitable asymptotic conditions which enable the conformal method to be used to construct initial data sets on an asymptotically hyperbolic manifold (eg. a CMC hypersurface meeting $\mathcal{J}^+$). Such initial data sets are called hyperboloidal.

• Characterise those hyperboloidal initial data arising from a spacetime with a smooth conformal null infinity. Describe a class of LCBY data $(\gamma_{ab}, \lambda^c{}_d, \tau)$ which can be used to construct such hyperboloidal initial data.

• Classify the various kinds of smoothness properties which hyperboloidal initial data may have at (a possibly generalised) conformal infinity. In particular, find the weakest conditions under which a proof of positivity of the Bondi mass can be completed (cf [BR2;CP2;OM1] for the AF case).

• Can the space of (globally hyperbolic, vacuum) Einstein metrics on $V = \mathbb{R}\times M^3$ have more than one connected component? Gromov-Lawson [GL] have shown that the space of positive scalar curvature metrics on S^7 does have more than one component (in fact, there are infinitely many components [CR]). It is known [CBM] that the mass function on $\mathfrak{P}\mathfrak{M}$ (the set of maximal AF initial data sets satisfying the conditions of the positive mass theorem [BR6]) has only one smooth critical point, at $\mathbb{R}^3$. This suggests there is only one component in the space of AF spacetimes admitting a maximal Cauchy surface. What is needed here is some compactness/Palais-Smale condition for the mass function on $\mathfrak{P}\mathfrak{M}$.

Uniqueness/Rigidity Theorems (static and stationary metrics)

A surprising feature of the Einstein equations is the restrictions it places on solutions satisfying apparently non-restrictive conditions. The classical example of this is Birkoff's theorem: a spherically symmetric vacuum metric is necessarily Schwarzschild. Other examples are the Israel-Robinson uniqueness theorem for Schwarzschild, recently given a particularly elegant and general proof by Bunting and Masood-ul-Alam [IW1;RD1;BMuA], the uniqueness of the Kerr-Newman spacetime [CaB1,2;RD2;BG;MP], and the Eschenburg-Yau splitting theorem [YS;EJH;Ga2;NR;BEMG].

- What conditions on the stress-energy tensor are needed to show that a static, AF metric is necessarily spherically symmetric? For perfect fluid stress-energy, this is the conjecture that a static star is spherically symmetric. For constant density, this follows from [BMuA], see also [MuA].

- Find a "good" local characterisation of the Kerr solution amongst stationary metrics (preferably independent of the slicing used). Several characterisations of Schwarzschild are known: spacelike slices conformally flat, or spherically symmetric, or $|\nabla V|$ is a function of the lapse V [RD1;IW1].

- Suppose (M_i,g_i,K_i), i=1,2, are disjoint Cauchy surfaces in an AF (vacuum) spacetime, which are isometric (i.e. there is a map $\varphi:M_1\rightarrow M_2$, such that $\varphi^*g_2 = g_1$, $\varphi^*K_2 = K_1$). Show that the spacetime is stationary (admits a timelike Killing vector field). For compact Cauchy surfaces, this follows easily from the Eschenburg-Yau splitting theorem, but positive mass makes it impossible to construct a globally maximising timelike line in the AF case. A result for analytic metrics, periodic at null infinity, is shown in [GS]. An affirmative answer to this question for vacuum Einstein would be strong evidence against the existence of bound state solutions (geons [BH]).

- Give a general discussion of the possible types of singularities which may occur for solutions to the static and stationary vacuum Einstein equations, starting with a description of the possible singularities of the spacelike slices. Since such metrics cannot arise from smooth initial data, this is not a question directly about cosmic censorship, but more about the geometry of the static/stationary vacuum equations. The detailed analysis of the Curzon metric by Scott and Szekeres [SS] indicates that the behaviour can be quite surprising.

- Recently it has been shown numerically that the static, spherically symmetric Einstein-Yang-Mills (EYM) equations have non-singular, AF solutions [BM]. This raises many interesting questions, for example:

(i) prove existence of the static spherically symmetric EYM solutions;

(ii) show there is no static spherical solution with non-zero electric field;

(iii) analyse the time dependent spherically symmetric equations, analogous to the analysis of the spherically symmetric Einstein-scalar field equations in [CD2];

(iv) find axially symmetric static or stationary solutions generalising the Kerr-Newman solution;

(v) find an EYM generalisation of the Majumdar-Papetrou superposition of extremal RN solutions [HH];

(vi) show that a static black hole solution of EYM must be Reissner-Nordstrøm (An EYM black hole has no YM hair [YP;GE]).

Approximations

Motivated by the need to make predictions about real physical phenomena such as the solar system, millisecond binary pulsars, gravitational waves, colliding stars, etc., there has been a lot of work devoted to constructing metrics which approximately satisfy the Einstein equations. This work consists of both numerical computation of the Einstein evolution, and the many asymptotic expansion/ linearisation/matching techniques. The reader is referred to survey papers in the literature (eg. [SBF1,2;DT]) for details of current work; here I will restrict myself to outlining the problems.

- Prove any result of the form:

Conjecture: *Let* g_ε *be an "approximate solution" of the (vacuum) Einstein equations (i.e.* $\|\mathrm{Ric}(g_\varepsilon)\| \leq \varepsilon$*, in some suitable norm). Then there is a metric* g *such that* $\mathrm{Ric}(g) = 0$ *and* $\|g - g_\varepsilon\| \leq \varepsilon$.

This question was also suggested by Ehlers [EJ]. Such a result (assuming the norms $\|\cdot\|$ are sufficiently general) would be of fundamental importance, since it would provide a good way to evaluate approximate/asymptotic and numerical solutions. Typical applications are described in the following questions.

- Show that a solution of the linearised Einstein equations (linearised about Minkowski space) is close to a (non-flat!) exact solution. A global result here would give a new proof of the result of [CK2] on global existence of small data, radiating solutions.

- Asymptotic expansion methods develop series solutions in a small parameter ($c^{-1} \to 0$ is post-Newtonian, $G \to 0$ is post-Minkowskian, cf. [FS;BD; BlL;MTW]). Although there is general agreement that these expansions give realistic results for situations of physical interest, there is certainly doubt about their range of validity — a major cause for concern is the reliance on the De Dondeur (harmonic) gauge condition [CB2]. Show precise conditions under which the post-Newtonian and post-Minkowskian expansions converge, or at least approximate, a global solution. It may be necessary to work with a non-flat background metric (cf. the gauge source functions of Friedrich [FH2]).

• Prove the existence of a limit in which solutions of the Einstein equations reduce to Newtonian spacetimes. (A careful expansion analysis in the limit with matter sources going to zero has been made by Futamase and Schutz [FS], see also [CCBN]).

• Find a simple, usable approximation scheme which allows one to determine (accurately) the gravitational radiation produced by an isolated astrophysical system. In particular, prove the quadrupole radiation formula. Explicit exact radiative solutions are rare: the best available are the boost-symmetric spacetimes [BJ], which however have singularities on $\mathcal{I}^+$ and zero ADM mass, but do provide explicit smooth metrics near parts of $\mathcal{I}^+$.

• Show that test particles (very small, but finite, bodies) move on spacetime geodesics. This is a famous problem, and there has been extensive investigation of asymptotic expansions — see the discussion in [EJ].

• In what sense does a Regge spacetime [RT],[BrL] — a piecewise linear (PL) manifold with PL metric (=leglengths) — approximate a smooth vacuum spacetime? A similar question applies to the spacetimes constructed by numerical relativity. In [CMS] (see also [FFLR]) it is shown that various curvature measures (including the Riemannian Einstein-Hilbert action), defined combinatorially as distributions from PL decompositions of a fixed Riemannian manifold, converge, in an average fashion only, to the corresponding smooth curvature measures. It is not clear that this is the type of convergence that is natural here, since the spacetime is generally not given apriori, but is to be "constructed" from the Regge scheme. In any case [CMS] says nothing about the convergence of the Ricci tensor.

• A problem encountered in numerical relativity is that of ensuring that the constraint equations are preserved by the evolution [SJ]. Morally this problem arises because a lattice formulation does not naturally encode the diffeomorphism invariance of the equations. Devise a reasonable scheme which can be shown to preserve the constraints (cf. [DS]). This may also be thought of as a step towards showing convergence of some numerical scheme to a (long time) solution of the Einstein equations.

Maximal and Prescribed Mean Curvature Surfaces

The basic elliptic *a priori* estimates of [BR1,3,4;GC1] reduce the questions of existence and regularity of maximal (and CMC) hypersurfaces to questions about the existence of coordinates satisfying weak uniformity conditions. However, the satisfaction of these conditions requires knowledge about the long-time behaviour of the spacetime. Since failure of the uniform interior condition of [BR1] in particular implies the interior metric is undergoing unbounded change in some sense, it has been argued that the existence theorems may be turned around to form a "singularity theorem" [COMB],[BR5]. Much more work is needed to understand the conditions.

- Maximal Cauchy surfaces are known to be unique if the timelike convergence condition (TCC) is satisfied [BF,COMB], or if the spacetime admits a timelike Killing vector. There is an unsatisfactory gap between these two conditions, which includes the dominant energy condition. Prove a uniqueness theorem for maximal surfaces, assuming only DE.

- Generalise the estimates of [BR1] to show the existence of a CMC hypersurface asymptotic to a given cut of $\mathscr{I}^+$ in an AF spacetime. What are the weakest asymptotic conditions on $\mathscr{I}^+$ for which this holds? A complete description of the CMC hypersurfaces in Minkowski space is given by [TA;CT].

- Let V be an AF spacetime, having a Cauchy surface without horizons. Show there is a maximal Cauchy hypersurface in V. (The point is we have no coordinate conditions, but the known obstructions to existence of maximal surfaces [Br;WD; BR5] are absent).

- Show that a maximal surface in a boost-type domain [COM] is necessarily AF and must coincide with the maximal slices which can be constructed by [BR1; COMB]. (This is a non-flat generalisation of the Cheng-Yau Bernstein theorem [CY]).

Causality and Singularities

- Is there a timelike geodesically complete, inextendible Lorentz manifold satisfying an energy condition and having a partial Cauchy surface which contains a trapped surface? This must be acausal: the possibilities are restricted [TF].

- Show that a weak Cauchy surface (i.e. a pointwise limit of a sequence of Cauchy surfaces) in a globally hyperbolic spacetime satisfying suitable energy conditions, cannot contain an inextendible null geodesic. This is motivated by the regularity of variationally maximal surfaces [BR3]: such hypersurfaces are smooth and strictly spacelike, except on null geodesics which are inextendible and contained entirely within the hypersurface.

- A spacetime satisfies condition (G) if there is a point whose domain of influence meets every inextendible timelike curve [Ge1;Ga1;BR5]. Show that if condition (G) fails or there is a "hidden infinity" [BR5] in a cosmological spacetime V satisfying TCC, then there is a trapped surface in V. This would give a proof of

Conjecture: *Let* V *be a cosmological spacetime satisfying the timelike convergence condition. Then either* V *is timelike geodesically incomplete or* V *splits as* $\mathbb{R}\times$M *isometrically (and thus* V *is static).*

If V contains a timelike line, then this is the Eschenburg-Yau splitting theorem.

- Prove a singularity theorem assuming the dominant energy condition rather than the timelike convergence condition. The DE condition is the most natural energy condition from a physical standpoint [HE], but unfortunately TCC seems to be the most useful from the standpoint of proving singularity theorems.

- If $g_{ab} \in C^{1,1}$ then the curvature tensor is L^∞ , and the initial value problem (IVP) for geodesics has a unique solution. Apparently, $C^{1,1}$ is essentially the weakest condition for uniqueness to hold [HP]. However, length maximising timelike curves will exist if the metric is merely C^0, and these will be $C^{1,\alpha}$ ($0<\alpha<1$) if the metric is $C^{0,\alpha}$. Is a maximising geodesic unique in this case? Is there a physical interpretation of non-uniqueness of the IVP? In particular, should a metric for which uniqueness fails for the IVP for timelike geodesics, be considered singular?

Initial Value Problem (Cosmic Censorship)

Everyone seems to have their own version of what "cosmic censorship" means. Metaphysically, the aim is to show that the Einstein equations are "physically reasonable". Since there are already exact solutions with closed timelike lines (eg Gödel, NUT), we make a restriction: consider only globally hyperbolic spacetimes (ie. solutions of the IVP). This excludes solutions with gross causality violations, but still leaves some singular possibilities. The Hawking-Penrose singularity theorems [HE] show that singularities (i.e. timelike geodesic incompleteness, Cauchy horizons) are to be expected in general, so the problem reduces to showing that these are physically acceptable. The usual statement is then that a typical observer does not see a singularity (but observers heading for oblivion may be permitted a glimpse at the unthinkable — this is weak CC). Technically, seeing a singularity involves crossing a Cauchy horizon, but this viewpoint is not common when discussing the AF question [ME]. Of course, this discussion supposes we have agreed what constitutes a singularity, but this is not obvious — see the questions on existence/uniqueness for the IVP for geodesics and the vacuum equations. There have been many attempts to prove CC by showing that singularities satisfying certain conditions are not naked. Although some information can be gained from these approaches, this line of argument seems to be avoiding the real issue, which is showing that the "certain conditions" are in fact satisfied by solutions of the Einstein IVP. This amounts to proving long-time estimates for the evolution equations.

- Show a long-time existence theorem for the vacuum AF Einstein equations in the maximal slicing gauge [ME]. For nearly flat initial data this is major work of Christodoulou-Klainerman [CK2]. For large data this would imply the absence of globally naked singularities and that any singularities formed would be "shielded" from the maximal surface evolution by a maximal surface barrier analogous to that found in the Schwarzschild and Kerr solutions [ES]. Christodoulou has proved comprehensive results about spherically symmetric spacetimes with dust [DC1] and massless scalar field [DC2]. One line of attack is to first consider axially symmetric spacetimes, where the vacuum Einstein equations reduce to a coupled Einstein-harmonic map system [KSMH].

- Determine conditions on AF initial data which ensure that the null infinity of the resulting solution of the IVP is sufficiently regular that the Penrose extended manifold exists, with at least C^3 metric so the peeling theorem holds [PR1]. Friedrich has shown that sufficiently small hyperboloidal initial data, satisfying a system of "conformal constraint equations", generates a smooth $\mathcal{I}^+$ which extends to a smooth ι^+ [FH3].

Estimates for the linearised equations [CK1;PS;SB] indicate that the decay rate of the data near ι^0 is important. Finite energy is not enough to provide estimates consistent with peeling [CK1;HL], whilst the conformal method [CBC; CD2] gives solutions which necessarily satisfy peeling, but must have faster decay at ι^0. The results we have for the full vacuum Einstein equations [FH2,3;CK2] appear consistent with these linear results.

- Give an exact "purely radiative spacetime", i.e. a solution to the Einstein vacuum equations which has complete smooth $\mathscr{I}^{\pm}$, regular $i^{\pm}$, and positive mass, or show the existence of such a spacetime. One method is to show the existence of time-symmetric initial data sets which are arbitrarily close to flat and are exactly equal to Schwarzschild near infinity, and then apply results of Friedrich [FH3]. For Einstein-Maxwell this has been carried out by [CW] (this also works for Einstein-Yang-Mills). This is also motivated by the above question about the regularity of $\mathscr{I}^{+}$.

- Find the weakest possible regularity for a vacuum metric [CC;HKM;KT]. As observed by G. Huisken, the Ricci tensor is defined distributionally when the metric satisfies merely $g, g^{-1} \in L^{\infty}$, $\partial g \in L^2$, so a metric satisfying only these regularity conditions could conceivably satisfy the (distributional) vacuum Einstein equations. There are a number of problems of interpretation here, not least of which is deciding whether such a metric should be considered singular. Since the coordinates in which the metric is given originally are not restricted, there is the problem of determining the optimal coordinates (="best differentiable structure"). Results on the regularity of variational maximal surfaces [BR3] may be useful here. The analogy is the use of harmonic coordinates in Riemannian geometry, which fails in the Lorentzian case since the regularity theory cannot generalise.

- By analogy with the IVP for geodesics, there may be regularity conditions for the vacuum Einstein IVP which guarantee existence of a solution, but not uniqueness [CBY] (see the previous question). Should such a (globally hyperbolic!) spacetime be considered singular? Is there an example of (smooth) initial data whose vacuum evolution develops a singularity, but which can be extended, non-uniquely but still satisfying global hyperbolicity, beyond this singularity? This would imply that the maximal Cauchy extension is not unique in general and depends on the function space in which the IVP is posed.

- Show long-time ($\mathrm{tr}K \to \pm\infty$) existence in the CMC slicing gauge for the cosmological spacetime vacuum equations (this is the compact Cauchy surface version of a previous question). For Gowdy spacetimes ($\mathbb{T}^2$ symmetry) this is Moncrief [MV1]. Singularities where $\mathrm{tr}K \to \pm\infty$ are called *crushing*, or of big bang/crunch type [GC]. Note there is a (non-vacuum) cosmological spacetime which does not admit constant mean curvature Cauchy surfaces [BR5].

- Prove that every globally hyperbolic, maximally extended spacetime solution of the Einstein or Einstein-Maxwell equations on $V = \mathbb{R}\times S^3$ contains a maximal hypersurface (and also both a big bang and big crunch). Are there any vacuum or Einstein-Maxwell solutions on $\mathbb{R}\times S^3$ which cannot be foliated by CMC hypersurfaces? (This is a restatement of the previous question). Special case: Prove this result for the spatially homogeneous solutions (Bianchi IX).

- Show that, in an appropriate sense, the set of spacetime metrics which are (smoothly?, distributionally?) extendible across Cauchy horizons are "measure zero" in the set of all spacetimes ([ME;MI], generally asked about spacetimes with compact Cauchy surfaces). Taub-NUT [MC] is the classic example of a spacetime with a very unpleasant, but smooth, Cauchy surface, and which would be branded "non-generic" by such a result.

- Find an exact solution of the Einstein equations which represents two orbiting bodies. Is the 2-body system unstable in Einstein gravity? This is probably the most embarrassing indictment of our (lack of) understanding of the Einstein equations. See eg. [DP] for an asymptotic analysis.

- Determine the class of spacetimes which are, in an appropriately defined sense, asymptotically velocity dominated near the singularity [BKL;ELS].

- Show that the only solution of the vacuum Robinson-Trautman equations on $S^2 \times \mathbf{R}$ with mass > 0, is the Schwarzschild metric [RA;SB2;LPPS]. Can a similar theorem for the Einstein-Maxwell RT equations be shown?

- Physical and linearisation arguments suggest that a perturbation of the Schwarzschild (and Kerr) solution decays exponentially (black holes are stable); see Chandresekhar [Ch] for a comprehensive review. Prove this (cf. [CD1,3] for the spherically symmetric dust and scalar field equations).

- Show that a cosmological spacetime with CMC initial data having positive Ricci 3-curvature, has an evolution which preserves Ricci positivity (S.T. Yau).

Quasi-Local Mass

The obvious generalisation of the Newtonian mass of a body is not useful in Einstein gravity, since it does not detect energy in the vacuum field. The problem of defining the total energy of an isolated system was essentially settled in [ADM], but the correct definition of the energy content of a bounded region in spacetime is still not settled. A number of candidates have been suggested [HS;PR2;LV1;BR6], but for none of these has it been possible so far to verify all the properties one would like a quasi-local mass definition to have [ED]. The payoffs from a good definition would be considerable: a precise measure of gravitational binding energy, the energy content of gravitational waves, ...

- Find a sensible notion of quasi-local mass which appears in non-trivial theorems such as: If a 3-dimensional region with volume V has QL-mass $\geq M_0(V,...)$, then any spacetime containing this region must have a trapped surface and thus a black hole.

- Show the set $\mathcal{PM}$ of AF 3-manifolds which satisfy the conditions of the positive mass theorem [SY1,2;WE] has some weak compactness property, eg. a sequence $\{M_k\}$ with bounded mass and bounded geometry has a convergent subsequence (cf. [OM2] and these proceedings). What regularity might we expect in the limit manifold?

- Let $\Omega \subset (M,g)$ be a bounded region in a $\mathcal{PM}$ manifold, with connected boundary. Show there is another $\mathcal{PM}$ manifold $(M^*,g^*) \supset \Omega$, isometrically, such that (M^*,g^*) is *static* outside Ω (cf. [CW]). This is part of the static metric conjecture of [BR6], which further conjectures that (M^*,g^*) has least ADM mass amongst all those $\mathcal{PM}$ extensions of Ω having no horizon outside Ω.

- Construct a proof of the positive mass theorem, based on Geroch's foliation by 2-spheres idea [Ge2]. There are several variations of this argument [JK;JP; LV2], but these all require the existence of a foliation with special properties — the (highly non-trivial) problem then being that of showing the existence of such foliations. Recently existence results have been given for the inverse mean curvature flow near spatial infinity [CG2;HG;UJ] and for foliations near infinity by constant mean curvature ("round") 2-spheres [HY].

- Show the Bartnik quasi-local mass [BR6] is strictly positive for non-flat data. Show that the Penrose quasi-local mass is non-negative on reasonable data (there are examples [TP] with negative mass).

- Explain the relation between the various definitions of quasi-local mass.

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