

## INVERSE PROBLEMS IN SEISMIC SURVEYS

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## 1. INTRODUCTION

Seismic surveys are commonly used in the exploration for oil and natural gas to help picture the subsurface structure in the region of interest. One way of displaying the data recorded is by means of an unmigrated stacked section (see fig. 1). However, this gives a distorted picture of the subsurface structure because of the refractive effects produced by the variations in velocity. The next step is to determine a good coarse-scale velocity model to use in "migrating" the data to its true location. In this paper, a generalised linear inversion technique is used to derive the coarse-scale velocity model. [1, 2] have used a similar approach in their study of this problem although they worked with a different selection of data from the seismic survey compared with that outlined in this paper. Such an inversion technique addresses a problem inherent in many existing interval velocity modelling methods: velocity errors cumulative with depth. It also permits input data to be weighted according to its accuracy or some other criteria. Weighting of data becomes desirable especially in the over-determined problem and also when mixed data sets are used such as a combination of stacking velocities and two-way travel times from the unmigrated stacked section of seismic data as in this investigation.

## 2. METHOD.

The basic equations are set up in the usual way with a linearised Taylor expansion of the forward modelling procedure (ray tracing) about the current estimate of the model giving:

$$(1) \quad \mathbf{d} - f(\mathbf{m}) \doteq \mathbf{g} - \mathbf{A}\mathbf{x}$$

where  $\mathbf{d}$  denotes the observed data,  $f$  denotes the forward modelling function,  $\mathbf{m}$  denotes the model parameters,  $\mathbf{g}$  is the mismatch in data,  $\mathbf{A}$  is the matrix of partial derivative values and  $\mathbf{x}$  is the correction required to the current estimate of the model.

In a weighted least square approach, we then minimise the weighted sum of the squared residual

$$(2) \quad S = (\mathbf{g} - \mathbf{A}\mathbf{x})^T \mathbf{W}(\mathbf{g} - \mathbf{A}\mathbf{x})$$

with respect to  $\mathbf{x}$ .  $\mathbf{W}$  is a matrix of weighting factors reflecting the accuracy of the data or the relative importance of each type of data to the solution.

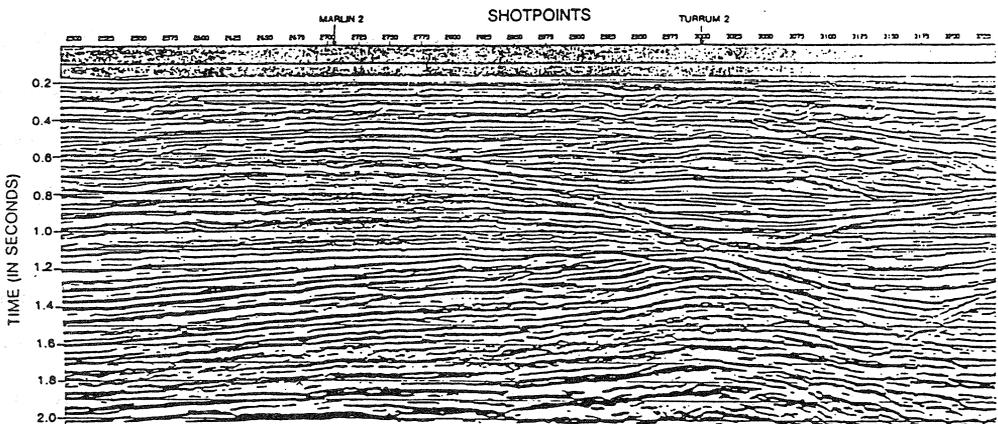


Figure 1. Unmigrated stacked section from seismic survey in Gippsland Basin, an area offshore South-East Australia.

In the overdetermined problem, the solution may be written

$$(3) \quad \mathbf{x} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} \mathbf{g}$$

and one notices the similarity with part of the stochastic inverse solution when the weighting factor is  $C_D^{-1}$ , the inverse of the covariance matrix for the data. [3] has shown that for statistically independent data, the choice of  $\mathbf{W}$  as a diagonal matrix whose elements are the reciprocals of the variances of the data causes the residual for each data point to be compared with its expected error.

### 3. SEISMIC DATA

The data shown in fig. 1 comes from an unmigrated stacked section of a seismic survey in the Gippsland Basin (an area off-shore South-East Australia). The seismic interpreter has identified 8 horizons of interest (including the water bottom) and then performed stacking velocity analyses every 50 shotpoints (that is, at intervals of 1125 metres) between shotpoints 2488 and 3288. "Stacking" adjusts the record to show properties of hypothetical normal rays, that is, the rays which would be reflected at right angles from the reflecting boundaries.

In industry, it has been customary to use the two way travel times  $T_0$  and the stacking velocities  $v_{NMO}$  determined at the 16 stacking analysis sites for the 8 horizons of interest to determine the velocity model for the region underneath the line. The standard technique in industry involves a top down ray-tracing procedure and thus the error in the determination of the interval velocity is cumulative with increasing depth.

Instead, we use a generalised linear inversion technique in order to try and minimise this difficulty. Several iterations are usually required to obtain the solution because of the non-linearity of the forward modelling procedure. Lateral variations in interval velocity are permitted. Assuming the deviation from constant velocity within each layer is small, the curved rays are replaced by

*straight rays with corrections* [5]. The formulae for the corrections to straight ray geometry are obtained by applying the method of small perturbations to the ray and energy transport equations [4]. Nevertheless, the model of the medium is always chosen to have fewer parameters than the number of input data to ensure an overdetermined problem.

#### 4. RESULTS

In earlier studies [6, 7], the seismic line was divided into overlapping sections which were processed independently; the appropriateness of a set of weighting factors for the problem was determined by the criterion that it should enhance the lateral consistency between the solutions obtained all the way across the seismic line. Indeed quite good consistency was found between solutions obtained across the section for both depth and velocity profiles. Furthermore, there were significant similarities in the solutions obtained with different weighting matrices, indicating what might be called the essential features of the model.

The procedure is now being modified to handle data from an entire seismic line simultaneously. Spline representations of boundaries and velocity profiles are being incorporated into the earlier procedure. This should cause significant improvement in the resulting velocity profile because of the reduction of the distorting edge effects which were inherent in the earlier work with its small lateral dimensions. Preliminary results indicate that the procedure gives quite good indication of the shapes of the boundaries producing the reflections of interest in the unmigrated stacked section and so has value as a diagnostic tool, e.g., distinguishing between an apparent anticlinal structure and "pull up" due to a high velocity zone.

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