Workshop in Stochastic Analysis

26-30 November 2018, ANU

ABSTRACTS

Maximal regularity for stochastic Volterra integral equations

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In this talk we discuss an approach to obtain maximal regularity estimates for solutions of stochastic Volterra integral equations driven by a multiplicative Gaussian noise. To achieve that, we mainly focus on suitable estimates for deterministic and stochastic convolution operators. Starting with the scalar-valued case, we use functional calculi results to lift the corresponding estimates to the operator-valued setting. Once maximal regularity estimates for convolutions are obtained, appropriate Lipschitz and linear growth assumptions on the nonlinearities will lead to unique mild solutions with Hölder continuous trajectories. This is joint work with Boris Baeumer and Petru Cioica-Licht.

Quantifying uncertainty in Lagrangian coherent structures in unsteady flows

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Over the past several decades, there has been considerable interest in tracking structures in unsteady flows which 'remain coherent,' as well as the boundaries of such structures. Such 'Lagrangian coherent structures' are fundamental in transport (of heat, salinity, humidity, pollutants, bioorganisms); examples are the Gulf stream, the Antarctic Circumpolar Vortex ('the ozone hole'), hurricanes, oceanic eddies within which there is high plankton concentration, the region on the west coast of Florida to which the

Deepwater Horizon oil spill did not reach, and separated globules in a microfluidic device which resist mixing. Standard techniques for tracking these involve numerically advecting initial conditions by using the velocity field (usually available only from data), and then extracting from this flow map coherent structures based on a range of techniques. None of these explicitly address uncertainties (noise) in the velocity, which should be of concern because the data typically has very low resolution (oceanic measurements are on grids of size 100 km).

In this talk, I develop a stochastic differential equation model for analytically computing uncertainties of eventual trajectories, which allows for the velocity to be unsteady (the system is nonautonomous), and the noise to be not just time, but also location-dependent ('multiplicative noise'). Combining stochastic calculus techniques such as the Burkholder-Davis-Gundy inequality and Ito's isometry, with ordinary differential equations methods such as the derivation strategy of Melnikov's method, I derive the anisotropic variance of the eventual location explicitly. The 'stochastic sensitivity' field I additionally obtain enables the identification of evolving flow regions which are robust with respect to a user-specification of length-scale and noise-level in the data. It is expected that these will be invaluable new tools in ascribing uncertainties to Lagrangian coherent structures.

PDEs with stochastic memory and reaction

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Based on a basic continuous time random walk model we carefully derive a general PDE with memory (aka Volterra integral equation). We show how a physical reaction term will impact the memory term and end up with a stochastic Volterra equation where the noise is part of the memory.

Regularity and approximation of SPDEs

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The speed of an approximation method is related to the regularity of the target function. As a rule of thumb, the convergence rate of classical uniform methods is governed by the Sobolev regularity, whereas adaptive methods correlate with the regularity in special scales of Besov spaces.

In this talk, I am going to present some results concerning the spatial regularity of second order SPDEs in such scales. The main difficulty comes from the fact that the scales do not consist of Banach spaces but merely of quasi-Banach spaces.

One way out is to extend the stochastic integration theory from UMD Banach spaces to proper classes of quasi-Banach spaces. Another strategy is to show how the singularities of the solution can be handled by appropriate weights and embed suitable weighted Sobolev spaces into the quasi-Banach Besov spaces of interest. I am going to elaborate on how far we can get (so far) in each direction. In particular, I will focus on what happens if the underlying domain is not smooth.

(*) The part on stochastic integration theory is joint work with Mark C. Veraar (TU Delft) and Sonja G. Cox (U Amsterdam). The part on the weighted Sobolev regularity is joint work with Kyeong-Hun Kim (Korea University), Kijung Lee (Ajou University), and Felix Lindner (Universität Kassel).

Sharp Strichartz estimates for Schrödinger equation

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We consider the $L_t^2 L_x^r$ estimates for the solutions to the wave and Schrödinger equations in high dimensions. For the homogeneous estimates, we show $L_t^2 L_x^\infty$ estimates fail at the critical regularity in high dimensions by using stable Lévy process in $\mathbb{R}^d$. Moreover, we show that some spherically averaged $L_t^2 L_x^\infty$ estimate holds at the critical regularity. As a by-product we obtain Strichartz estimates with angular smoothing effect. For the inhomogeneous estimates, we prove double L_t^2-type estimates. This is a joint work with Ji Li, Kenji Nakanishi and Lixin Yan.

Schauder estimates and stochastic PDEs

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The talk has three parts: First in part 1, we introduce the standard Schauder estimates for elliptic PDEs, including Possion's equation and Monge-Ampere equations. Then in part 2, we move to stochastic settings where the Schauder estimate was an open problem raised by Krylov in 1999. Last in part 3, we state our recent work on the Schauder estimates for SPDEs and some applications, which is a joint work with Kai Du.

Stochastic PDE's and mixed PDE/Monte-Carlo methods for derivatives pricing

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In the spirit of [1] we propose a pricing method for derivatives when the underlying diffusion is given by a set of stochastic differential equations, with the objective of reducing the computing time. The numerical method is based on a joint use of Monte-Carlo simulations, PDE or analytical formulas. We show that this method has an natural interpretation in terms of stochastic pde's and from this observation propose a new way of implementing it. We also show how to implement the Least Square Monte-Carlo method proposed in [2] together with the mixed PDE/Monte-Carlo method.

[1] G. Loeper and O. Pironneau A Mixed PDE /Monte-Carlo Method for Stochastic Volatility Models CRASS 2009

[2] F.A. Longstaff and E.S. Schwartz Valuing American Options by simulations:A Simple Least-Squares Approach. Working Paper Anderson Graduate School of Management University of California 1998.

Strong solutions of stochastic models for viscoelastic flows of Oldroyd type

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In this talk, I will provide basic difference between Stratonovich integral and Ito integral for general stochastic partial differential equations. We then move to the study of stochastic Oldroyd type models for viscoelastic fluids in $\mathbb{R}^d, d = 2, 3$. In the current work, we have shown existence and uniqueness of strong local maximal solutions when the initial data are in ^{s} for $s > d/2, d = 2, 3$. Probabilistic estimate of the random time interval for the existence of a local solution is expressed in terms of expected values of the initial data.

Stochastic maximal regularity for rough time-dependent problems

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To solve non-linear parabolic SPDE, one often uses a simple fixed point argument, based on a subtle regularity property of the linear part, called a stochastic maximal regularity estimate. Such estimates have a long history, including the 2012 milestone result of van Neerven-Veraar-Weis (NVW) taught in the mini-course. In this talk, I will present an extension of this result to situations where the coefficients are L^∞ in time. It combines the NVW operator theoretic approach with Krylov's PDE approach, to deduce sharp results for coefficients that are continuous in the spatial variables. It also introduces a new method that combines the NVW approach with harmonic analysis to deduce stochastic maximal regularity estimates for coefficients that are mereley L^∞ in BOTH space and time. This is joint work with Mark Veraar (Delft).

Ergodicity for stochastic dispersive equations

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In this talk, we study the long time behaviour of some stochastic partial differential equations (SPDEs).

After introducing the notions of ergodicity, unique ergodicity and convergence to equilibrium, we will discuss how these have been proven for a very large class of parabolic SPDEs.

We will then shift our attention to dispersive SPDEs, where the general strategy for the parabolic case fails. We will describe this failure for the wave equation on the 1-dimensional torus and present a recent result that settles unique ergodicity even in this case.

Topics in Gaussian Harmonic Analysis

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In the first part of the talk we briefly review the basic notions of Gaussian harmonic analysis: semigroups, maximal functions, Littlewood-Paley functions, spectral multipliers and singular integrals. In the second half we discuss new developments on continuity of those operators on variable exponent Lebesgue spaces.

Burkholder–Davis–Gundy inequalities and stochastic integration in UMD Banach spaces

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In this talk we will present Burkholder–Davis–Gundy inequalities for general UMD Banach space-valued martingales. Namely, we will show that for any UMD Banach space X, for any X-valued martingale M with $M_0 = 0$, and for any $1 \leq p < \infty$

$$\mathbb{E} \sup_{0\leq s\leq t} \|M_s\|^p \eqsim_{p,X} \mathbb{E}\gamma([\![M]\!]_t)^p, \quad t \geq 0,$$

where $[\![M]\!]_t$ is the covariation bilinear form of M defined on $X^* \times X^*$ so that

$$[\![M]\!]_t(x^*, y^*) = [\langle M, x^*\rangle, \langle M, y^*\rangle]_t, \quad x^*, y^* \in X^*,$$

and $\gamma([\![M]\!]_t)$ is the L^2-norm of a Gaussian measure on X having $[\![M]\!]_t$ as its covariance bilinear form.

As a consequence we will extend the theory of vector-valued stochastic integration with respect to a cylindrical Brownian motion by van Neerven, Veraar, and Weis, to the full generality.