LONG BEACH PROBLEMS

The following is a report on what the participants in this conference knew concerning the progress on problems posed at the Long Beach conference, July 1981. The numbers refer to the section Open Questions, pages 460 – 470, of volume 975, Springer Lecture Notes. Only problems on which some change was reported are listed.

3. The answer to the first question is no, see J. C. Candeal and J. E. Galé, On the existence of analytic semigroups bounded on the half–disc in some Banach algebras, *Bull. London Math. Soc.*, to appear 1989.

Some partial results concerning the second part have been obtained by J. E. Galé, Compactness properties of a locally compact group and analytic semigroups in the group algebra, preprint, 1988.

5. There is an obvious misprint – ν should be assumed to be *dis*continuous. Some partial results are in H. G. Dales and R. J. Loy, Prime ideals in algebras of continuous functions, *Proc. Amer. Math. Soc.*, 98 (1986), 426 – 430.

6. The answer to the first part is yes, H. G. Dales and J. P. McClure, Non-standard ideals in radical convolution algebras on a half–line, *Can. J. Math.*, 39 (1987), 309 – 321. See also problem 10 below.

7. No answer yet, but the question motivated the work of F. Ghahramani, S. Grabiner and J. P. McClure, Standard homomorphisms and regulated wights on weighted convolution algebras, *J. Functional Analysis*, to appear.

8. (a) This problem was part of the motivation for S. Grabiner, Homomorphisms and semigroups in weighted convolution algebras, *Indiana Univ. Math. J.*, 37 (1988), 589 – 615.

10. (a) For $l^1(\omega)$ this has been solved in M. P. Thomas, A non-standard ideal of a radical Banach algebra of power series, *Acta Math.*, 152 (1984), 198 – 217, and for $l^p(\omega)$ in M. P. Thomas, Quasinilpotent strictly unilateral weighted shift operators on $l^p(\omega)$ which are not unicellular, *Proc. London Math. Soc.*, (3) 51 (1985), 127 – 145. The weights in this latter paper are regulated, and so the condition of regularity of the weight is not sufficient to ensure that all closed ideals in $l^1(\omega)$ are standard. It is shown in S. Grabiner and M. P. Thomas, Non–cellular strictly cyclic quasinilpotent shifts on Banach spaces, *J.*

Operator Theory, 13 (1985), 163 – 170, that every Banach space contains strictly cyclic, non-unicellular shifts.

(c) Such an example is given in M. P. Thomas, A nonstandard closed subalgebra of a radical Banach algebra of power series, *J. London Math. Soc.,* (2) 29 (1984), 153 – 163.

12. (b) An element $f \in l^1(\mathbb{Q}^+, \omega)$ is *primary* if $f(s) \neq 0$, where $s = \inf \text{supp}\,(f)$. It is not too difficult to deal with closed ideals generated by primary elements. In this direction, let ω be a star shaped weight on $\mathbb{Q}^+$. Then each closed ideal in $l^1(\mathbb{Q}^+, \omega)$ generated by a primary element is standard. This is proved in M. P. Thomas, Weighted convolution algebras associated with subsemigroups of the positive real axis, *J. London Math. Soc.,* (2) 32 (1985), 283 – 296, when a certain growth condition is imposed on the weight, and in the general case by Y. Domar, On the unicellularity of $l^p(\omega)$, *Mh. Math.,* 103 (1987), 102 – 113. It seems to be much harder to deal with closed ideals generated by elements which are not primary, and only fragmentary results are known.

A serious difficulty when one is trying to grapple with convolution algebras over discrete semigroups is the absence of a decent Titchmarsh convolution theorem. A result in this direction is in Y. Domar, Convolution theorems of Titchmarsh type on discrete $\mathbb{R}^n$, *Proc. Edinburgh Math. Soc.,* to appear.

13. (a) No, G. F. Bachelis and S. Saeki, Banach algebras with uncomplemented radical, *Proc. Amer. Math. Soc.,* 100 (1987), 271 – 274, W. G. Bade, The Wedderburn decomposition for quotient algebras arising from sets of non-synthesis, *these proceedings,* 25 – 31.

14. G. A. Willis, The norms of powers of functions in the Volterra algebra, II, *these proceedings,* 350 – 351, gives an example of a weight sequence (ω_n) which cannot be attained as $(\| a^n \|)$ for some $a \in (L^1[0, 1], *)$.

16. The question remains open even when B and $\overline{\vartheta(B)}$ are assumed to be C*-algebras.

17. Any epimorphism from a C*-algebra onto a commutative Banach algebra is continuous, K. B. Laursen, Continuity of homomorphisms from C*-algebras into commutative Banach algebras, *J. London Math. Soc.,* (2) 36 (1987), 165 – 175. It seems likely that any epimorphism from an AF C*-algebra is continuous. Partial results have been obtained in the Masters theses of D. Loch (Saarbrücken) and B. Gudjonsson (Copenhagen).

20. No, K. B. Laursen and M. M. Neumann, Decomposable operators and automatic continuity, *J. Operator Theory* , 15 (1986), 33 – 81, example 4.2.

22. (b) All derivations from $L^1(G)$ into a commutative $L^1(G)$–module are continuous, G. A. Willis, The continuity of derivations and module homomorpisms, *J. Austral. Math. Soc. Ser.* A, 40 (1986), 299 – 320.

(c) The paper just mentioned shows that if all $l^1(G)$ – module homomorphisms from $L^1(G)$ are continuous, then continuity of derivations reduces to the case of discrete groups. The former condition in fact holds true, G. A. Willis, preprint, 1989.

23. No, L. Stedman, Banach algebras with one dimensional radical, *Bull. Austral. Math. Soc.*, 27 (1983), 115 – 119.

24. There are related results in B. Forrest, Amenability and derivations of the Fourier algebra, *Proc. Amer. Math. Soc.*, to appear.

26. Misprints : the implication (7) ⇒ (9) should be removed, and the implication (9) ⇒ (10) should be inserted because it is trivial. G. A. Willis, Examples of factorization without bounded approximate units, preprint, 1989, has shown that (3) fails to imply (2) even for commutative separable *A*. See also P. G. Dixon, Factorization and unbounded approximate identities in Banach algebras, preprint, 1989.

27. (a) G. A. Willis, Probability measures on groups and some related ideals in group algebras, *J. Functional Analysis*, to appear, has shown (Corollary 3.10) that every element of $I_0(\mathbb{F}_2)$ is the sum of two products. Related results may also be found there.

28. (a) The answer to the first part is no, S. Saeki, Discontinuous translation invariant functionals, *Trans. Amer. Math. Soc.*, 282 (1984), 403 – 414.

(b) Considerable progress here :

B. E. Johnson, A proof of the translation invariant form conjecture for $L^2(G)$, *Bull. Sci. Math.*, (2) 107 (1983), 301 – 310.

J. R. Rosenblatt, Translation invariant linear forms on $L^p(G)$, *Proc. Amer. Math. Soc.*, 94 (1985), 226 – 228.

G. A. Willis, Translation invariant linear functionals on $L^p(G)$, *J. Austral. Math. Soc. Ser.* A, 41 (1986), 237 – 250.

J. Bourgain, Translation invariant linear forms on $L^p(G)$, $1 < p < 2$, *Ann. Inst. Fourier, Grenoble*, 36 (1987), 97 – 104.

G. A. Willis, Continuity of translation invariant linear functionals on $C_0(G)$ for certain locally compact groups G, *Mh. Math.*, 105 (1988), 161 – 184.

(c) Some recent developments here :

J. Krawczyk, Continuity of operators commuting with translations for compact groups, *Mh. Math.*, 107 (1989), 125 – 130.

G. H. Meisters, Linear operators commuting with translations on $D(\mathbb{R})$, preprint, 1988.

29. Entries in the second column of the table, questions 4 and 5, should be "yes".

30. Some work on this question is in P. G. Dixon and J. Esterle, Michael's problem and the Poincare-Fatou-Bieberbach phenomenon, *Bull. Amer. Math. Soc.*, 15 (1986), 127 – 187. Their reformulation in terms of complex variables is answerable within ZFC (H. Woodin).

In the paper of J. Esterle in the same Springer Lecture Notes, p. 62, there is a list of questions. The answer to #3 is yes, L. Stedman, The algebraic structure of topological algebras, PhD thesis, ANU, 1985. An example is also given there to show that class III \ class IV $\neq \varnothing$.