# CARBON CYCLE MODELLING: ILLUSTRATIONS OF MODELLING PROBLEMS IN IGBP STUDIES

*I.G. Enting*

## 1. INTRODUCTION

The International Geosphere-Biosphere Program (IGBP) is a proposed multi-disciplinary study of the geosphere-biosphere system. The various elements of this system are the atmosphere, hydrosphere, terrestrial and aquatic biota, soils and sediments. The main elements of the IGBP defined in the 1986 ICSU report [1] are:

- Studies of interactive processes that govern global change.
- Development of a new generation of coupled models of the environment.
- Design of suitable tests to guide the development of these models and the understanding of the processes.
- Programs of observations tailored to provide data needed for these activities.

The present report explores some of the issues involved in modelling and other theoretical studies within the IGBP. The perspective reflects the author's experience in atmospheric science; the global carbon cycle is used to illustrate many of the issues.

The proposed IGBP core projects include two modelling projects: *Modelling Global Biogeochemical Cycles* and *Geosphere-Biosphere Models* [2]. The biogeochemical modelling is introduced as a guide to the more complicated geosphere-biosphere modelling and because the carbon cycle in particular will 'be the core element of any truly global geosphere-biosphere model' [2].

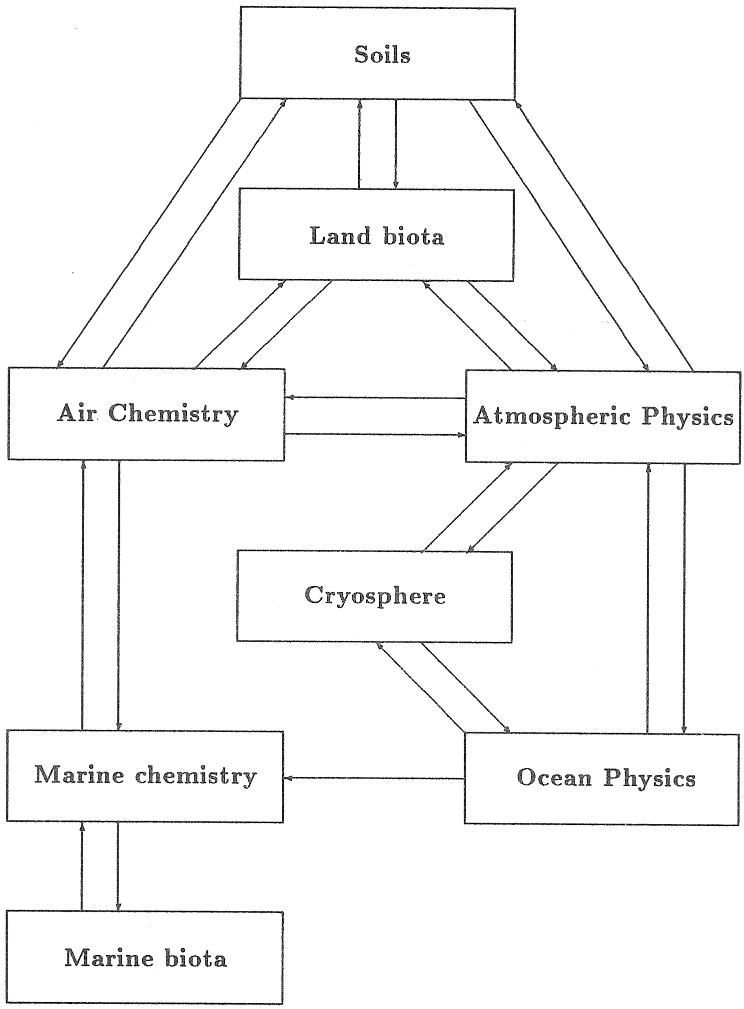


Figure 1: Schematic representation of major components of the geosphere-biosphere system.

The IGBP is an attempt to obtain an overall view of global change as an alternative to piecemeal approaches that have been used to date. Most studies of aspects of global change have been on a problem-by-problem basis, e.g. ozone depletion by supersonic transports, ozone depletion by nuclear testing, ozone depletion by chlorofluorocarbons, the ozone hole, the $CO_2$ greenhouse effect and the augmenting of the greenhouse effect by methane and chlorofluorocarbons. The proposed US Global Tropospheric Chemistry Program [3] is designed to give a degree of understanding of atmospheric chemistry that is adequate for dealing promptly with new concerns as they arise. In the area of ocean chemistry and dynamics a series of integrated programs such as GEOSECS, TTO and the proposed World Ocean Circulation Experiment (WOCE) [4] have been developed to provide data bases for ocean modelling. These integrating programs have been within single disciplines. The IGBP aims to take the process one step further and integrate different disciplines. The aim of this integrative activity could be regarded as Lovelock's 'Geophysiology' which he refers to as 'the earth science' (singular) rather than the traditional terminology of 'earth sciences' (plural) [5].

In the description of the elements of the IGBP, the key word is 'interactive'. It is the interaction between the various components of the geosphere-biosphere system than make a multi-disciplinary approach necessary. Interaction between the various components of the system allows the possibility of complex feedback loops.

The importance of identifying the feedbacks in the system can be demonstrated by comparing two extreme views of the system. The first view is the traditional single-discipline approach in which the various components shown in Figure 1 are studied individually, generally at a finer level of description than the lumped form used in Figure 1. In such studies, the other components are regarded as a specified environment (often fixed in time) that acts on the subsystem of interest but which is not significantly influenced in return. In the absence of feedback loops such an approach could be used to obtain a description of the behaviour of the system as a whole.

An alternative to the 'single-discipline' view of the geosphere-biosphere system is embodied in the 'Gaia' hypothesis of Lovelock [6]. This approach regards the biota as controlling the composition of the atmosphere in such as way as to preserve the habitibility of the planet in the face of perturbing influences. In later work, Lovelock [5] has noted that in systems where such negative feedback leads to homeostasis, it is frequently possible for the feedback processes to 'saturate' so that a sufficiently strong perturbing influence can overwhelm the controls and cause the system to change abruptly to some new state. Broecker [7] has warned that the relation between $CO_2$ content and deposition temperature obtained from polar ice-cores indicates that such an instability may occur in the subsystem involving the dynamics and $CO_2$ content of the atmosphere and the dynamics and carbon content of the oceans.

The bulk of this report (Sections 4 to 7) explores various problems that can be expected in modelling the geosphere-biosphere system. Section 2 lists these problems. Some of them were identified in the 1986 ICSU report [1]. Others are suggested by experience in the development of atmospheric composition studies and carbon cycle studies in particular. Carbon cycle modelling is discussed in Section 3; it is used to illustrate many of the problems that are discussed.

Sections 4, 5, 6 and 7 explore, respectively, the problems of different model type, mismatch in time- and space-scales, the difficulties of ill-posed problems and the possibility of new modes of chaotic behaviour. Section 8 reviews some significant problems involving linkages between different components of the geosphere-biosphere system.

## 2. PROBLEMS

The 1986 ICSU report [1] identified two important problems in the study of the geosphere-biosphere system:

- The differences between the spatial scales usually involved in studies of individual components of the system.
- Differences in the time-scales involved in the various components.

There are, however, a number of other methodological problems that can be expected when undertaking studies of the geosphere-biosphere system:

- Many studies are going to involve the solution of ill-posed problems in which one is trying to deduce properties that are extremely sensitive to small errors in measurements and small uncertainties in our knowledge of the system behaviour.
- There is a severe mismatch between the types of model possible for the different components of the geosphere-biosphere system. Karplus [8] suggested classification of models along a spectrum ranging from 'black-box' curve-fitting models through to 'white-box' fully-deductive models. Currently models of various components of the geosphere-biosphere system range right across this modelling spectrum.
- It is possible that the existence of the feedback loops shown in Figure 1 allows the system to exhibit new modes of chaotic dynamics that are additional to the chaotic behaviour that occurs in individual components of the system. Such chaotic behaviour is a quasi-random evolution of deterministic systems. Chaotic behaviour has been extensively studied, especially in connection with the theory of turbulence and so is well known in atmospheric and oceanic dynamics. A number of key references were collected by Cvitanovic [9].

| Reservoir | Size, Gt C | Turnover, y | Rate of change |
|---|---|---|---|
| Atmosphere | 614 | 4 | 3 |
| Land biota | ≈700 | ≈10 | ? |
| Soils | ≈1500 | | |
| Ocean surface | 1000 | ≈8 | ≈0.5 |
| Deep oceans | 30000 | > 100 | ? |
| Fossil fuel | 7000 | ? | -5 |
| Reactive $CaCO_3$ sediments | 2500 | 10000 | ? |
| $CaCO_3$ | $3 \times 10^7$ | $10^7$ | ? |

Table 1: The main reservoirs involved in the global carbon cycle, together with current carbon contents (in Gt C), and turnover time and net carbon exhanges (in Gt C $y^{-1}$)

- The development of the 'new generation of coupled models' is likely to encounter considerable difficulty and require new methodologies for calibrating and validating models when the calibration process is inherently ill-conditioned and the system may exhibit chaotic dynamics.

These various difficulties are described in more detail in subsequent sections, and illustrated by relevant examples from previous studies of the carbon cycle and other components of the geosphere-biosphere system. It is important to note this list of problems is based on such experience. The possibility remains that studies of the geosphere-biosphere system may encounter further methodological problems that are qualitatively different from any encountered in the study of particular subsystems.

## 3. CARBON CYCLE MODELLING

Table 1 lists the various components involved in the global carbon cycle, and using data based on the study by Sundquist [10] gives the approximate size of the carbon reservoirs, their estimated rates of change ( i.e. net flux into the reservoir) and their turnover times, defined as the ratio of reservoir size to gross carbon exchanges with other reservoirs.

Three-dimensional models of atmospheric transport have been used to look at the seasonal variation of $CO_2$ [11]. These models compute the transport using either meteorological data or else the transport fields calculated by a general circulation model (GCM) of atmospheric dynamics. Ocean general circulation models have been used to calculate the uptake of $CO_2$ into the deep oceans [12]. Because of the very different time-scales involved in the atmosphere and the ocean, these two types of GCM-based transport models of aspects of the carbon cycle have not been run coupled together and it seems inappropriate to do so; in each case a parameterised form of the other model's behaviour can be used as a boundary condition of the other.

Two-dimensional models of atmospheric transport are more flexible than the corresponding three-dimensional models. In particular it is possible to run such models in an inverse mode, specifying a zonally averaged surface $CO_2$ concentration and calculating the corresponding surface sources [13, 14]. When run in this mode, the models can be run as 'atmosphere-only' models. When atmospheric transport models are run in the forward mode, the surface sources need to be specified. At the very least, an ocean surface model is needed as in the work of Pearman and Hyson [15]. Even in this case the treatment of transfer of carbon into the sub-surface layers of the ocean must be parameterised in some way.

Traditionally the combined global carbon system in the atmosphere, the oceans and the land biota has been modelled using coarse lumped 'box-models' [16, 17]. These are used to study changes on time-scales of decades to centuries. Sundquist [10] presents a range of models of this type, including several that include the carbonate sediments and which are therefore applicable to geological time-scales.

## 4. THE MODELLING SPECTRUM

Karplus [8] defined a spectrum of mathematical modelling according to the amount of inductive as opposed to deductive information embodied in the models. This spectrum

| Quantity | Reservoir | Space | Time interval | Coverage |
|---|---|---|---|---|
| $CO_2$ conc | Atmosphere | 30 sites | $\leq$ 1 month | $\approx$1980 onwards |
| $CO_2$ conc | Atmosphere | 2 sites | $\leq$ 1 month | 1958 onwards |
| $CO_2$ conc | Air in ice | several sites | $\approx$10 y | 1700 – present |
| $CO_2$ conc | Air in ice | few sites | $\approx$1000 y | 160000 BP to present |
| $^{13}C$:$^{12}C$ | Atmosphere | Several sites | $\leq$ 1 month | last 5 years |
| $^{14}C$:$^{12}C$ | Atmosphere | Several sites | $\approx$1 month | 1960 – present |
| $^{14}C$:$^{12}C$ | ocean surface | Several regions | $\leq$ 1 month | 1906 on |
| $^{14}C$:$^{12}C$ | Ocean | Few cross sections | one-off | 1973 |
| $^{14}C$:$^{12}C$ | Tree rings | Many sites | $\leq$ 5 y | several centuries |

Table 2: Typical data for carbon cycle modelling

runs from fully-inductive 'black-box' modelling through various shades of grey to fully-deductive 'white-box' models. One important property associated with the spectrum is the range of validity of a model. An inductive model can only be expected to be valid for those conditions from which it was calibrated. In contrast, models that are developed by systematic deduction from general physical laws can be expected to be valid over the whole domain of applicability of such laws.

Karplus noted that particular subject areas tended to occupy specific positions on the modelling spectrum. However, a contrasting viewpoint was given by Enting [18] who gave examples of aspects of the carbon cycle using models spanning virtually the whole range of the modelling spectrum.

As noted in Section 2, the models that have usually been applied to individual components of the geosphere-biosphere system have come from widely scattered points on the modelling spectrum. A few examples will illustrate the range.

*Atmospheric dynamics* In principle the dynamical behaviour of the atmosphere is close to being a white-box system. The behaviour of the atmosphere is governed by basic physical laws: Newton's laws of motion, the equation of state for air as a function of moisture content and laws of radiative transfer. The boundary conditions at the earth's surface, involving friction, radiation and exchange of water pose some difficulties. One of the most serious difficulties in modelling atmospheric dynamics is that the spatial scale of atmospheric motions ranges from turbulent motions on the scale of metres to organised hemispheric circulations with corresponding variations in time scales of minutes to years. Because of the non-linear interactions, the effects of such sub-grid scale turbulence must be parameterised empirically. Another parameterisation is involved in the description of cloud cover in global-scale models, because clouds generally form on scales smaller than any currently realistic grid-scale. Empirical parameterisations can be derived inductively but, as noted above, this can limit the extent to which the model can be applied under different climatic conditions. The questions of whether such parameterisations remain valid under changed climatic conditions represents one of the major doubts concerning the applicability of general circulation models of the atmosphere to studies of climatic change. In spite of these problems, general circulation models of the atmosphere are close to the 'white-box' end of the spectrum.

*Atmospheric chemistry* Modelling the behaviour of chemically active constituents of the atmosphere is subject to a number of difficulties. Almost invariably very large numbers of distinct chemical reactions are taking place and often the rate constants (and their dependence on temperature) are poorly known. It is generally necessary to truncate the system to 100 or so reactions without there being any systematic technique for evaluating the importance of those reactions that are excluded. The unpredicted appearance of the so-called 'ozone-hole' is an example of the surprises that may be in store in this area. (The 'ozone-hole' is apparently the result of chemistry on ice in polar stratospheric clouds — such inhomogeneous processes had not been included in atmospheric models at the time the hole was detected.)

*Ocean dynamics* The development of ocean general circulation models from first principles is at a less developed stage than the development of general circulation models of the atmosphere. There are a number of reasons for this. One is that both temperature and salinity variations give density changes that can drive ocean circulation. A more important difference is the smaller scale of the most dynmaically important eddies. This not only makes modelling more difficult but also has the further effect of limiting the representativeness of observational records which are in any case far sparser than the corresponding atmospheric records.

A consequence of the difficulties involved in modelling the circulation of the ocean is that most studies of ocean chemistry have used highly simplified parameterisations of ocean transport of minor constituents. A prominent example is the 'box-diffusion model' of Oeschger et al. [16] which was developed for global carbon cycle studies. This model simply represents the world ocean as a single, spatially uniform surface layer overlying a water mass with purely diffusive transport. This approach to modelling ocean uptake of atmospheric constituents corresponds very much to a grey-box region of the spectrum of models. It is not entirely inductive since it is assumed that different tracers can be taken up in similar ways. In particular, these models are often calibrated using the response to the $^{14}C$ from nuclear tests. Broecker et al. [19] suggested recalibrating the diffusion parameter on the basis of tritium from thermonuclear weapons tests.

At the 'black-box' end of the spectrum, statistical modelling has an important role. Much of the analysis of the carbon cycle uses signals that have been obtained by statistical processing of observational data (often as time series). Proper statistical analysis and processing requires the use of a statistical model (either implicitly or explicitly) and such models need to be validated for that task. This is not often done in studies of atmospheric composition. For $CO_2$, the WMO Hilo meeting [20] represents a useful step towards a sounder approach to statistical analysis.

## 5. MISMATCH BETWEEN SPACE AND TIME SCALES

The differences in space and time scales between the various components of the geosphere-biosphere system was noted as a problem in the 1986 ICSU report [1]. There is already a large body of experience with problems of discordant time scales that should give some guidance in dealing with these problems. Hopefully some of the techniques can be used to deal with mismatched space-scales. Various previously studied contexts in which discordant time-scales are involved are:

*Atmospheric chemistry* Problems with time-scales in atmospheric chemistry arise from the large variations in rate constants for chemical reactions. In some cases these problems can be handled by numerical techniques designed for 'stiff' differential equations. In other cases the behaviour can be approximated by assuming that a set of 'fast' reactions can always be regarded as having proceeded to equilibrium and then the resulting equilibrium conditions can be used to specify the concentrations of constituents involved in the 'slow' reactions.

*Atmospheric dynamics* There are a number of contexts in which problems in atmospheric dynamics involve a very wide range of time scales. As an example, Holton [21] describes how numerical forecasting calculations have to remove the fastest dynamical modes of the atmosphere (sound and gravity waves) allow sufficeintly large time step in the numerical solution.

*The global carbon cycle* Sundquist [10] presents an analysis of an hierarchial series of carbon cycle models describing the periods ranging from decadal time scales for the atmosphere/ocean/biosphere system through to time-scales of $10^5$ years or longer. A matrix-eigenvector analysis was used to relate various models in the sequence which were derived by successive combination of reservoirs.

In general the mis-match between space and time scales becomes important when non-linear interactions are involved — for linear interactions, we can work with averages. Unfortunately some of the most strongly non-linear interactions occur between the biota and the atmosphere (both physical and chemical) where much of the atmospheric influence comes from relatively rare extreme events.

## 6. ILL-CONDITIONED INVERSE PROBLEMS

There has been an increasing recognition of the importance of ill-conditioned inverse problems in studies of many components of the geosphere-biosphere system. In appreciating the role of inverse problems it is necessary to make a distinction between descriptive or forward modelling as opposed to deductive or inverse modelling. In this context, 'deductive' refers to the way the model is used whil in Section 4, it refers to the way the model is constructed. Thus forwardmodelling is involved with the 'whiter' end of the modelling spectrum defined in Section 4 above. A system is modelled by assuming a complete specification of all the model processes and the outputs are compared to real-world observations. In inverse modelling, some or all of the processes are incompletely specified and the comparison between the model outputs and real-world observations is used as a basis for adjusting the model until agreement is reached. Allison [22] has described the reasons why this deductive inverse approach is potentially unstable. The argument is that for numerical or mathematical modelling to be practical, the system must be such that it can be usefully approximated. The outputs of the model must be relatively insensitive to incompleteness or inaccuracy in the model specifications and to incompleteness or errors in the inputs driving the model. Relatively large amounts of error or incompleteness in both the model and its inputs should give only small changes in the outputs. It follows that if a model with these properties is being used in the inverse mode of determining aspects of the model structure or its inputs from a comparison between observations and model outputs then small amounts of error or incompleteness in the observations will lead to large errors in the quantities determined in the inverse calculation. Allison [22] noted that in mathematical terms, the ability to sensibly approximate a system corresponds to the requirement that the behaviour of the system be described by a compact operator. The difficulties with the inverse problem arise from the fact that no compact operator has a bounded inverse.

Craig and Brown [23] discuss the methodological problems of fields of study in which inverse problems are common. It is usually the case that if only indirect information is available about some physical system then only a projection of the system's characteristics will be obtainable. A description of a general mathematical approach to the linear inverse problem is given by Jackson [24]. Techniques for dealing with ill-conditioned inverse problems are well-advanced in seismology and other fields involving remote sensing [25]. There is also a significant history of using temperature and salinity variations to deduce ocean circulation. A more recent biogeochemical study attempted to use multiple tracers to determine the broad scale features of ocean circulation [26]. This attempt encountered some consistency problems involving phosphorus distributions. Mansbridge and Enting [27] re-examined the calculations to see whether the problem might have arisen from insufficient smoothing but they concluded that the problem must lie in some limitation of the model.

In the time-domain the most common inverse problem is the deconvolution problem. For a linear system the amount of tracer remaining in a reservoir such as the atmosphere at a time $t$ after a unit impulse source is given by a response function $R(t)$. This allows one to relate the time history of concentrations $c(t)$ to a time history of sources by

$$c(t) = c(t_0) + \int_0^t R(t-t')s(t')dt'. \tag{1}$$

Enting and Mansbridge [28] discussed the inversion of (1) in connection with the interpretation of $CO_2$ data from ice-cores [29]. In particular they noted:

(i) there is an explicit inversion relation of the form

$$s(t) = \dot{c}(t)/R(0) + (c(t) - c(0))\dot{R}(0)/R(0)^2 - \int_0^t K(t-t')[c(t') - c(0)]dt' \tag{2}$$

where $K(t)$ is a smoothly varying inversion kernel.

(ii) the instability in the inversion arises solely from the occurrence of a derivative of $c(t)$ in the first term of (2);

(iii) approximating $R(t)$ by sums of exponentials leads to $K(t)$ being also a sum of exponentials.

(iv) the inversion relation (2) shows clearly how uncertainties about past $CO_2$ concentrations will influence estimates of present $CO_2$ sources.

For spatial problems a greater diversity of inverse problems occurs; some of these were discussed by Enting (1985). A more comprehensive analysis is given by Newsam and Enting (1988). They considered the problem of deducing surface sources of a tracer such as $CO_2$ from measurements of atmospheric concentrations approximating atmospheric transport as a purely diffusive process. They found that the the noise amplification depends roughly linearly on the wave number $k$. Enting [30] and Enting and Mansbridge [13] noted the linear $k$-dependence for inversions using a two-dimensional advective-diffusive model.

Newsam and Enting [31] also considered the inversion of high-altitude data e.g. upper troposphere $CO_2$ data. The inversion problem is worse-conditioned than any order of differentiation: the noise amplification grows as $[k \sinh(k)]^{1/2}$. Thus, the concentrations are only significantly affected by the global-scale features of the sources. Therefore, so long as the inversion is constrained to produce only such global-scale features, they can be deduced without being concerned about interference from smaller-scale details. Enting and Newsam [32] also considered the extent to which ground-based observations can give information about processes in the free atmosphere.

Enting [33] and Enting and Pearman [34] have pointed out the poorly-conditioned nature of the calibration of carbon cycle models — often the data are insufficient to distinguish between alternative processes. They used a Bayesian approach to model calibration. Another interesting aspect of that work (see also [35]) was their use of the models to analyse the extent to which new observations could resolve the uncertainties. They did this by performing model calibrations, combined with sensitivity studies while including various items of 'hypothetical data' to see if such data could reduce the sensitivity of the model. Since we can expect many aspects of the geosphere-biosphere system to be poorly-defined, this type of modelling may be used to plan observational programs and therfore could play a key role in the IGBP.

## 7. CHAOTIC DYNAMICS

In recent years it has been recognized that many deterministic dynamical systems can exhibit apparently random behaviour. This can occur in quite simple systems: the minimum requirements are three first-order differential equations with a non-linear coupling. A classic example of this type is given in the study by Lorenz [36]. In such chaotic systems states that are close initially tend to diverge exponentially with time implying that the detailed evolution of the system is essentially unpredictable after some characteristic period. Lorenz [37] has studied the implications of this type of behaviour on the limits of weather forecasting; the existence of chaotic behaviour in the atmsophere places very severe limitations on our ability to predict the weather at a specified time.

However this limitation does not automatically make it impossible to compute climatic averages under either present or future climatic forcing. A more serious problem for the prediction of climatic change is the role of chaotic behaviour in the ocean or in the combined atmosphere-ocean system. The role of such variability is currently unclear. The field of chaotic dynamics is currently an extremely active field of research particularly in connection with the problems of turbulence.

Within the geosphere-biosphere system it is obvious that chaotic dynamics occurs in many of the individual components. The atmosphere has already been noted. Turbulent eddies also play an important role in the oceanic circulation. Chaotic dynamics can also occur in biotic systems of the predator-prey type. If, as is common, time-lagged interactions are considered as representing the maturing of individuals then chaotic behaviour can occur with fewer than three components [38].

As implied by the examples given above, this chaotic behaviour is relatively well-known in many of the relevant subject areas. The most important question for the IGBP is whether coupling the various subsystems allows any new modes of chaotic behaviour as opposed to merely shifting the parameters describing the behaviour of individual subsystems.

The behaviour of the carbon cycle itself, seems to be too highly damped to support chaotic oscillations. There is however, a suggestion by Ghil [39] that chaotic behaviour may be involved with the interactions between $CO_2$ and climate. The feedback path is discussed further in the following Section but it may be summarised as 'climate affects ocean circulation which affects ocean chemistry which affects atmospheric $CO_2$ which affects climate'. This feedback provides a way of amplifying the relatively weak radiative effects of variations in the earth's orbit to a sufficient degree to cause the ice ages. Ghil suggests that the natural behaviour of this interacting system could be some form of chaotic behaviour so that the astronomical forcing would produce a response that varied from one time period to the next. This suggestion could give an explanation of some of the anomalies observed in the relation between paleoclimatic records and the astronomical cycles. The first anomaly is that the relations are apparent in the frequency domain but are less obvious in the time domain — this could be expected if the behaviour of the system corresponded to periodic forcing of a chaotic oscillator. The second anomaly is that the relative importance of the various frequencies in the paleo-climatic record does not reflect the ordering of the relative radiative effects of the astronomical variations – again this could be explained by reference to the instrinsic dynamics of the $CO_2$-climate system.

## 8. CURRENT PROBLEMS

A problem of current concern is the possibility of a closed feedback loop in the $CO_2$ climate connection. Such a possibility is suggested by the fact that $CO_2$ concentrations (as determined from bubbles trapped in polar ice) were typically 200 ppmv during glacial periods. Since $CO_2$ is an important greenhouse gas, this result implies that the reduction in $CO_2$ contributed to the lowering of temperatures. However the correspondence between glacial periods and the characteristics of the earth's orbit suggest a primary astronomical forcing (subject to the qualifications in the previous section). The $CO_2$ feedback can resolve the problem that orbital changes alter the radiation input to the earth by an amount that is insufficient to account for the temperature differences between glacial and interglacial periods. In addition, the attribution to $CO_2$ of a major role in the glacial inter-glacial cycle explains why glaciations were synchronous between the hemispheres even though the primary astronomical forcing had opposing phases in each hemisphere.

Various mechanisms have been proposed for the lowering of $CO_2$. Currently the most plausible involve changes of ocean circulation giving changes of ocean nutrient balance causing changes in the marine biota and thus in the carbon distribution in the ocean surface and the atmosphere.

This feedback is of current concern for two related reasons. Firstly, a positive feedback between $CO_2$ and climate could amplify the greenhouse effect due to fossil fuel use. Secondly there is a growing body of evidence that the ocean circulation has at times changed quite abruptly, particularly around the end of the last glaciation. It is thus conceivable that such abrupt changes could occur in response to anthropogenic $CO_2$. As noted above, such abrupt change can be characteristic of the response of homeostatic systems when subjected to a forcing that exceeds the adaptive limits of the system.

In connection with these problems, an important question is whether the ocean circulation and chemistry is currently represented by an equilibrium steady state plus a chemical perturbation from fossil carbon. There are a number of aspects of this question that suggest further investigation. Firstly, Enting and Mansbridge [40] showed that there was no possible linear steady-state ocean model that could reconcile the biotic $CO_2$ release estimates by Houghton et al. [41] with a history of $CO_2$ concentrations from ice cores obtained by Neftel et al. [29]. Enting and Mansbridge favoured the explanation that biotic release estimates were either in error or missing some processes but noted the possibility of a time-varying ocean circulation, possibly representing a recovery from little ice-age conditions. Given the importance of phosphorus in many theories of the climate-$CO_2$ feedback it is interesting to note that the multitracer inversion of Bolin et al. [26] (which assumed a steady-state ocean circulation) had most difficulty accounting for the observed phosphorus distribution. Further theoretical studies of these questions seem desirable. In addition it would be valuable to conduct more detailed studies using ice-cores to establish $CO_2$ changes through the onset, duration and recovery from the little ice-age. Such a study might provide an example of the climate $CO_2$-feedback as well as helping answer the question of whether the ocean circulation and chemistry is (i) in a steady state; (ii) still recovering from changes connected with the little ice age, or (iii) responding to climatic consequences of the increases in $CO_2$. This last possibility seems unlikely at present given the lack of clear detection of a $CO_2$-induced increase in atmospheric temperature. However it is a possibility that must be given increasing consideration as time goes on.

A recent study that raises the possibility of such changes is that of Tans et al. [42]. Their argument has three main steps;

- Inversion of atmospheric $CO_2$ data suggests that the southern oceans are only a weak sink of $CO_2$ (in proportion to their area) while there is a strong sink at mid latitudes of the northern hemisphere

- Ocean $p_{CO2}$ data indicates that northern oceans are not a strong $CO_2$ sink, implying that the sink must be in the terrestrial biota

- Integration over the oceans gives a net sink of 0.5 to 1.0 Gt C $y^{-1}$

There are three distinct interpretations of these results:

- The analysis is incorrect.

- The oceans have always been a weak sink of $CO_2$. This seems to be inconsistent with radiocarbon data from the period after nuclear testing unless some key process has been neglected.

- The oceanic uptake has changed so that the oceans are taking up less $CO_2$ than previously.

One fact that points towards a possible change in oceanic $CO_2$ uptake is that the modelling study by Pearman and Hyson (1986) found that the atmospheric $CO_2$ data were consistent with the southern oceans being a strong sink while the studies by Enting and Mansbridge [13] and Tans et al. [14] indicated a weak southern ocean sink. While there were important differences between the models used, a major determinant of the differing conclusions was differences in the $CO_2$ data. The earlier data analysed by Pearman and Hyson [15] had a much greater change between the south pole and the equator than was apparent in the later data. This may indicate a genuine change but it could equally well reflect calibration problems in the earlier records.

## 9. CONCLUSIONS

The conclusion of this review is that theoretical studies within the IGBP will require the development of new methodologies for modelling. There is a need for development in the fields of numerical modelling, mathematical analysis and statistical modelling.

The development in numerical modelling must deal with the problems of mismatches in space and time scales. Mathematical analysis is needed as a guide to the appropriate use of numerical models. In the area of fluid dynamics of the atmosphere and ocean there is a large body of analytic work that can provide paradigms for interpreting the behaviour of numerical models in terms of such concepts as internal gravity waves, Rossby waves, Kelvin waves, etc. In other fields such as atmospheric chemistry such a tradition is lacking although simplified low-resolution models can be of some use in interpreting the results of large numerical models.

## 10. ACKNOWLEDGEMENTS

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CSIRO Division of Atmospheric Research
Private Bag No.1,
Mordialloc, Victoria, 3195,
Australia