FIRE SPREAD AS A MOVING BOUNDARY

R.O. Weber

1. INTRODUCTION

Attempts to model fire spread go back at least as far as Fons [1]. Most of the models (see Weber [2] for a review) consider only the heat transfer and hence can be classified as Stefan-type models of fire spread. However, following the "combustion theory" approach (e.g. Williams [3]) one can formulate a fire spread model which considers chemical kinetic effects as well as heat transfer.

The two types of fire spread models will be compared using examples and by refering to experimental observations. Including chemical kinetics offers several advantages; in particular the potential to predict extinction phenomena. However, it is difficult to justify favouring one type of model over the other because of the many parameters in each type of model which can only be roughly estimated. Combined experimental and theoretical work is required to test either type of model.

2. STEFAN FORMULATION

Cekirge [4], Fujii *et al* [5] and Pagni [6] have all proposed a formulation of fire spreading which is mathematically similar to an ablation problem (e.g. Crank [7]). The simplest version of this is

$$\rho c u_t = k u_{xx}, \quad s(t) < x < \infty, \; t > 0 \tag{1}$$

$$u(x \to \infty, t) = u_a \tag{2}$$

$$u(x = s(t), t) = u_i \tag{4}$$

$$\rho L \frac{ds}{dt} = k \frac{\partial u}{\partial x}\bigg|_{x=s(t)} + Q \tag{4}$$

The heat input at the boundary between burnt and unburnt matter, Q, drives the fire

front as can be seen from the exact solution for the temperature u, and the speed $\dot{s}$:

$$u = u_a + (u_i - u_a)e^{-\rho c\dot{s}(x-s(t))/k} \tag{5}$$

$$\dot{s} = \frac{ds}{dt} = \frac{Q}{\rho c(u_i - u_a)} \tag{6}$$

The speed $\dot{s}$ is constant. The constant is the ratio of the energy input across the front per unit area of the front to the energy required to ignite a unit mass per volume of the combustible matter. Notice that $\dot{s}$ is independent of k.

In more sophisticated versions of this model the conductivity k is replaced by a non-linear function to model radiation, e.g. Fujii *et al* [5], replace k with $4\sigma\delta u^3$, and append cooling and other terms. However, this does not alter the qualitative behaviour as $k \to 4\sigma\delta u^3$ can be bounded above and below by a positive constant.

The comparison of the results of this type of model with experiments is revealing.

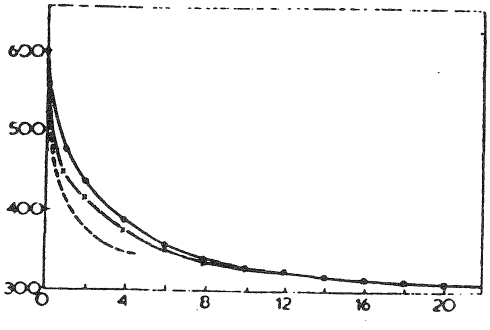

Figure 1: Temperature (°C) versus time (sec) curves from Fujii *et al* [5]. Dashed line is experiment, solid lines are two theoretical models.

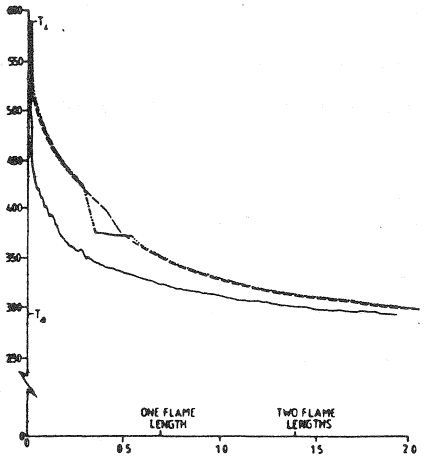

Figure 2: Temperature (K) versus distance from flame (m) curves from de Mestre *et al* [8]. Solid line is experiment, broken lines are two different models.

Figures 1 and 2 show the comparisons made by Fujii *et al* [5] and de Mestre *et al* [8]. In each case it was possible to predict the speed $\dot{s}$ by a judicious choice of parameters. However, it was not possible to reproduce the temperature curves. The experimental curves are always much steeper near ignition. Baines [9] has modified the ignition temperature to 526K (from 593K) to match the temperature curves, but there is no real justification for this *ad hoc* approach. On the other hand, this highlights a significant weakness of these fire models; namely, that there are many parameters which are only approximately known - often only upto a factor of two.

3. INFLECTION POINT MODEL

The temperature-time curves in Figs.1 and 2 for a fuel bed are only given in the interval from ambient to ignition temperature. Therefore, it is of interest to seek temperature-time information during and after the ignition period. Fig.3 shows a full temperature-time curve for a fire spreading on a single, horizontal, ponderosa pine needle.

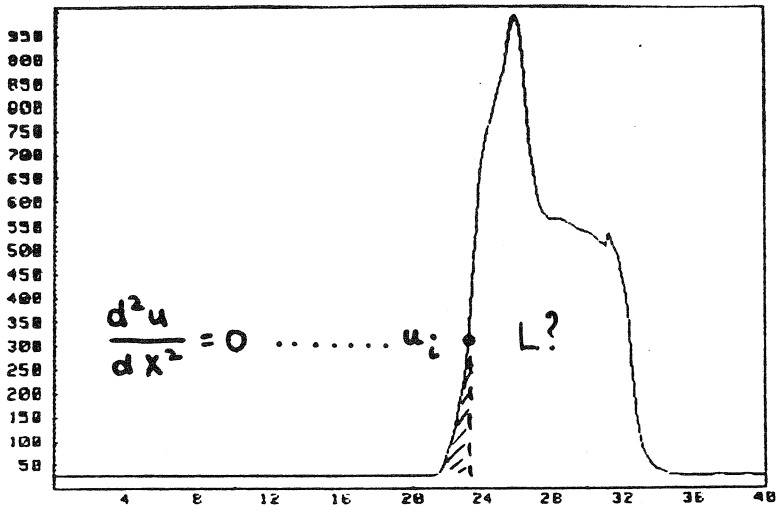

Figure 3: Temperature (°C) versus time (sec) curves for fire spread on a single, horizontal ponderosa pine needle

Notice that at the ignition temperature of approximately 320°C (593K) there is no evidence of any "latent heat" associated with the transition to flaming combustion. This is to be compared with a phase change (such as ice to water) where the temperature-time curve has a flat portion at the phase change temperature.

The point in the temperature-time curve at which ignition occurs appears to be an inflection point (Weber, [10]). This would also tie in nicely with asymptotic methods for determining the flame spreading velocity in reaction-diffusion models, e.g. Williams [3].

An example (which can be compared with the models in section 2) is

$$\rho c u_t = k u_{xx} + Q\delta(x - s(t)) \tag{7}$$

$$\lim_{x\to\infty} u(x,t) \to u_a \tag{8}$$

$$u(s(t),t) = u_i \tag{9}$$

$$u_{xx}\Big|_{x=s(t)} = 0 \tag{10}$$

Obviously, the last equation replaces the Stefan condition of eq.(4), and the heat release at the fire front Q must be placed in the governing partial differential equation, eq.(7).

Transforming to a reference frame which is moving with the fire front,

$$X = x - s(t), \tag{11}$$

and looking for a steady solution in this frame one obtains the fire spreading velocity

$$\dot{s} = \frac{Q + [Q^2 + 4k(u_i - u_a)Q]^{\frac{1}{2}}}{2\rho c(u_i - u_a)}. \tag{12}$$

Comparing this with the prediction from the Stefan model, eq.(6), one notices that now the conductivity k influences the fire spread rate. Furthermore, eq.(6) is obtained from eq.(12) if the limit $k \to 0$ is taken.

This forces the remarkable conclusion that in the Stefan models of fire spread, the spread rate is independent of any diffusive mechanism of heat transfer; while in the inflexion point model the spread rate increases as the strength of the diffusive mechanism increases. Such a sharp distinction should be amenable to experimental tests, if only through glowing combustion such as in a cigarette.

4. THE REACTION REGION

So far, only the preheat region, $u_a \leq u \leq u_i$, has been modelled (even though implications of the whole temperature-time curve were used in section 3). An obvious way to include the reaction region is by a reaction term in the energy equation:

$$\rho c u_t = k u_{xx} + F(u). \tag{13}$$

It is no longer necessary to include a heat release term Q anywhere in the model, as the reaction $F(u)$ provides the energy needed to drive the front. The reaction $F(u)$ must at least be bounded for the system to reach a steady state and it is preferable if it is zero or very small at ambient temperature and zero at some higher final temperature. A nice example of this was studied by Fisher [11] and Kolmogorov *et al* [12] in the context of genetics:

$$\rho c U_t = k U_{xx} + AU(1-U). \tag{14}$$

Here the temperature U has been scaled so that $U = 0$ corresponds to $u = u_a$ and $U = 1$ to $u = u_b$ (burnt or final temperature). A is a constant which measures the energy released when a unit amount of substance is consumed. As proved conclusively by Hagan [13], the initial value problem for eq.(14):

$$U(x,0) = f(x) \tag{15}$$

$$U(x,t) \to 0 \text{ as } x \to \pm\infty \tag{16}$$

has a unique "diffusive wave", and associated with it a unique spreading velocity

$$\dot{s} = 2\sqrt{kA}/\rho c_p \tag{17}$$

provided the initial function $f(x)$ in eq.(15) is non-zero in only a finite range of x; i.e. if $f(x)$ has compact support (other forms of $f(u)$ do not change the qualitative behaviour).

One can see this numerically, e.g. in fig.4 with $k/\rho c = 0.1$, $A/\rho c = 1.0$ and $f(x) = 0.1$ $\mathrm{sech}^2 10x$, whence $s = 2\sqrt{0.1}$. One notices that the inflection point, in the steady front which is ultimately reached, occurs at $U = 0.5$. This corresponds to $u_i = \frac{1}{2}(u_a + u_b)$; the mean of the ambient and the burnt temperatures.

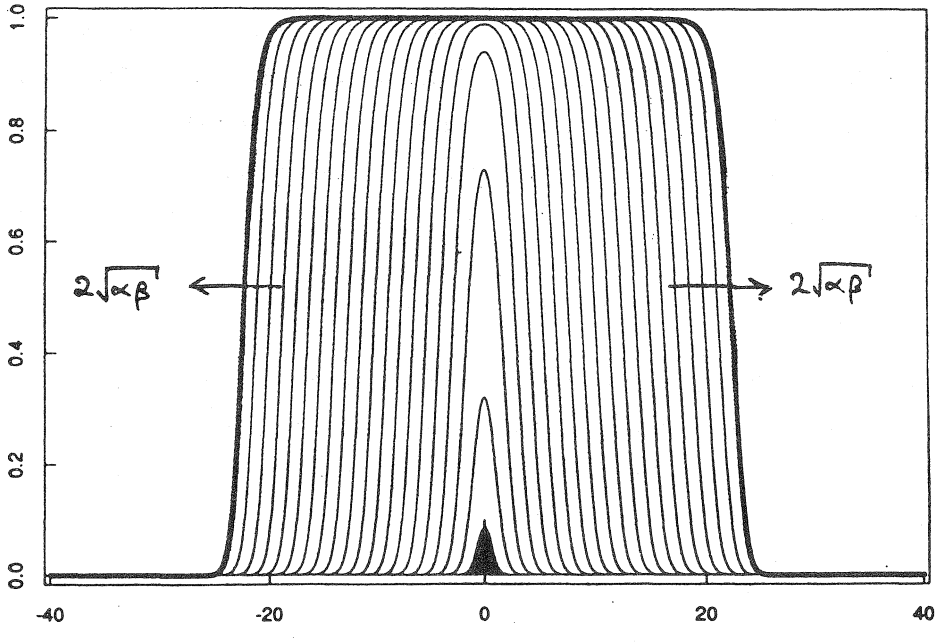

Figure 4: Numerical solution of eq.(14) from Tang and Weber [19].
Each time is represented by one curve

The spreading velocity can be found analytically by linearising about $U = 0$:

$$\rho c U_t = k U_{xx} + AU, \tag{18}$$

then

$$U \sim e^{\frac{A}{\rho c}t} e^{-x^2/(4kt/\rho c)} \tag{19}$$

Balancing these exponentials yields eq.(17).

It is also of interest to note that we have passed from a conductivity independent model in section 2, through an intermediate in section 3, to a model in which the fire does not spread if $k = 0$.

Comparing temperature profiles shows that the experimentally observed rise to a maximum temperature is reasonably well modelled (parameter fiddling will ensure this). However, thereafter the two curves will differ, presumably because the model of eq.(13) ignores any heat loss mechanism such as convective cooling.

Finally, note that the rather imprecise quantity Q has been replaced by A. This latter quantity is also difficult to determine, but at least it is in principal amenable to direct measurement in the laboratory.

5. TOWARDS MORE REALISTIC MODELS : INCLUDING WIND

It is an easy matter to include the effect of bulk advection of hot material in the previous models. One simply includes a term $\rho c \omega u_x$ in the partial differential equation, where ω is the speed of the bulk advective process (wind speed). This merely provides an additional change of reference frame when one calculates the fire spreading velocity. Hence one finds a linear law for the dependence upon windspeed. This seems to work very well for high wind speeds and for elevated fuels, see Catchpole [14]. However, for low wind speeds and ground based fuels there is usually a more complex dependence of fire speed upon windspeed, e.g. Rothermel [15]. The real physical reason for this is not known, however, there are at least two possibilities.

(1) Flame spread over a solid, such as a single pine needle, is at least a two, if not three, dimensional process. Therefore, only when the windspeed is independent of x, y and z will a one-dimensional model suffice. Conversely where there is a significant wind profile, such as near the ground, a two or three-dimensional model will be necessary. This should be valid for fuel beds as well as for single solid fuels (the latter have been studied by di Blasi *et al* [16]).

(2) The models so far presented are only based upon energy conservation equations in the solid phase. However, many studies of fire spread (e.g. Weber and de Mestre [17]) have realised the importance of the interaction of the gas and solid phases. This necessitates the inclusion of at least one other equation, that for energy conservation in the gas phase, together with some coupling between the solid and gas phases. A complicated example has been introduced by Grishin *et al* [18] but the large number of variables obscures the influence of the coupling. Let us therefore consider the following model:

$$\rho_s c_s \frac{\partial u_s}{\partial t} = k_s \frac{\partial^2 u_s}{\partial x^2} + h(u_g - u_s) \tag{20}$$

$$\rho_g c_g \frac{\partial u_g}{\partial t} = k_g \frac{\partial^2 u_g}{\partial x^2} - h(u_g - u_s) + A_g e^{-\Gamma_g/u_g}. \tag{21}$$

The subscripts s and g denote solid and gas respectively, and partial derivatives are indicated in full. A linear coupling with strength given by the parameter h is assumed. The reaction really only occurs in the gas phase so there is only one reaction term and it is in eq.(21). Any analysis of eqs.(20) and (21) is now already much harder to carry out. However, linearising by expanding u_g and u_s about u_a, yields

$$\rho_s c_s \frac{\partial \epsilon_s}{\partial t} = k_s \frac{\partial^2 \epsilon_s}{\partial x^2} + h(\epsilon_g - \epsilon_s) \tag{22}$$

$$\rho_g c_g \frac{\partial \epsilon_g}{\partial t} = k_g \frac{\partial^2 \epsilon_g}{\partial x^2} - h(\epsilon_g - \epsilon_s) + \frac{A_g}{u_a} e^{-\Gamma_g/u_a} \left(1 + \frac{\Gamma_g}{u_a}\epsilon_g\right) \tag{23}$$

where $\epsilon_s = (u_s - u_a)/u_a$, $\epsilon_g = (u_g - u_a)/u_a$.

From eq.(23) one can read off immediately that a fire will not propagate if

$$h > \frac{A_g \Gamma_g}{u_a^2} e^{-\Gamma_g/u_a}. \tag{24}$$

This is just from the terms with ϵ_g and noting that a positive coefficient is required if ϵ_g is to grow. This is already interesting as it provides an extinction condition. Namely, the fire will not propagate if the coupling between the solid and gas is too large, because then the gaseous combustion does not produce enough energy to overcome the heat sink effect of the solid.

This example demonstrates the possibilities offered by models with coupled equations for modelling complex phenomena such as firespread. However, as a final caveat, it should be mentioned that the associated increase in the number of parameters increases the experimental work needed to "calibrate" the model. It is for this reason that one wishes to construct a model with a minimal set of parameters, and that the single reaction-diffusion equation of section 4 deserves further attention.

REFERENCES

[1] W.L. Fons, Analysis of Fire Spread in Light Forest Fuels. *J. Agr. Res.* **72** (1946), 93-121.

[2] R.O. Weber, Modelling Fire Spread through Fuel Beds. *Prog. En. Combust. Sci.* (1990) in press.

[3] F.A. Williams, *Combustion Theory* 2nd ed. Benjamin/Cummings, Menlo Park CA, 1985

[4] H.M Cekirge, Propagation of Fire Fronts in Forests. *Comp. Math. Applic.* **4** (1978), 325-332.

[5] N. Fujii, J. Hasegawa, L. Pallop and Y. Sakawa, A Non-stationary Model of Fire Spreading. *Appl. Math. Modelling* **4** (1980), 176-180.

[6] P.J. Pagni, Flame Spread over Thin Solid Fuelds. *J. Heat Transfer* **97** (1975), 153-155.

[7] J. Crank, *Free and Moving Boundary Problems.* OUP, 1984.

[8] N.J. de Mestre, E.A. Catchpole, D.H. Anderson and R.C. Rothermel, Uniform Propagation of a Planar Fire Front without Wind. *Combust. Sci. Tech.* **65** (1989), 231-244.

[9] P.G. Baines, Physical Mechanisms for the Propagation of Surface Fires. *Math. Comput. Modelling* (1990) in press.

[10] R.O. Weber, Thermal Theory for Determining the Burning Velocity of a Laminar Flame, Using the Inflection Point in the Temperature Profile. *Combust. Sci. Tech.* **64** (1989), 135-139.

[11] R.A. Fisher, The Wave of Advance of Advantageous Genes. *Ann. Eugen.* **7** (1936), 355-369.

[12] A. Kolmogorov, I. Petrovsky and N. Piscounoff, Study of the Diffusion Equation with Growth of the Quantity of Matter and its Application to a Biology Problem. As reprinted in *Dynamics of Curved Fronts*, ed. P. Pelcé, Academic Press, 1988.

[13] P.S. Hagan, Travelling Wave and Multiple Travelling Wave Solutions of Parabolic Equations. *SIAM J. Math. Anal.* **13** (1982), 717-738.

[14] W.R. Catchpole, Heathland Fuel and Fire Modelling. PhD thesis, UNSW, 1987.

[15] R.C. Rothermel, A Mathematical Model for Predicting Fire Spread in Wildland Fuels. Forest Service USDA Res. Paper INT-30, 1972.

[16] C. di Blasi, S. Crescitelli and G. Russo, Near Limit Flame Spread Over Thick Fuels in a Concurrent Forced Flow. *Combust. Flame* **72** (1988), 205-215.

[17] R.O. Weber and N.J. de Mestre, Flame Spread Measurements on Single Ponderosa Pine Needles : Effect of Sample Orientation and Concurrent Flow. *Combust. Sci. Tech* **70** (1990), 17-32.

[18] A.M. Grishin, A.D. Gruzin and V.G. Zverev, Mathematical Modelling of the Spreading of High Level Forest Fires. *Sov. Phys. Dokl.* **28** (1983), 328-330.

[19] S.Tang and R.O. Weber, Numerical Study of Fisher's Equation by a Petrov-Galerkin Finite Element Method. Submitted to *J. Austral. Math. Soc. Ser. B* (1990) in press.

Department of Mathematics
University College
University of New South Wales
Australian Defence Force Academy
Canberra ACT 2600 Australia