Baxter 2020: Frontiers in Integrability

Mathematical Sciences Institute, The Australian National University
MSI Special Year in 2020

Program & Title and Abstract booklet
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<th>Time</th>
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<th>Speaker</th>
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<tr>
<td>8:15</td>
<td>Registration</td>
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<tr>
<td>9:00</td>
<td>Welcome</td>
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<td>9:05</td>
<td>Barry McCoy, Stony Brook University</td>
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<td>Title: Ising correlation $C(M,N)$ for $\nu=-k$</td>
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<td>9:45</td>
<td>Charlotte Kristjansen, Niels Bohr Institute, University of Copenhagen</td>
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<td>Jean Michel Maillet, CNRS &amp; ENS Lyon</td>
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<td>Title: On quantum separation of variables</td>
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<td>14:00</td>
<td>Atsuo Kuniba,</td>
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<td>Title: Generalized hydrodynamics in box-ball system</td>
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<td>Benjamin Doyon, King's College London</td>
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<tr>
<td>16:00</td>
<td>Andrew Kels, SISSA</td>
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<td>Title: New developments for integrability and the star-triangle relations</td>
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<tr>
<td>16:30</td>
<td>Robert Weston, Heriot-Watt University</td>
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<td>Title: Baxter's Q Operator for Open Quantum Spin Chains</td>
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<tr>
<td>17:00</td>
<td>Paul Pearce, University of Melbourne</td>
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<td>Title: Extended T-Systems, Q Matrices and T-Q Relations for $sl(2)$ Models at Roots of Unity</td>
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<tr>
<td>9:00</td>
<td>Vladimir Bazhanov, Australian National University</td>
<td>Venue: 1.33 Hanna Neumann Building</td>
<td>On the scaling behaviour of the alternating spin chain</td>
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<tr>
<td>9:45</td>
<td>Paul Fendley, University of Oxford</td>
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<td>Free fermions and parafermions in disguise</td>
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<tr>
<td>11:00</td>
<td>Konstantin Zarembo, Nordita</td>
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<td>Matrix models and supersymmetric gauge theories at large-N</td>
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<td>11:45</td>
<td>Didina Serban, Institut de Physique Théorique Saclay</td>
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<td>An integrable deformation of the Haldane-Shastry model and its affine quantum symmetry</td>
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<tr>
<td>14:00</td>
<td>Tony Guttmann, The University of Melbourne</td>
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<td>Moment sequences in combinatorics</td>
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<tr>
<td>14:30</td>
<td>Filippo Colomo, INFN, Sezione di Firenze</td>
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<td>Arctic curves and limit shapes</td>
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<td>15:00</td>
<td>Jan De Gier, The University of Melbourne</td>
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<td>Limit shapes for the asymmetric five vertex model</td>
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<td>16:00</td>
<td>Jules Lamers, University of Melbourne</td>
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<td>A fresh look at Inozemtsev's elliptic spin chain</td>
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<tr>
<td>16:30</td>
<td>Alexandr Garbali, The University of Melbourne</td>
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<td>An integrable model for the quantum toroidal gl. 1</td>
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<td>17:00</td>
<td>Murray Batchelor, Australian National University</td>
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<td>Towards a loop model for the superintegrable chiral Potts model</td>
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<tr>
<td>9:00</td>
<td>Kevin Costello</td>
<td>University of Cambridge</td>
<td>Integrable field theories with higher germs spectral curve</td>
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<tr>
<td>9:45</td>
<td>Masahito Yamazaki</td>
<td>Kavli IPMU, University of Tokyo</td>
<td>Chiral Potts Models, Gauge Theories and Beyond</td>
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<td>Morning Tea</td>
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<td>11:00</td>
<td>Nalini Joshi</td>
<td>The University of Sydney</td>
<td>When applied mathematics collided with algebra</td>
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<tr>
<td>11:45</td>
<td>Ivan Kostov</td>
<td>Institut de Physique Theorique - CEA Saclay</td>
<td>Four-points correlation functions and octagons</td>
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<td>14:00</td>
<td>Yang Shi</td>
<td>Flinders University</td>
<td>Normalizer theory of Coxeter groups and discrete integrable systems</td>
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<tr>
<td>14:30</td>
<td>Paolo Rossi</td>
<td>Università degli Studi di Padova</td>
<td>The non-commutative KdV equation and enumerative geometry</td>
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<tr>
<td>15:00</td>
<td>Ian Marquette</td>
<td>The University of Queensland</td>
<td>Painlevé transcendents in quantum mechanics</td>
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<td>Afternoon Tea</td>
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<tr>
<td>16:00</td>
<td>Clare Dunning</td>
<td>University of Kent</td>
<td>Coefficients of Wronskian polynomials</td>
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<tr>
<td>16:30</td>
<td>Pieter Roffelsen</td>
<td>International School for Advanced Studies (SISSA)</td>
<td>Wronskians of Hermite polynomials, anharmonic oscillators and Painlevé IV</td>
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<tr>
<td>9:00</td>
<td>Andrei Marshakov, CAS Skoltech</td>
<td><strong>Title:</strong> Cluster integrable systems and supersymmetric gauge theories</td>
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<tr>
<td>9:45</td>
<td>Rei Inoue, Chiba University</td>
<td><strong>Title:</strong> R-matrices in Cluster Algebras</td>
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<tr>
<td>11:00</td>
<td>Chihiro Matsui, The University of Tokyo</td>
<td><strong>Title:</strong> Nonequilibrium physics in integrable systems and spin-flip non-invariant conserved quantities</td>
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<tr>
<td>11:45</td>
<td>Paul Zinn-Justin, The University of Melbourne</td>
<td><strong>Title:</strong> The six-vertex model on random lattices revisited</td>
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<tr>
<td>14:00</td>
<td>Angela Foerster, INSTITUTO DE FISICA DA UFRGS</td>
<td><strong>Title:</strong> Exactly solved models: past, present and future</td>
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<tr>
<td>14:45</td>
<td>Bernard Nienhuis, University Leiden</td>
<td><strong>Title:</strong> Conserved charges of the XXZ chain in the Temperley-Lieb representation</td>
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<td>15:30</td>
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<td><strong>Closing remarks</strong></td>
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There are many ways to get around Canberra. Below is some useful information about bus and taxi transport around the ANU, the airport and surrounding areas.

**Taxi**

To book a taxi from ANU to the airport request to be collected from the Ian Ross Building or the ANU gym on North road. A taxi to the airport usually costs around $40 and travel time is approximately 15 minutes. Pricing and time may vary depending on traffic.

- Canberra Elite Taxis: 13 22 27
- ACT Cabs: 6280 0077
- Silver Service Canberra: 133 100

**Buses**

Canberra buses are a cheap and easy way of getting around town once you’re here.

To view the bus timetable, go to: https://www.transport.act.gov.au/getting-around

**Canberra Light Rail**

If you are staying in the North of Canberra, the Light Rail would be a suitable option for you. The train runs from Gungahlin marketplace to the Canberra city centre.

To travel just use your MyWay card or purchase a ticket from the vending machines located at all the stops and bus interchanges. You can purchase a single trip ticket which includes a free 90 minute transfer period.

Conference wifi information is listed below. If you have any issues logging into ANU Access please contact Admin.research.msi@anu.edu.au

USERNAME: Baxter2020
PASSWORD: SpecialYear2020

ANU Access

ANU Access is only available to staff and affiliates, and to connect you will require an ANU ID.

To connect to ANU Access:

1. Make sure the wireless network port is active.
2. Select ANU Access from the list of available wireless networks.
3. Enter your ANU ID and password, and press login.

Once connected, it is not necessary to log out of ANU Access—you will be automatically logged out after a certain period of inactivity. However, if you have javascript enabled on your browser, a logout window will pop-up.

It is strongly recommend that you enable your operating system’s firewall, use up-to-date anti-virus software and read the Wireless Security Recommendations.

Eduroam

Eduroam is a wireless network that allows visiting students, staff and academics to connect to and access a University’s wireless network using their home university login. Eduroam is available on a range of operating systems including iPhone/iPad, Macintosh, Windows XP, Windows Vista, Windows 7, Blackberry, Android and Linux, and is available wherever you find ANU Secure. Usually the participating institution will allow users to open a Virtual Private Network (VPN) connection to your home institution.

All ANU staff and students can be eduroam users when at a participating national and international institutions, but should not attempt to log on to the eduroam network when at ANU.
CANBERRA MARKETS

Canberra Hospital Markets
Cnr Yamba & Hindmarsh Drives, Woden
Phone: 6254 6140
5th Sunday each month, 9am – 2pm. Home-made crafts and plants. Money raised grants wishes to cancer patients.

Gorman House Markets
Cnr Ainslie Ave & Hesse St, Canberra City
Phone: 6247 3202
Saturday, 10am – 4pm
New & used clothes, books, vegies, craft, furniture, multicultural food, live music.

Hall Markets
Hall Showground (near Victoria St)
Phone: 6260 5555
1st Sunday each month, 10am – 3pm, gold coin entry
Home produce, crafts, food, live music, kids activities and more.

Old Bus Depot Markets
Wentworth Avenue Kingston
Phone: 6292 8391
www.obdm.com.au
Sunday, 10am – 4pm
Quality art & craft, clothing, jewellery, plants, home wares, produce, food, etc.

Belconnen Fresh Food Markets
Lathlain St, Belconnen
Phone: 6251 1680
Wednesday – Sunday, 8am – 6pm
Some stores are open 7 days

Food Co-Op
Phone: 6257 1186
Open Tues & Thurs 10am – 7pm
Wed, Fri & Sat 10am – 4pm
www.anu.foodco-op.com

Canberra Region Farmers Market
Exhibition Park (EPIC), Northbourne Ave
Phone: 0419 626 234
Saturday, 7:30am – 11:30am

Fyshwick Fresh Food Markets
Cnr Mildura & Dalby St, Fyshwick
Phone: 6295 0606
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<th>Places of Interest</th>
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| **Bush Walking in the Capital**  
Canberra has great bush walking trails for all levels.  
For more information:  
[www.anumc.edu.au](http://www.anumc.edu.au) |
| **The National Portrait Gallery**  
(2 locations)  
1. Commonwealth Place, Parkes  
2. Old Parliament House, Parkes  
Phone: 6270 8236  
| **Australian National Botanic Gardens**  
Clunies Ross St, Acton  
Phone: 6250 9540  
| **Black Mountain**  
Clunies Ross St, Acton  
Phone: 6207 2500  
National Gallery of Australia  
Parkes Place, Parkes, Phone: 6240 6502  
Free entry to the permanent collection and varying charges for other exhibitions.  
[www.nga.gov.au](http://www.nga.gov.au) |
| **The National Library**  
Parkes Pl, Parkes, Phone: 6262 1111  
Telstra Tower  
Black Mountain Drive, Acton  
Phone: 6219 6111  
| **Cockington Green Gardens**  
11 Gold Creek Road, Nicholls ACT 2913  
(02) 6230 2273  
| **National Zoo and aquarium**  
999 Lady Denman Dr, Weston Creek ACT 2611  
(02) 6287 8400  
| **Canberra Glassworks**  
11 Wentworth Avenue, Kingston Foreshore  
Kingston, Australian Capital Territory  
Australia, 2604  
Phone: 02 6260 7005  
[www.canberraglassworks.com](http://www.canberraglassworks.com) |
| **The National Museum of Australia**  
Lawson Cres, Acton Peninsula, Acton.  
Phone: 6208 5000 or 1800 026 132  
Free entry to the standard museum and varying charges for temporary exhibitions.  
| **Old Parliament House**  
King George Tce, Parkes, Phone: 6270 8222  
Entry: adult $2, Con $1, Family $5  
| **The Australian War Memorial and Anzac Parade**  
Anzac Parade, Phone: 6243 4211  
| **Lake Burley Griffin**  
A short walk or cycle from the ANU or civic. The great thing about the Lake is that you can access it from a variety of different locations. (Parking – turn off Commonwealth Avenue)  
Mt Ainslie & Remembrance Nature Park  
Slopes of Mt Ainslie  
Phone: 6207 2113 |
| **Mt Ainslie & Remembrance Nature Park**  
Slopes of Mt Ainslie  
Phone: 6207 2113 |
| **National Film and Sound Archive**  
McCoy Circuit, Acton  
Phone: 6248 2000  
| **The National Arboretum Canberra**  
Forest Drive (off Tuggeranong Parkway), Acton, Australian Capital Territory, Australia, 2611  
| To find out more fun things to do in Canberra go to:  
Abstracts

Baxter 2020 - Frontiers in Integrability

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Ising correlation $C(M,N)$ for $v=-k$

Barry McCoy
Stony Brook University

We show that the Ising correlation two-point function on the anisotropic lattice $C(M,N)$ where the anisotropy $\sinh 2E_h/kT/\sinh 2E_v/kT$ is related to the modulus for $T < T_c$ of $k = (\sinh 2E_v/kT \sinh 2E_h/kT)^{-1}$ by $\gamma = -k$ satisfies the same Painlevé VI equation satisfied by the Toeplitz determinants of Forrester-Witte. For $T > T_c$ with $\gamma = -k > = -\sinh 2E_v/kT \sinh 2E_h/kT$ we find that $C(M,N) = 0$ for $M + N$ odd and that $C(M,N)$ factorizes for $M + N$ even.

***

Spin Chain overlaps and the AdS/CFT Correspondence

Charlotte Kristjansen
Niels Bohr Institute, University of Copenhagen

We explain how overlaps between Bethe eigenstates of integrable spin chains and matrix product states contain information about observables of the AdS/CFT correspondence in setups where defects are present. Furthermore, we present novel techniques to obtain such overlaps as well as novel results in the form of closed formulas for large classes of such overlaps. Finally, we explain how the overlap formulas provide us with a positive test of the AdS/CFT correspondence in a situation where supersymmetry is completely broken.

***

On quantum separation of variables

Jean Michel Maillet
CNRS & ENS Lyon

A new approach to construct the separate variables bases leading to the full characterization of the transfer matrix spectrum of quantum integrable lattice models will be presented. The bases are generated by the repeated action of the transfer matrix itself on a generically chosen state of the Hilbert space. The fusion relations satisfied by the transfer matrix,
stemming from the Yang-Baxter algebra properties, provide the necessary closure relations to define the action of the transfer matrix on such bases, leading to a separate transfer matrix spectral problem. As a first example of this approach, the construction of such a basis for integrable models associated to $Y(gl_n)$ will be given. Then this general scheme will be applied concretely to fundamental models associated to the $Y(gl_2)$ and $Y(gl_3)$ R-matrices leading to the full characterization of their spectrum. Other examples that can be treated by this method like trigonometric spin chains, open chains with general integrable boundaries, and further higher rank cases will be briefly discussed.

***

Exact solution of the spin-1/2 $XXX$ chain with off-diagonal boundary field

Andreas Kluemper
University of Wuppertal

The spin-1/2 Heisenberg chain with periodic boundary conditions is a seminal model of integrable resp. exactly solvable systems. It is known that the Heisenberg chain with arbitrary boundary fields is still integrable, but so far defied an explicit solution for the case of off-diagonal fields which break the $U(1)$ symmetry. As the magnetization is no longer a good quantum number, the direct application of the Bethe ansatz fails.

Here we show how the problem can be solved by a set of non-linear integral equations (NLIEs). Instead of two NLIEs as in the case of the periodically closed chain, we find a set of three NLIEs from which the eigenvalues of the Hamiltonian can be obtained. The numerical iterative treatment of the equations is involved due to non-trivial monodromy properties of the solution functions. However, an appropriate choice for the location of the kinks in the initial data renders the iterative procedure convergent. For the simpler case of parallel boundary fields, we managed to derive CFT data analytically.

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Generalized hydrodynamics in box-ball system

Atsuo Kuniba
University of Tokyo

Box-ball system (BBS) is a distinguished example of soliton cellular automaton in one dimension. Its integrability has been studied extensively from the viewpoint of quantum groups, Bethe ansatz, ultradiscretization and tropical geometry. In this talk I introduce the generalized Gibbs ensemble of BBS solitons and present several results related to
thermodynamic Bethe ansatz and generalized hydrodynamics. They include exact solutions to the speed equation for solitons and the stationary current, a novel connection of the speed to a scaled period matrix of the tropical Riemann theta function, explicit description of the density plateaux that emerge from a domain wall initial condition and so forth. The content is based on a joint work with Gregoire Misguich and Vincent Pasquier.

***

Hydrodynamic projections in integrable systems

Benjamin Doyon
King’s College London

Evaluating dynamical correlation functions in thermal states and generalised Gibbs ensembles of integrable systems is a formidable problem. Hydrodynamic projection methods give powerful tools for their asymptotics. After reviewing the theory of generalised hydrodynamics (GHD), I will explain how, combined with hydrodynamic projections, exact expressions are obtained for such asymptotics, for Drude weights and for diffusion.

***

New developments for integrability and the star-triangle relations

Andrew Kels
SISSA

The star-triangle relation (STR) is a central equation for integrability of 2-dimensional Ising-type lattice models of statistical mechanics, which implies the commutation of transfer matrices of a lattice model, that can be used to find an exact solution through the pioneering methods of Baxter. Recently there has been found an unexpected connection of the STR’s to rather different types of integrable systems, namely, systems of integrable partial difference equations, which may be considered as discrete versions of integrable soliton equations, such as the KdV equation. In modern times, a widely used integrability condition for partial difference equations is known as consistency-around-a-cube (CAC), which implies a Lax pair, as well as the consistency of an equation when extended into higher dimensional lattices. A classification of equations based on CAC was given by Adler, Bobenko, and Suris (ABS), resulting in a list of equations that included most of the important examples (such as discrete KdV and Hirota equations), as well as some new ones.
In this talk, it will be shown how the STR’s themselves may be interpreted as quantum (in a path-integral sense) counterparts of the equations in the ABS list, and that the entire ABS list may be generated through the degenerations and quasi-classical (asymptotic) expansions of these STR’s. This is a remarkable connection, particularly since the classification of ABS does not involve any form of the STR, and is the basis of a wider correspondence between integrable lattice models of statistical mechanics, and systems of integrable difference equations. Some of the more recent results obtained from this correspondence include, a new set of classical R-matrices (Yang-Baxter maps) satisfying the set-theoretical Yang-Baxter equation, and a new CAC type condition for difference equations, which can naturally be called consistency-around-a-face-centered-cubic (CAFCC).

***

**Baxter’s Q Operator for Open Quantum Spin Chains**

Robert Weston
Heriot-Watt University

I will first review the purpose and algebraic construction of Baxter’s Q-operator for closed quantum spin chains. I will then move on to a parallel discussion for open spin chains; here the algebraic construction is more subtle due to the four different algebras that are in play (the quantum group, two Borel subalgebras and a coideal subalgebra). I will navigate this sea of algebras and give a new construction of the Q-operator for the open XXZ chain with diagonal boundary conditions. This is joint work with Bart Vlaar.

***

**Extended T-Systems, Q Matrices and T-Q Relations for sl(2) Models at Roots of Unity**

Paul Pearce
The University of Melbourne

Holger Frahm, Alexi Morin-Duchesne, Paul A. Pearce

The mutually commuting $1 \times n$ fused single-row transfer matrices $T^n(u)$ of the critical six-vertex model are considered at roots of unity $q = e^{i\lambda}$ with crossing parameter $\lambda = \frac{(p' - p)\pi}{p'^r}$ a rational fraction of $\pi$. For diagonal twisted boundary conditions, we find explicit closure relations for the T-system functional equations and obtain extended sets of bilinear T-system identities. We also define extended $Q$ matrices as linear combinations of the fused
transfer matrices and obtain extended matrix $T$-$Q$ relations. Using our extended $T$-system and extended $T$-$Q$ relations for eigenvalues, we deduce the usual scalar Baxter $T$-$Q$ relation and the Bazhanov-Lukyanov-Zamolodchikov decomposition of the fused transfer matrix $T^{p-1}(u+\lambda)$ in terms of the product $Q^+(u)Q^-(u)$ or $Q(u)^2$. It follows that the zeros of $T^{p-1}(u + \lambda)$ are comprised of the Bethe roots and complete $p'$ strings. We also clarify the formal observations of Pronko and Yang–Nepomechie–Zhang and establish, under favourable conditions, the existence of an infinite fusion limit $n \to \infty$ in the auxiliary space of the fused transfer matrices.

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On the scaling behaviour of the alternating spin chain

Vladimir Bazhanov
Australian National University

In this talk I will report the results of the study of a 1D integrable alternating spin chain whose critical behaviour is governed by a CFT possessing a continuous spectrum of scaling dimensions. I will review both analytical and numerical approaches to analyzing the spectrum of low energy excitations of the model. It turns out that the computation of the density of Bethe states of the continuous theory can be reduced to the calculation of the connection coefficients for a certain class of differential equations whose monodromy properties are similar to those of the conventional confluent hypergeometric equation. The finite size corrections to the scaling are also discussed.

***

Free fermions and parafermions in disguise

Paul Fendley
University of Oxford

I describe how to compute the exact spectrum and partition function for several models not obviously free. In these models this calculation is possible only with free or fixed boundary conditions – periodic boundary conditions are much more complicated. The first is a quantum chain whose Hamiltonian is comprised solely of local four-fermi operators and a supersymmetry.
By a non-local and highly non-linear map, I construct free-fermion raising and lowering operators, and find that the spatially uniform system is gapless with dynamical critical exponent $z = 3/2$. The second is the superintegrable classical chiral Potts model in two dimensions, and a closely related “inverse” model known as the $\tau_2$ model (and which ought to be called the Korepanov- Baxter-Bazhanov-Stroganov model). Here I solve it by constructing free-parafermion operators in a simpler and more general way than in earlier work.

***

Principal chiral field at large-N: a new non-critical string

Konstantin Zarembo
Nordita

The PCF is often pictured as the closest cousin of QCD in 2d, featuring asymptotic freedom, dimensional transmutation and ’t Hooft’s large-N limit. At the same time the model is integrable and its large-N expansion can be explored non-perturbatively. The Bethe ansatz solution at large-N unveils striking similarities to the $c=1$ non-critical string in its matrix model guise, including the double-scaling limit that combines $1/N$ with the strong-coupling expansion.

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An integrable deformation of the Haldane-Shastry model and its affine quantum symmetry

Didina Serban
Institut de Physique Théorique Saclay

The subject of integrable long-range deformations of spin chains is understudied, in spite of its relevance from the point of view of the physical applications, but also of the underlying mathematical structures. I am going to present some results concerning the spectrum and the wave functions of the XXZ-like deformation of the Haldane-Shastry spin chain, the corresponding symmetries and the link to Macdonald polynomials.
Moment sequences in combinatorics

Tony Guttmann
The University of Melbourne

Joint work with Alin Bostan, Andrew Elvey Price, Jean-Marie Maillard

A number of combinatorial sequences have coefficients that can be represented as moments of a nonnegative measure on \([0, \infty)\). Such sequences are known as Stieltjes moment sequences, and have a number of useful properties, such as log-convexity, which in turn enables one to rigorously bound their growth constant.

I will discuss some classical combinatorial sequences that possess this property, and focus on some pattern-avoiding permutations \(\text{Av}(P)\), counting permutations of \(\{1, 2, \ldots, n\}\) that avoid some given pattern \(P\). For increasing patterns \(P = (12 \ldots k)\), we show that the corresponding sequences, \(\text{Av}(123 \ldots k)\), are Stieltjes moment sequences, and we explicitly find the underlying density function, either exactly or numerically, by using the Stieltjes inversion formula.

In the challenging case of the \(\text{Av}(1324)\) sequence I give compelling numerical evidence that this too is a Stieltjes moment sequence. Accepting this, I show how rigorous lower bounds on the growth constant of this sequence can be constructed, which are stronger than existing bounds.

Arctic curves and limit shapes

Filippo Colomo
INFN, Sezione di Firenze

We shall review the state of the art in the field, and, time permitting, illustrate some of its numerous connections with other branches of mathematics and physics, ranging from algebraic combinatorics to quantum quenches.
Limit shapes for the asymmetric five vertex model

Jan De Gier
The University of Melbourne

We derive the local surface tension for the asymmetric five vertex model as a function of both local horizontal and vertical magnetisation. As a consequence, limit shapes for 5V configurations in finite geometries can be determined via the Euler-Langrange equation, of which the general solution is parametrised by a single analytic function. This work is in collaboration with Rick Kenyon and Samuel Watson, and is an extension of general limit shape theory beyond the free fermion (dimer) case.

A fresh look at Inozemtsev’s elliptic spin chain

Jules Lamers
The University of Melbourne

Three decades ago Inozemtsev discovered an isotropic long-range spin chain with an elliptic pair potential that interpolates between the Heisenberg and Haldane–Shastry spin chains while admitting an exact solution throughout, based on a connection with the \((g = 2)\) elliptic quantum Calogero–Sutherland model. Although Inozemtsev’s spin chain is widely believed to be quantum integrable, the underlying algebraic reason for its exact solvability is not yet well understood.

As a step in this direction we clarify various aspects of its exact solution. We refine Inozemtsev’s ‘extended coordinate Bethe ansatz’ motivated by the connection with the well-known exact solutions of the limiting spin chains. We identify more suitable quasimomenta in terms of which the M-particle energy is close to being (functionally) additive, as one would expect from the limiting models; our expression is additive iff the energy of the elliptic Calogero–Sutherland system is so. We define a (two-body) S-matrix that is independent of positions, and enables us to clarify the limit to the Heisenberg scattering phases and Bethe-ansatz equations. Our reformulation furthermore allows us to rewrite the elliptic M-particle energy and Bethe-ansatz equations on the elliptic curve, turning the spectral problem into an algebraic problem.

This is work in progress together with Rob Klabbers (Nordita, Stockholm).
An integrable model of the quantum toroidal $gl_1$

Alexandr Garbali
The University of Melbourne

The quantum toroidal $gl_1$ algebra has an exciting representation theory and multiple connections to various problems in mathematical physics. An interesting direction is the study of the integrable model given on the Fock representation of this algebra.

In my talk I will focus on the problem of calculation of the R-matrix for this integrable model.

Towards a loop model for the superintegrable chiral Potts model

Murray Batchelor
Australian National University

The superintegrable chiral Potts model occupies a special place in the pantheon of exactly solved models. The hamiltonian of the $N$-state superintegrable chiral Potts chain can be written in terms of a coupled algebra defined by $N-1$ types of Temperley-Lieb generators. I will present a pictorial representation of this coupled algebra which involves a generalisation of the well-known pictorial representation of the Temperley-Lieb algebra to include $N-2$ lines around which loops can become entangled. The pictorial representation provides a graphical proof of the algebraic relations. A crucial ingredient in the resolution of diagrams is a crossing relation for loops encircling a line. It is anticipated that this approach may lead to further progress in understanding various aspects of the superintegrable chiral Potts model.

This talk is based on work with Remy Adderton and Paul Wedrich.
Integrable field theories with higher germs spectral curve

Kevin Costello
University of Cambridge

I will discuss a class of two-dimensional integrable field theories I introduced with Masahito Yamazaki. These are integrable s-models whose target is a moduli space of bundles on a Riemann surface, and whose spectral parameter lives in the same Riemann surface. If time permits, I will mention the connection between the RG flow of these models and billiards dynamical systems.

Via video link

Chiral Potts Models, Gauge Theories and Beyond

Masahito Yamazaki
Kavli IPMU, University of Tokyo

The celebrated chiral potts model is a mysterious integrable lattice models with a higher-genus spectral curve. In this talk I will discuss how this model (along with their generalizations) can be obtained naturally in gauge theories, and along the way comment on connections with a number of different topics in mathematics and physics.

When applied mathematics collided with algebra

Nalini Joshi
The University of Sydney

Imagine walking from one tile to another on a lattice defined by reflections associate with an affine Coxeter or Weyl group. Examples include triangular or hexagonal lattices on the plane. Recently, it was discovered that translations on such lattices give rise to the Painlevé equations, which are reductions of integrable systems that are more familiar to applied mathematicians and mathematical physicists. I will explain this surprising
development through introductory examples and explain the background to the discovery of continuous and discrete Painlevé equations.

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**Four-points correlation functions and octagons**

Ivan Kostov  
Institut de Physique Theorique - CEA Saclay

Remarkably, a class of four-point functions of heavily charged BPS operators in $N = 4$ supersymmetric multicolor Yang-Mills theory can be computed analytically for any value of the gauge coupling. The computation boils down to that of a special form factor - the octagon. In dual string picture at strong coupling, the octagon is a kind of twist operator creating a “half-branch point” - a defect with negative curvature $-\pi$ on the world sheet of the string. The octagon depends on the coordinates and the polarisations of the four operators via two pairs of cross ratios. I will explain the representation of the octagon as an expectation value for a system of free fermions. The resulting expression for the octagon in terms of a Fredholm determinant allows one to obtain explicit expressions in the weak/strong coupling limits or for special kinematics.

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**Normalizer theory of Coxeter groups and discrete integrable systems**

Yang Shi  
Finders University

The normalizer theory of Coxeter groups was developed in Howlett (1980), Brink and Howlett (1999). Here we look at how it can be used to understand some peculiar cases in Sakai's classification of discrete Painlevé equations; and to clarify the relationships between various higher-dimensional generalisations of discrete integrable systems.
The non-commutative KdV equation and enumerative geometry

Paolo Rossi
Università degli Studi di Padova

In the past years, in a joint project with A. Buryak, we have developed a general framework to construct classical and quantum field systems (in one space and one-time dimensions) from intersection theory of the moduli space of stable curves. Recently a particularly interesting example came under our attention, which actually produces an integrable system of Hamiltonian PDEs in two space and one-time dimensions. Computations of quadratic double ramification integrals, an entirely natural problem in algebraic geometry, turn out to produce an interesting generalization of the KdV hierarchy to 2 non-commutative space dimensions (KdV on a noncommutative Moyal torus).

Painlevé transcendents in quantum mechanics

Ian Marquette
The University of Queensland

In recent years, progress toward the classification of two-dimensional superintegrable systems with higher order integrals of motion has been made. In particular, a complete classification on Euclidean space of all exotic potentials with a third or a fourth order integrals, and allowing separation of variables in Cartesian coordinates. It has been demonstrated how the Chazy class of third order differential equations plays an important role in solving determining equations. It has been conjectured that all quantum superintegrable potentials that do not satisfy any linear equation satisfy nonlinear equations having the Painlevé property.

In addition, it has been discovered that their integrals naturally generate finitely generated polynomial algebras and the representations can be exploited to calculate the energy spectrum. Moreover, one dimensional Hamiltonians related to various operator algebras defined as Abelian, Heisenberg, Conformal and Ladder case of operator algebras were connected to Painlevé transcendents. More recently, a method has been developed for the analysis and classification of higher order superintegrable systems on any 2D Riemannian space. New models with the six Painlevé transcendent on 2-sphere and 2-hyperboloid have been obtained.

BK Berntson, I Marquette, W Miller Jr, a new approach to analysis of 2D higher order superintegrable systems, arXiv:1909.08654
Coefficients of Wronskian polynomials

Clare Dunning
University of Kent

Wronskian Hermite polynomials arise in studies of the Painlevé equations and higher-order analogs, random matrix theory, vortex dynamics, supersymmetric quantum mechanics, integrable models, orthogonal polynomials etc.

I will describe a combinatorial interpretation of the coefficients of Wronskian Hermite polynomials in terms of cores and quotients of partitions. We prove an asymptotic result on the location of the zeros of the polynomials in the complex plane, and make some conjectures on a long open problem regarding simplicity of the zeros.
I will also briefly discuss results for (p-1)-orthogonal polynomials with zeroes on the p-star.

The talk is based on arXiv:1909.03874, co-authored with N Bonneux and M Stevens.
Wronskians of Hermite polynomials, anharmonic oscillators and Painlevé IV

Pieter Roffelsen
International School for Advanced Studies (SISSA)

Wronskians of Hermite polynomials, anharmonic oscillators and Painlevé IV Arguably, the most striking property of Wronskians of consecutive Hermite polynomials is that they define Painlevé IV tau-functions with highly degenerate associated monodromies. About fifteen years ago, Peter Clarkson published numerical investigations on the zero distributions of such Wronskians in the complex plane and posed the problem of giving an analytic explanation of his numerical findings. In this talk, I will present an asymptotic answer to Clarkson’s problem obtained in recent joint works with Davide Masoero. I will discuss how zeros of such Wronskians are intimately related to certain anharmonic oscillators satisfying two quantisation conditions simultaneously. This allows for the derivation of exact and asymptotic properties of the former by means of the application of Nevanlinna theory and complex WKB methods to the latter.

Cluster integrable systems and supersymmetric gauge theories

Andrei Marshakov
CAS Skoltech

The construction of integrable systems on cluster varieties is related with recent results in super-symmetric gauge theory. Various aspects of this correspondence are discussed from the perspectives of classical problems of analysis and representation theory.

R-matrices in Cluster Algebras

Rei Inoue
Chiba University, Japan

In this talk I introduce two types of `R-matrices' realized as sequences of cluster mutations, based on joint works with Kazuhiro Hikami [2], Thomas Lam, Pasha Pylyavskyy [4], Tsukasa Ishibashi and Hironori Oya [3].
The cluster algebra was introduced around 2000 by Fomin and Zelevinsky. The heart of the algebra is algebraic operations called 'mutations' acting on a quiver and a set of variables on its vertices. This algebra has been variously applied in mathematics and mathematical physics. One idea is to find an interesting sequence of mutations which preserves a quiver Q, from which we define a rational map for the variables on the vertices. Such sequences of mutations constitute a group called the cluster modular group $\Gamma_Q$.

The first $R$-matrices are defined with a quiver $Q_m(An)$ on a cylinder. We have $n$ mutation sequences $R_1; R_2; \ldots; R_n$ which preserve $Q_m(An)$ and generate the action of Weyl group $W(An)$ of $An$-type [4]. The second $R$-matrices are defined with a quiver $D_p(An)$ on a disk with $p$ punctures and two marked points at the boundary. We have $p-1$ mutation sequences $b_1; b_2; \ldots; b_{p-1}$ which preserve $D_p(An)$ and generate the action of $p$-Braid group $B_p$ [2, 6]. The $b_i$ really come from the universal $R$-matrix of the quantum group $U_q(An)$, and geometrically $b_i$ realizes the half Dehn twist of the $i$th and $(i+1)$st punctures.

A remarkable facts are that the quivers $Q_{n+1}(An)$ and $D_1(An)$ are mutation equivalent, and that $D_p(An)$ is obtained by 'amalgamating' $p$ copies of $D_1(An)$. According to the higher Teichmüller theory introduced by Fock and Goncharov, the $R_i$ generate the geometric $W(An)$-action around a puncture in $D_1(An)$ [1], which extends to that around $p$ punctures in $D_p(An)$. Finally it follows that $B_p(An) \rtimes W(An)^p \subset \Gamma D_p(An_i)$.

This story is not only for $An$ but also for finite dimensional simple Lie algebras [3, 5].

References
Nonequilibrium physics in integrable systems and spin-flip non-variant conserved quantities

Chihiro Matsui
The University of Tokyo

Recently found spin-flip non-invariant (SFNI) conserved quantities play important roles in discussing nonequilibrium physics of the XXZ model. The representative examples are the generalized Gibbs ensemble (GGE) and the ballistic transport of the spin current. In spite of big progress in understanding nonequilibrium physics of integrable systems, the general framework to determine a minimal complete set of conserved quantities which describes the long-time steady state has not yet been found.

In my talk, I show that the GGE of the gapless XXZ model consists of functionally independent conserved quantities rather than linearly independent. At the same time, the physical meaning of SFNI conserved quantities is provided. I also discuss that there exist ballistic channels of the spin current supported by non-quasilocal conserved quantities. The saturation of the lower bound for the Drude weight by quasilocal conserved quantities reads the linear dependence of non-quasilocal conserved quantities on quasilocal ones. I show that their (generalized) linearly dependence relation is consistent with the statement that the GGE consists of functionally independent conserved quantities without containing all linearly independent conserved quantities.

The six-vertex model on random lattices revisited

Paul Zinn-Justin
The University of Melbourne

We consider the six-vertex model on dynamical random lattices, or equivalently, the weighted enumeration of planar 4-valent maps equipped with an Eulerian orientation. This problem was solved exactly 20 years ago by I. Kostov, but has reemerged in the recent combinatorics literature. We reformulate in purely combinatorial terms Kostov's solution, then analyze its modular properties, leading to explicit linear differential equations for the corresponding generating series.
Exactly solved models: past, present and future

Angela Foerster
INSTITUTO DE FISICA DA UFRGS

In this talk we begin with a brief historical overview of the application of exactly solved models to physical systems. Then we discuss our present research project, a general construction of exactly solved models for boson tunneling in multi-well systems. As an application we show how to engineer an atomtronic switching device by exploring the triple well model [1]. Subsequently we study the generation of entangled states in this triple well case [2]. We conclude with a vision for the future and some challenges in the area.


Conserved charges of the XXZ chain in the Temperley-Lieb representation

Bernard Nienhuis
University Leiden

Conserved charges of the XXZ chain in the Temperley-Lieb representation

Conservation laws play a central role in physics. In integrable systems, the number of conserved quantities is as great as the number of degrees of freedom. We studied the structure of the quasi-local conserved quantities (or charges) of the XXZ chain, as a sequence of expressions of increasing degree in terms of generators of the Temperley-Lieb (TL) algebra. We calculated the first ten conserved quantities by direct calculation of the k-th derivative of the logarithm of the transfer matrix, with respect to the spectral parameter.

Based on the expressions we found, we observed a number of remarkable and simplifying properties of these expressions. If our observations are valid they predict the the entire sequence of conserved charges. Thus we predicted the sequence up to 17, limited only by (modest) computational power. Indeed all of these commute with the Hamiltonian, which, besides the degree in terms of TL generators, is the essential defining property. We note that
the computational complexity is determined by generating all terms, and not by computing their coefficients.

In conclusion we conjecture the form of all conserved charges of the XXZ chain. The work reported was done in collaboration with my then master student Onno Huygen.