SIMILARITY SOLUTIONS FOR NONLINEAR DIFFUSION AND RELATED PHENOMENA

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ABSTRACT

The nonlinear diffusion equation with general diffusivity $D(c)$, namely

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D(c)\frac{\partial c}{\partial x}\right), \tag{1}$$

arises in many important areas of science and technology and considerable attention has been given to the power law diffusivities $D(c) = \alpha(c+\beta)^m$ where α, β and m denote constants or with appropriate change of variables simply $D(c) = c^m$. It happens that the few known exact solutions of

$$\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(c^m\frac{\partial c}{\partial x}\right), \tag{2}$$

turn out to be similarity solutions, even though originally they were not derived as such. In this talk we present an overview of recent results relating to similarity solutions for nonlinear diffusion and related phenomena.

In particular we consider the general nonlinear diffusion equation

$$\frac{\partial c}{\partial t} = x^{\ell}\frac{\partial}{\partial x}\{c^m x^n |\frac{\partial c}{\partial x}|^p \frac{\partial c}{\partial x}\}, \tag{3}$$

which includes a wide variety of different physical processes, where ℓ, m, n and p denote arbitrary constants. We also consider high order nonlinear evolution equations of the form

$$\frac{\partial c}{\partial t} = \sum_{j=1}^{n} \alpha_j \frac{\partial^j}{\partial x^j}(c^{m_j}), \tag{4}$$

for certain real constants m_j and α_j denote any real constants. Equations of this general form include a number of standard equations such as the nonlinear diffusion equation, Burger's equation, the Korteweg-de Vries equations and other equations which have been proposed for the flow of a thin viscous fluid sheet on an inclined bed and fluid flow in porous media.

All of the above equations admit similarity solutions of the general form

$$c(s,t) = x^{\alpha}\phi(x/t^{\beta}), \tag{5}$$

for certain constants α and β and the given partial differential equation may be reduced to an ordinary differential equation. For particular values of α and β this ordinary differential equation admits a first integral and the question arises as to why special integrals exist for these particular parameter values. We show that these first integrals can be alternatively derived making use of the first order partial differential equation for $c(x,t)$ rather than the explicit functional form (5). This approach presents similarity solutions in an entirely new light and by equal utilization of both the partial differential equation to be solved and the first order partial differential equation we are able to deduce previous integrals virtually immediately without reference to the underlying ordinary differential equation.

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