Dynamics of Fusion and Stellar Plasmas

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Centre for Plasmas and Fluids

RSPE Seminar, 17 November 2016

Acknowledgement: Australian Research Council, ANU
Dynamics of Fusion and Stellar Plasmas

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Fusion, the power of the sun and the stars, is one option

“…Prometheus steals fire from the heaven”

On Earth, fusion could provide:

- Essentially limitless fuel, available all over the world
- No greenhouse gases
- Intrinsic safety
- No long-lived radioactive waste
- Large-scale energy production
Stellar cf terrestrial fusion

**Proton-proton cycle**

e.g. PP1 branch

Overall produces 26.732 MeV

**D-T and D-D reactions:**

Fusion  \((D^2 + T^3 \rightarrow He^4 + n \) \)  17.6 MeV  
\((D^2 + D^2 \rightarrow He^3 + n \) \)  3.27 MeV  
\((D^2 + D^2 \rightarrow T^3 + H^1 \) \)  4.03 MeV

D-T reaction co-discovered by Australian Sir Marc Oliphant 1932
Conditions for *stellar* fusion power

- Classically, the Sun shouldn’t burn
- works by quantum tunneling of wavefunctions through the repulsive barrier
- Probability is small, but the Sun’s volume is enormous

Sun’s core temperature ~ 1.3keV (15C million)
Conditions for *terrestrial* fusion power

- Achieve sufficiently high ion temperature \( T_i \)
  \[ \Rightarrow \text{exceed Coulomb barrier} \]
  \[ \text{density } n_D \propto \text{energy yield} \]
  \[ \text{energy confinement time } \tau_E \]

\[ \tau_E = \text{insulation parameter: e.g. time taken for a jug of hot water to lose energy to the surroundings} \]

- "Lawson" ignition criteria: Fusion power > heat loss

Fusion triple product \[ n_D \tau_E T_i > 3 \times 10^{21} \text{ m}^{-3} \text{ keV s} \]

- Steady-state access requires confinement

\[ \approx 600 \text{ million } ^\circ \text{C} \]
Toroidal Magnetic Confinement

- Magnetic fields cause charged particles to spiral around field lines. Plasma particles are lost to the vessel walls only by relatively slow diffusion across the field lines.

- Only charged particles (D⁺, T⁺, He⁺...) are confined. Neutrons escape and release energy.

- Toroidal (ring shaped) device: a closed system to avoid end losses.

- The most successful Magnetic Confinement device is the TOKAMAK (Russian acronym for ‘Toroidal Magnetic Chamber’).
Components of a Tokamak
Fields in confining plasma

\[ \bigcirc_B \quad \nabla B \quad \text{ion} \quad \text{electron} \]
Fields lie in flux surfaces

- In an “perfect” tokamak field lines lie in flux surfaces
- If magnetic field sufficiently strong ions and electrons bound to flux surfaces
- Different flux surfaces are ~ thermally insulated
- Flux surfaces support pressure gradient
- Tokamaks maximise core pressure, needed to initiate fusion

⇒ *bottles the plasma*
How to obtain the extreme temperatures?

**Ohmic heating:** $\sigma \propto T^{3/2} \Rightarrow$ limited to $T \sim 3$ keV, additional heating needed, which also drives current:

- **Positive ion beams:** $E \sim 100$ keV
- **Negative ion beams:** $E \sim 1$ MeV
"MHD with anisotropy in velocity, pressure"

- Pressure different parallel and perpendicular to field due mainly to directed neutral beam injection
“MHD with anisotropy in velocity, pressure”

- Pressure different parallel and perpendicular to field due mainly to directed neutral beam injection

⇒ Pressure is a tensor
\[ P = p_\perp I + \Delta B B / \mu_0, \quad \Delta = \frac{\mu_0 (p_\parallel - p_\perp)}{B^2} \]

- Inclusion of anisotropy and flow in equilibrium MHD equations


\[ \nabla \cdot (\rho v) = 0, \quad \rho v \cdot \nabla v = J \times B - \nabla \cdot \bar{P}, \quad \nabla \cdot B = 0 \]

\[ \mu_0 J = \nabla \times B, \quad \nabla \times (v \times B) = 0, \]
Tokamak Equilibria with anisotropy & flow

• **New EFIT TENSOR** reconstruction code
  ➢ Add physics of flow/ anisotropy
  ➢ Adds kinetic constraints to magnetic-only constraints of EFIT
  ➢ Showed $J_\phi$ a strong function of transport model
    

• **HELENA+ATF** Add physics of flow/ anisotropy to fixed boundary & profile solver HELENA
  ➢ Written to enable stability studies (Computes equilibrium in appropriate metric)
  ➢ Decomposed $J_\phi$ into
    
    $$J_\phi = R \frac{B_p^2}{B^2} \left( \frac{\partial p_\parallel}{\partial \Psi} \right)_B + R \frac{B^2}{B_p^2} \left( \frac{\partial p_\perp}{\partial \Psi} \right)_B + \frac{1 - \Delta}{2R} \left( \frac{\partial (RB_\phi)^2}{\partial \Psi} \right)_B - R \nabla \cdot \frac{\Delta \nabla \Psi}{R^2}$$

    and showed $J_{p\perp}$ dominates of $J_{p\parallel}$
  ➢ Computed parametric scans of changes in to equilibrium with anisotropy – most significant change is to $J_{pol}$
    
Tokamak Stability Zoo

A whole zoo of modes. Can divide them as:

- **Most-serious (disruptive):** e.g. external modes such as the \((n, m) = (1,1)\) *external kink*, driven by gradients in pressure and current density

- **Serious but tolerable (performance-limiting):**
  - *Sawteeth*, internal kink, \((n, m) = (1,1)\) – reconnection of core. Periodic collapse of central temperature
  - *Alfven eigenmodes*, wave-particle resonance driven. Loss of fast particle confinement
  - *Edge-Localised Modes (ELMs)*, which occur for moderately high \(m\) and \(n\).

  ELM mitigation / suppression demonstrated by application of resonant magnetic perturbation coils, that deliberately perturb edge
Stability: ANU single adiabatic model

• Compressional
  ➢ *Double-adiabatic* (CGL)
    • Collisionless, \( p_\parallel \) and \( p_\perp \) do **independent** work
    • No streaming particle heat flow
    • Does not reduce to MHD in the isotropic limit
  ➢ *New ANU single adiabatic* (SA) model
    • \( p_\parallel \) and \( p_\perp \) doing **joint** work
    • Account for the isotropic part of the perturbation
    • Can reduce to MHD in isotropic limit
      [Fitzgerald, Hole, Qu, PPCF 57 (2015) 025018 ]

• Incompressional

\[
p_{\parallel 1} = -\xi_n \left[ \frac{\partial p_\parallel}{\partial n} - (p_\parallel - p_\perp) \frac{\partial \ln B}{\partial n} \right] \\
p_{\perp 1} = -\xi_n \left[ \frac{\partial p_\perp}{\partial n} - (2p_\perp + \dot{\epsilon}) \frac{\partial \ln B}{\partial n} \right]
\]

* A B Mikhailovskii, Instabilities in a confined plasma, IOP publishing (1998)

• Implemented in CSCAS (CSMIS-A) and MISHKA (MISHKA-A)
  [Qu, Hole, Fitzgerald, PPCF 57 (2015) 095005]
Wave-particle drive: (non)linear dynamics

Evolution of unstable modes in realistic geometry and with simulated distribution functions uses drift-kinetic wave-particle interaction codes, such as HAGIS. [Pinches et al, Comp. Phys. Comm. 111 (1998) 133-149]

Inclusion of anisotropy:
- Formal: change the magnetic structure, equations of motion
- Approximate: use a mapping to match $q(\psi)$ and $l_{\text{total}}$
  ✓ Works provided particle orbits are similar.
Anisotropy on MAST: #29221

- MAST #29221
- 1.6MW NB heating
- \( I_p = 0.9 \text{MA}, \beta_n \sim 3 \)
- Magnetics shows TAEs, tearing modes fishbones, long-lived modes

![Graph showing MAST shot 29221 with various parameters over time.](image)

- Magnetics
  - possible TAE
- Long-lived modes
  - \( n=1 \)
Beam + thermal population: \( p_{\parallel} / p_\perp \approx 1.7 \)

HELENA+ATF / EFIT TENSOR: \( p^* = (p_{\parallel} + p_\perp)/2 \) (isotropic)

HELENA+ATF / EFIT TENSOR: \( p_{\parallel}, p_\perp \) (anisotropic)

\( p_{\parallel}/p_\perp = 1.7 \) at \( s=0.5 \) outboard
Beam + thermal population: $p_{\parallel}/p_{\perp} \approx 1.7$

HELENA+ATF / EFIT TENSOR: $p^* = (p_{\parallel} + p_{\perp})/2$ (isotropic)

- What is the impact on stability due to this q profile?

Beam + thermal population: $p_{\parallel}/p_{\perp} \approx 1.7$ at $s=0.5$ outboard

$\frac{p_{\parallel}}{p_{\perp}} = 1.7$ at $s=0.5$ outboard
Incompressible continuum for MAST

$\text{ANU PTM student Z. Qu}$

MAST #29221 at 290ms.

$n=1, \gamma=0$

isotropic

$R_{\text{mag}} = 0.914$

$f_A(R_{\text{mag}}) = 280\text{kHz}$

anisotropic

$p_{||}/p_{\perp} = 1.7$ at $s=0.5$ outboard

$p_{||}/p_{\perp} = 1.7$

$R_{\text{mag}} = 0.928$

$f_A(R_{\text{mag}}) = 260\text{kHz}$

$\Delta f_{\text{TAE}}^{\text{isotropic}} < \Delta f_{\text{TAE}}^{\text{anisotropic}}$

$\Rightarrow$ anisotropic modes less susceptible to continuum damping
Incompressible continuum for MAST

Z. Qu

MAST #29221 at 290ms.

n=1, γ=0

\[ f = 81.3\text{kHz} \]

Isotropic core mode
\[ f = 83.5\text{kHz} \]

Isotropic global mode
\[ f = 88.9\text{kHz} \]

\[ f_{A0} = 260\text{kHz} \]

\[ f_{A0} = 280\text{kHz} \]
Mode profile broader with anisotropy

Z. Qu

- Might saturation level be different for two cases?
Resonance maps from HAGIS

ANU PTM postdoc B. Layden

- Use linear eigenfunction from MISHKA+ATF to compute resonance maps ($\Omega=0$) of wave/particle evolution

$$\Omega = \omega - n\omega_\phi - p\omega_\theta$$

isotropic,
n=1, p=1,
f = 88.9kHz, $\Lambda = 0.3$

anisotropic,
n=1, p=1,
f = 81.3kHz, $\Lambda = 0.3$
Saturation level for (an)isotropic cases

- Distribution function is slowing down in energy, Gaussian in \( s \) (centred at \( s=0 \)) and delta function in \( \Lambda \) (\( \Lambda = 0.3 \)).

- Anisotropic drive 35% larger than isotropic mode
- Anisotropic saturation 15% weaker than isotropic mode

- First wave-particle interaction study in anisotropic plasmas
- *Lots more to do*: relax delta function in \( \Lambda \)
Other Burning Plasma Results

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• Demonstrated perturbative approach used for last 20 years to compute continuum damping does not converge [G. W. Bowden, A. Könies, M. J. Hole, N. N. Gorelenkov, G. R. Dennis Phys. Plas. 21, 052508 (2014)]

• First calculation of continuum damping in a realistic stellarator (W7X, H-1). [G. W. Bowden, M. J. Hole, and A. Könies, Phys. Plas. 22, 092114 (2015)]

• Used singular finite elements (a first) to compute continuum damping. [G. W. Bowden and M. J. Hole, Phys. Plas. 22, 022116 (2015)]
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• Energetic geodesic acoustic modes associated with two-stream like instabilities in tokamak plasmas, [Z.S. Qu and M.J. Hole, M. Fitzgerald, 116, 095004, PRL]
  ➢ Discovered a new instability (EGAM)
  ➢ Describes wave activity during DIII-D after beam switch-on.
  ➢ one of the referees noted “the work will open up a new way to study and understand energetic particle-driven instabilities in fusion plasmas and make a big impact in fusion research.”
Components of a Stellarator

**Principle:**
- Confining field nearly all produced by external coils
- Twists the field so as to eliminate $\nabla B$ drift
- Intrinsically 3D (no continuous symmetry)

**Consequence:**
- Large toroidal current removed (stability ✓)
- Cross section changes shape with toroidal position (confinement ×)
Wendelstein 7-X

- Aim: evaluate future fusion reactor potential of stellarator
- Quasi-symmetry (lines of constant $B^2$ in natural field coords)
- Opened by Chancellor Merkel in February 2016
New ANU approach to 3D equilibria

• Simplest model to approximate global, macroscopic force-balance is magnetohydrodynamics (MHD).

\[ \nabla p = \mathbf{J} \times \mathbf{B}, \quad \nabla \times \mathbf{B} = \mathbf{J}, \quad \nabla \cdot \mathbf{B} = 0 \]

• Non-axisymmetric \( \Rightarrow \) field does not lie in nested flux surfaces unless surface currents allowed.

• Existing 3D solvers (e.g. VMEC) assume nested flux surfaces.

[CTH stellarator, Hanson et al, IAEA 2012]
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• Generalised Taylor relaxation model: Multiple Relaxed Region MHD (MRxMHD) supports full complexity of field: nested flux surfaces, magnetic islands, chaotic regions.

Volume: \( \nabla \times \mathbf{B} = \mu_0 \mathbf{B} \quad P_l = \text{constant} \)

Interfaces: \( [ [ P_l + B^2 / (2\mu_0) ] ] = 0 \quad \mathbf{B} \cdot \mathbf{n} = 0 \)
MRXMHD approaches ideal MHD as $N \to \infty$

ANU PTM postdoc G. Dennis

Stepped Pressure Equilibrium Code, SPEC

[ANU PTM alumnus S. Hudson]

Vector potential is discretised using mixed Fourier & finite elements

- Coordinates \((s, \varphi, \zeta)\)
- Interface geometry

\[
R_i = \sum_{l,m,n} R_{lmn} \cos(m, \varphi - n, \zeta), \quad Z_i = \sum_{l,m,n} Z_{lmn} \sin(m, \varphi - n, \zeta)
\]

- Exploit gauge freedom

\[
A = A_\varphi(s, \varphi, \zeta) \nabla \varphi + A_\zeta(s, \varphi, \zeta) \nabla \zeta
\]

- Fourier

\[
A_\varphi = \sum_{m,n} \alpha(s) \cos(m, \varphi - n, \zeta)
\]

- Finite-element

\[
a_\varphi(s) = \sum_i a_{\varphi,i}(s) \varphi(s)
\]

\& inserted into constrained-energy functional

\[
F = \sum_{l=1}^{N} \left( W_l - \mu_l H_l / 2 \right)
\]

- Derivatives wrt \(A\) give Beltrami field \(\nabla \times B = \mu B\)
- Field in each annulus computed independently, distributed across multiple cpu's
- Field in each annulus depends on enclosed toroidal flux, poloidal flux, interfaces \(\xi\)

Force balance solved using multi-dimensional Newton method

- Interface geometry adjusted to satisfy force balance \(\mathbf{F}[\xi] = \left[ p + B^2 / 2 \right]_{m,n} = 0 \)
- Angle freedom constrained by spectral condensation,
- Derivative matrix \(\nabla \mathbf{F}[\xi]\) computed in parallel using finite difference
Example: DIII-D with \( n=3 \) applied error field

[Hudson et al Phys. Plasmas 19, 112502 (2012)]

- 3D boundary, \( p, q \)-profile from STELLOPT reconstruction [Sam Lazerson]
- Irrational interfaces chosen to coincide with pressure gradients.

- Island formation is permitted
- No rational “shielding currents” included in calculation.

STELLOPT SPEC

formation of magnetic islands at rational surfaces
SPEC extended to vacuum and X points

- A prototype calculation performed for an illustrative (MAST-like) cross-section with a large perturbation applied in the vacuum.

  Edge field region chaotic by generating intersecting unstable manifolds’ (a homoclinic tangle) about the X-point.

  Some lobes can intersect the divertor target and result in the strike point splitting often observed during RMP experiments.

Spontaneously formed helical states
Dennis, Hudson, Terranova, Dewar, Hole, Escande

- The quasi-single helicity state is a stable helical state in RFP: becomes purer as current is increase

"Experimental" Poincaré plot
[Fig. 6 of P. Martin et al., Nuclear Fusion 49, 104019 (2009)]

- State can be described by a sequence of SPEC solutions, which are in a minimum energy state
Recent progress in MRxMHD

- Developed techniques to establish pressure jump a surface can support. [M. McGann, ANU PhD thesis, 2013]
Recent progress in MRxMHD

• Related helical bifurcation of a Taylor relaxed state to a tearing mode

• Explained spontaneously formed helical states in reverse field Pinch

• Reproduced MHD in infinite interface limit  [G. R. Dennis, S. R. Hudson, R. L. Dewar, M. J. Hole,
  Phys. Plasmas 20, 032509 (2013)]

• Generalized straight field line coordinates concept to fully 3D plasmas

• Developed theory of resonant current sheet formation and reconnection.

• Developed techniques to establish pressure jump a surface can support.
  [M. McGann, ANU PhD thesis, 2013]

• Extended MRxMHD to include non-zero plasma flow

• Extended MRxMHD to include non-zero plasma flow and anisotropy

• Stability of a two-volume MRxMHD model in slab geometry

• Stepped transform equilibria: transform everywhere irrational, arbitrary smooth pressure profile,
  continuously-nested flux surfaces.
  [Loizu, Hudson et al, Phys. Plas. 22, 090704, 2015]
MRxMHD *equilibria* of stellar plasmas

- Most coronal solar flare models assume the field is force-free, and adjust the field pitch to match local measurements of the photosphere field footprint. (nonlinear force free fields)
- boundary conditions are line-tied (solar) cf toroidal (fusion)

Solar magnetogram

e.g. Polarization, intensity of radio thermal free-free emission

*Iwai et al* Earth, Planets and Space, Volume 66:149, 10 pp.]
Current Sheets: Drivers of Nanoflares and Eruptive Flares in the Solar Corona

- Solar $T_{\text{core}} \sim 10^6$ K, $T_{\text{edge}} \sim 10^4$ K, $T_{\text{corona}} \sim 10^6$ K.

- Parker [E. N. Parker, Astrophys. J. 174, 499 (1972)] suggested current density in the corona must be surface currents (if volume currents, Ohmic heating can not compensate due to radiation loss)

- Current sheets can release their energy through “nanoflares” (possibly magnetic plasmoids) heating the corona

- Currents sheets mediate fast reconnection during coronal mass ejections, possibly by “breakout model” in which a fully 3D coronal configuration is driven by photospheric footpoint motion to form an extended thin current sheet which becomes violently unstable [S. Antiochos et al, Astrophys. J. 510, 485, 1999]
Multi-Region Relaxation Dynamics

DP170102606 Multi-Region Relaxation Dynamics — a new paradigm for fusion and stellar plasma physics, R. L. Dewar, M. J. Hole, S. R. Hudson; A. Bhattacharjee

Q1) What is the MRxMHD spectrum of normal modes, and what are the effects of field-line curvature and mass flow on their stability?

Q2) When are the MRxMHD current sheets topologically stable towards internal plasmoid formation?

Q3) When do unstable modes saturate at a low level or develop nonlinearly into explosive events?

• compare the effect of toroidal (fusion) and line-tied (solar) boundary conditions on wave spectrum and stability

• Develop codes to treat dynamics in full 3D: time-domain evolution code SPDC nonlinear numerical experiments and frequency-domain code SPECN to calculate linear normal modes

• Comparison of results with experimental and observational data
Some anticipated outcomes...

- Compute helical structure and magnetic islands induced by RMP coils, especially in the edge region. RMP fields are able to suppress ELMS, but there is still no accepted explanation.

- Use of SPECN to find “gap modes” in stellarators, with island / chaotic field structure

- Demonstration that a line-tied version of SPDC can simulate dynamics of solar flares,
Physics regimes and physics models

- **Gyrokinetic simulation** – simulate particle distribution functions
- **Particle in cell (PIC) simulation** – individual particles
- **Dielectric tensor**: often used for cold plasmas
- **MHD**: flowing plasma, single temperature for ions/electrons
- **Gyrokinetic simulation** – simulate particle distribution functions
What is $B$ for a tokamak, stationary plasma?

(1) $\mathbf{J} \times \mathbf{B} = \nabla p$ \implies \begin{align*}
\mathbf{B} \cdot \nabla p &= 0 \implies \text{No pressure gradient along } \mathbf{B} \\
\mathbf{J} \cdot \nabla p &= 0 \implies \text{Current flows within surfaces.}
\end{align*}

(2) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

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(2) $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

(3) $\nabla \cdot \mathbf{B} = 0$

Introduce poloidal magnetic flux function $\psi(R,Z)$ and co-ord. system $(R, \phi, z)$. In axisymmetry Eq. (1), (2) become Grad-Shafranov equation:

$$\nabla \cdot \frac{1}{R^2} \nabla \psi = -\frac{\mu_0 J_\phi}{R} = -\mu_0 p'(\psi) - \frac{\mu_0^2}{R^2} f(\psi) f'(\psi)$$

with $f(\psi)$ a toroidal flux function

$$f(\psi) = RB_\phi(\psi, R)/\mu_0$$

second order PDE for field and currents.
What is $B$ for a tokamak, stationary plasma?

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f(\psi) = RB_\phi(\psi,R)/\mu_0
\]

• To solve: prescribe $p'(\psi), f(\psi) f'(\psi)$ and boundary

• Solve numerically by current-field iteration
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2. $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

3. $\nabla \cdot \mathbf{B} = 0$

Introduce poloidal magnetic flux function $\psi(R,Z)$ and co-ord. system $(R, \phi, z)$. In **axisymmetry** $Z$, Eq. (1), (2) become **Grad-Shafranov equation:**

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second order PDE for field and currents.

With $f(\psi)$ a toroidal flux function

$$f(\psi) = RB_\phi(\psi, R) / \mu_0$$

- To solve: prescribe $p'(\psi), f(\psi) f'(\psi)$ and boundary
- Solve numerically by current-field iteration
- What is the solution to $\psi$? **Choose an experiment...**
MAST's mission:
• to explore the ST concept
• test low aspect ratio physics to strengthen general tokamak understanding.

Aspect ratio (A=R/a)

A=4.5  3.0  1.3

Achieved

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Mega Ampere Spherical Tokamak
“Kinetic” MAST equilibrium

$q_0 = 2.9$, $\beta_n = 4.97$, $q_{95} = 11.69$, $\beta_t = 5.03$, $q_{100} = 26.18$, $I_N = 1.01$, $l_I = 1.00$, $p(0)/\langle p \rangle = 1.98$

[M J Hole et al
Plasma Phys. Control.
Fusion 47 (2005) 581–613

#7085 at 290ms

Magnetic Surfaces

Safety factor

Parallel current

Poloidal flux function
What is $B$ for an accretion disc?

1. MHD/EM equations with gravity

$$
\nabla \cdot (\rho \mathbf{v}) = 0 , \\
\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} , \\
E + \frac{1}{c} \mathbf{v} \times \mathbf{B} = 0 , \\
\nabla \times \mathbf{E} = 0 , \\
\rho (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla p + \rho g + \frac{1}{c} \mathbf{J} \times \mathbf{B} , \\
\n\nabla \cdot \mathbf{B} = 0 ,
$$

2. Assume toroidal symmetry

$$
\mathbf{B} = \mathbf{B}_p + \mathbf{B}_\phi . \\
\mathbf{B}_p = \frac{1}{r} \nabla \Psi \times \hat{\phi} , \\
4\pi\rho \frac{|\mathbf{v}_p|}{|\mathbf{B}_p|} = F(\Psi) , \quad = r\Omega(\Psi)\hat{\phi} + \frac{F(\Psi)}{4\pi\rho} \mathbf{B} . \\
\text{3. angular momentum conservation} \\
- F(\Psi) r \mathbf{v}_\phi + r \mathbf{B}_\phi = H(\Psi) .
$$

4. Energy Flow conservation

$$
\int (d\rho/\rho) \bigg|_{\Psi = \text{const.}} + \frac{1}{2} |\mathbf{v}|^2 + \Phi_g - r \mathbf{v}_\phi \Omega(\Psi) = J(\Psi) , \\
\text{5. Assume flow is adiabatic} \\
S(p, \rho) = k_B(\Gamma - 1)^{-1} \ln (p/\rho^\Gamma) = S(\Psi)
$$

6. Generalised, nonrelativistic, Grad-Shafranov equation

$$
\left(1 - \frac{F^2}{4\pi\rho}\right) \Delta \Psi - F \nabla \left( \frac{F}{4\pi\rho} \right) \cdot \nabla \Psi = -4\pi\rho r^2 (J' + r \mathbf{v}_\phi \Omega') - (H + r \mathbf{v}_\phi F)(H' + r \mathbf{v}_\phi F') + 4\pi r^2 p(S'/k_B) ,
$$

ANU Theory and Modelling

- Supported by ~$2.5m in funding over last 8 years (ARC, ISL)

**Consortium**
- MAST (UK) compact
- KSTAR (Korea) superconducting
- ITER (Earth) ITPA
- W-7X (Germany) steady-state, reduced chaos
- RFX-mod (Italy) self-organising

Local experiments

- Burning Plasma Physics / Multi fluid models
- Basic Science
- Bayesian modelling
- MRxMHD

- Stellar dynamics

Other institutions and logos
January, 2005. Formation of Australian ITER Forum at AIP Congress with objectives

1. To promote an Australian involvement in ITER and articulate the benefits to Australia.
2. To promote the science of fusion energy.
3. To advance the recognition of fusion science and plasma physics in the wider scientific community.

fusion.ainse.edu.au
ITER

- Fusion power = 500MW
- Power Gain (Q) > 10
- Temperature ~ 100 million °C

- Growing Consortium

- Collaboration agreements with
  - International Atomic Energy Agency
  - CERN – world’s largest accelerator
  - Principality of Monaco
  - Australia 30/09/2017
  - Iran (4 July 2016, High-level Iranian delegation visits ITER worksite)

Construction +10 year operation cost ~$20 billion

**Powering Ahead**
A National Response to the Rise of the International Fusion Power Program

**Advisory committee**
- Dr Richard Garrett, ANSTO
- Dr Barry Green, UWA (formerly ITER design team)
- Em/Prof John Howard, ANU
- A/Prof Matthew Hole, ANU
- A/Prof Brian James, Univ Sydney (coordinator)
- Prof John O’Connor, Univ Newcastle

"The problem I hope scientists will have solved by the end of the century is nuclear fusion. It would provide an inexhaustible supply of energy without pollution or global warming."

Stephen Hawking

Co-released by Australian ITER Forum, ANU, and ANSTO
Key recommendations

(1) That the Australian government supports a national fusion program that would provide:

- A fusion program fellowships scheme – supporting ITPA, stellarator physics and advanced materials
- Funding for an Australian flagship contribution to ITER
- Ongoing support for the Australian Plasma Fusion Research Facility
- High-power enhancements to the MagPIE device
- Support for ITER engagement through the ITER research program of the International Tokamak Physics Activity (ITPA)

(2) That the Australian fusion science community through ANSTO establishes a Memorandum of Understanding with the ITER to formally enable Australian participation in ITER and the ITPA.

✓ Cooperation agreement between ANSTO / ITER enabling collaboration and ITPA participation signed 30 Sep., 2016
International Tokamak Physics Activity

• ITPA is a framework for internationally coordinated fusion research.
• operates under the auspices of ITER.
• the participants in the ITPA are the members of ITER.
• 7 topical groups:

**DIAGNOSTICS:** address issues that might arise both in plasma control and in the analysis of ITER plasmas and in reactor grade plasmas (e.g. DEMO)

**ENERGETIC PARTICLE PHYSICS:** tackle the qualitatively new physics element of ITER: dominant alpha particle heating.

**INTEGRATED OPERATION SCENARIOS:** establish operational scenarios in burning plasma experiments

**MHD, DISRUPTIONS & CONTROL:** establish β-limiting MHD instability thresholds and their active control; address disruption mitigation

**PEDESTAL & EDGE PHYSICS:** improve understanding of pedestal, focus on H-modes and suppression of ELMs

**SCRAPE-OFF-LAYER & DIVERTOR:** plasma-material interactions and their dependence on materials, hydrogen isotope recycling and their storage

**TRANSPORT & CONFINEMENT** develop a fundamental understanding of transport and confinement physics governing plasma performance,
Summary

• Plasma Theory and Modelling: a vibrant ANU pursuit developing theory for next generation fusion experiments, and supporting physics interpretation of existing experiments.
• Very strong international collaboration
• Research areas
  ➢ Burning plasma physics: anisotropy and flow, energetic particle driven modes
  ➢ Fully 3D toroidal physics and MRxMHD. Impacts of 3D structure on plasma. Dynamics of MRxMHD plasmas, *linkage to solar and toroidal fusion plasmas*
  ➢ *Bayesian inference of configurations*
    Bayesian inference (inversion) for fast particle velocity distribution
  ➢ Interpretation/modelling of international and domestic experiments
  ➢ *Not mentioned… wakefield accelerator physics, ELM statistics*
• Research synergies identified with ITER and ITPA. These align with Australian strategic planning for fusion science.
“CSIRO child care 2017 aims…”

Black Mountain Laboratory, CSIRO Care
17/11/2016

A
Acknowlegding theorists

I
Interpersonal relationships

M
Managing performance and mentoring