TITLE AND ABSTRACT

(Conference on geometric and nonlinear PDEs)
(4-10/2/2018, Murramarang)

Speaker: Ben Andrews

Title: Motion of hypersurfaces by harmonic mean curvature

Abstract: I will report on joint work with Xuzhong Chen (Hunan University) and Guoyi Xu (Tsinghua) on the motion of hypersurfaces in Riemannian manifolds by the harmonic mean of their principal curvatures. This flow appeared in 1994 in a proof of the sphere theorem, where convex hypersurfaces in positively curved background spaces were shown to contract to points. Here we observe that convex hypersurfaces remain convex in rather arbitrary background spaces, and provide a result which completely characterises their long term behaviour.

Speaker: Zbigniew Blocki

Title: Complex Monge-Ampère operator for plurisubharmonic functions with analytic singularities

Abstract: We discuss a problem of defining complex Monge-Ampère operator for non-smooth plurisubharmonic functions. It is much more subtle than in the real case when it is always possible for admissible functions (also for the Hessian operators, as shown by Trudinger and Wang). In particular, we will present results on plurisubharmonic functions with analytic singularities (this is a joint work with Mats Andersson and Elizabeth Wulcan).

Speaker: Xiaodong Cao

Title: Einstein 4-manifolds with pinched sectional curvature

Abstract: In this talk, I will start with an introduction to the Einstein 4-manifold. Then I will discuss some earlier result on classification of the positive case. Finally I will mention some recent development in this area.

Speaker: Jingyi Chen

Title: The space of compact shrinkers along Lagrangian mean curvature flow in the complex plane

Abstract: We will discuss rigidity, compactness, and finite quantization of entropy for the space of self-shrinking Lagrangian surfaces. This is based on joint work with John Man Shun Ma.

Speaker: Shibing Chen

Title: Stability results on the smoothness of optimal transport maps with general costs,

Abstract: Given domains Ω, Λ associated with densities f, g. Let u be the potential function of the optimal transport problem with cost function c. Suppose there is a sequence of cost functions c_n, densities f_n, g_n converging to c, f, g respectively, it is well known that the corresponding potential function u_n (up to a constant) converges to u in L^∞ norm. In this talk,I will discuss some results concerning the higher order ($C^{2,\alpha}$) convergence.

Speaker: Florica Cirstea

Title: Singular solutions to an elliptic equation with a sign-changing nonlinearity

Abstract: In this talk, we provide a full description of the profile of the positive solutions near an isolated singularity for a class of elliptic equations involving critical Hardy-Sobolev type potentials and a pure power nonlinearity. Because of the interaction between these two terms, we show that along with a Caffarelli-Gidas-Spruck type profile, the solution may develop two new singular behaviors. We also prove the actual existence of all these profiles. The results are based on works with F. Robert (University of Lorraine) and J. Vetois (McGill University).

Speaker: Panagiota Daskalopoulos

Title: Fully Nonlinear Geometric Flows

Abstract: We will discuss the evolution of entire graphs by Inverse Mean Curvature flow and Gauss Curvature flow. We will address the questions of long time existence and regularity.

Speaker: Vincenzo Ferone

Title: Weinstock inequality in higher dimensions

Abstract: We consider the problem of extending Weinstock inequality for the first nonzero Steklov eigenvalue to the case of sets in $\mathbb{R}^n$, for $n \geq 3$. A key result is a sharp isoperimetric inequality involving simultaneously the surface area, the volume and the boundary momentum of convex sets. Applications to isoperimetric inequalities for the first Wentzell eigenvalue will be also discussed.

Speaker: Nicola Fusco

Title: Stability results for some moving interface problems

Abstract: I will present a local minimality result for the sharp interface energy arising as limit of the Ohta-Kawasaki model. Using this result it is possible to show that three-dimensional strictly stable periodic configuration are exponentially stable for for the nonlocal Mullins-Sekerka and for the Hele-Shaw flow.

Speaker: Juanru Gu

Title: The sphere theorem for compact submanifolds inspired by Ejiri's rigidity theorem

Abstract: In 1970's, Simons-Chern-do Carmo-Kobayashi-Lawson, Yau, and Ejiri verified three well-known rigidity theorems for minimal submanifolds in spheres, respectively. During the past four decades, there has been important progress on this aspect. In 2013, Xu and I proved the generalized Eijiri's rigidity theorem for compact submanifolds with parallel mean curvature in space forms. Motivated by the above rigidity results, we proved an optimal topological sphere theorem for submanifolds under Ricci curvature pinching condition. Recently, Xu and I obtained a new optimal sphere theorem which improves our previous sphere theorem (GAFA, 2013).

Speaker: Daniel Hauer

Title: Non-convex superlevel sets of Robin eigenfunctions

Abstract: On a convex bounded Euclidean domain, the eigenfunction for the Laplacian with Neumann boundary conditions is a constant, while the Dirichlet eigenfunction is log-concave. The Robin eigenvalue problem can be considered as interpolating between the Dirichlet and Neumann cases, so it seems natural that the Robin eigenfunctions should have similar concavity properties. Log-concavity implies convexity of superlevel sets of a function, while the converse is not true. In this talk, I show that this is not the case: there exist convex domains, and small values of the Robin parameter $\alpha > 0$, for which the first Robin eigenfunction u_α fails to be log-concave and has some non-convex super-level sets.

The results presented in this talk are from [arXiv:1711.02779] and obtained in joint work with Ben Andrews and Julie Clutterbuck.

Speaker: Min-Chun Hong

Title: The gauge fixing theorem with applications to the Yang-Mills flow over Riemannian manifolds

Abstract: In 1982, Uhlenbeck established the well-known gauge fixing theorem, which has played a fundamental role for Yang-Mills theory. In this paper, we apply the idea of Uhlenbeck to establish a parabolic type of gauge fixing theorems for the Yang-Mills flow and prove existence of a weak solution of the Yang-Mills flow on a compact n-dimensional manifold with initial value A_0 in $W^{1,n/2}(M)$.

Speaker: Wenshuai Jiang

Title: The Structure of Noncollapsing Ricci Limit Spaces

Abstract: Let us consider $(M_i^n, g_i, p_i) \to (X, d, p)$ in Gromov-Hausdorff sense with $\mathrm{Vol}(B_1(p_i)) > v > 0$ and $\mathrm{Ric} \geq -(n-1)$. It is known from Gromov's precompactness theorem that X is a metric space. We will show that the singular set S of X is (n-2)-rectifiable. More generally,

for $0 \leq k < n$, the k-stratum $S^k = \{x \in X : \text{no tangent cone at } x \text{ splits a } R^{k+1} \text{ factor}\}$ is k-rectifiable. We will also discuss the quantitative estimate of S^k. This is joint work with Professor Jeff Cheeger and Aaron Naber.

Speaker: Slawomir Kolodziej

Title: Weak solutions of m-Hessian equations

Abstract: This is joint work with Cuong Ngoc Nguyen. We shall discuss weak solutions to the complex Hessian equation in subdomains of $\mathbb{C}^n$ and on compact Hermitian manifolds. In the first situation we prove the existence of the Hölder continuous solution to the Dirichlet problem when a Hölder continuous subsolution exists. On a compact Hermitian manifold (X, ω) of complex dimension n we solve the complex m-Hessian equation

$$(\omega + dd^c\varphi)^m \wedge \omega^{n-m} = cf\omega^n, \quad \omega + dd^c\varphi \geq 0,$$

where $0 \leq f \in L^p(X, \omega^n)$, $p > n/m$, and u is so called (ω, m) subharmonic function.

Speaker: Martin Li

Title: Desingularizing minimal surfaces

Abstract: Gluing constructions have proven to be powerful tools to construct solutions to various geometric problems. Pioneered by the work of Schoen on metrics with constant scalar curvature, Kapouleas made use of the methodology to construct new examples of minimal surfaces via doubling or desingularization constructions. In this talk, we will survey on some of the known examples which can be regarded as a desingularization construction. Then, we will discuss how these ideas can be applied to study free boundary minimal surfaces in the unit ball. This is joint work with N. Kapouleas. These work are partially supported by RGC grants from the Hong Kong Government.

Speaker: Qi-Rui Li

Title: Continuity of the optimal transport for Monge's original cost

Abstract: The optimal transportation problem was proposed by Monge in 1781. Since then the problem has been extensively studied and more general costs are allowed. The existence and regularity of optimal mappings have been established under certain conditions. But Monge?s original cost does not satisfy these conditions, and very little is known about the regularity of its optimal mapping. In this talk, we show that, in two dimensional case, the optimal mapping for Monge?s original cost is continuous. By a counter-example we also show that the mapping fails to be Lipschitz in general.

This is a joint work with F. Santambrogio and X.-J. Wang.

Speaker: Gregoire Loeper

Title: Reconstruction by optimal transport: applications in cosmology and finance

Abstract: Following the seminal work by Benamou and Brenier on the time continuous formulation of the Optimal Transport problem, we extend their approach to solve a problem of reconstruction in cosmology, and to a problem of volatility calibration in finance.

Speaker: Peng Lu

Title: Construction of ancient solutions of the Ricci flow on torus bundles

Abstract: We give an overview of the ancient solution of Ricci flow.

Speaker: Zhiqin Lu

Title: Hearing the shape of a trapezoid by its eigenvalues

Abstract: We shall prove that the shape of a trapezoid is determined by its Neumann eigenvalues. The Dirichlet eigenvalue case will also be discussed, with the classification on short families of closed geodesics on triangles and trapezoids. This is the joint work with Hamid Hezari and Julie Rowlett.

Speaker: Alexander Majchrowski

Title: Neck Detection for a 2-Convex Hypersurface Undergoing Exterior Flows.

Abstract: In 2009 Huisken and Sinestrari inspired by Hamiltons work on Ricci Flow described how to perform surgery on a 2-convex hypersurface immersed in Euclidean space undergoing mean curvature flow. I will discuss the process of neck detection in the mean curvature flow setting and how this can be extended to the fully nonlinear flow $G = \sum_{i<j} \frac{1}{\lambda_i+\lambda_j}$. This fully nonlinear flow G was studied by Brendle and Huisken to overcome the problems that mean curvature flow with surgeries of 2?convex hypersurfaces presented, mainly that 2-convexity was not preserved when the hypersurface was immersed in a general Riemannian manifold.

Speaker: Robert McCann

Title: On Concavity of the Monopolist's Problem Facing Consumers with Nonlinear Price Preferences

Abstract: The principal-agent problem is an important paradigm in economic theory for studying the value of private information; the nonlinear pricing problem faced by a monopolist is a particular example. In this lecture, we identify structural conditions on the consumers' preferences and the monopolist's profit functions which guarantee either concavity or convexity of the monopolist's profit maximization. Uniqueness and stability of the solution are particular consequences of this concavity. Our conditions are closely related to criteria given by Trudinger and others for prescribed Jacobian equations to have smooth solutions, while being simpler in many respects. By allowing for different dimensions of agents and contracts, nonlinear dependence of agent preferences on prices, and of the monopolist's

profits on agent identities, it improves on the literature in a number of ways. The same mathematics can also be adapted to the maximization of societal welfare by a regulated monopoly, This is joint work with PhD student Shuanjian Zhang.

Speaker: Jie Qing

Title: Strong rigidity for asymptotically hyperbolic Einstein manifolds

Abstract: In this talk I will talk on our recent work on asymptotically hyperbolic Einstein manifolds. I will present a proof for a sharp volume comparison theorem for asymptotically hyperbolic Einstein manifolds, which will imply not only the rigidity theorem for hyperbolic space in general dimension but also curvature estimates for asymptotically hyperbolic Einstein manifolds. In particular, as a consequence of our curvature estimates, one now knows that the asymptotically hyperbolic Einstein metrics with conformal infinities of sufficiently large Yamabe constant have to be negatively curved.

Speaker: Natasa Sesum

Title: Non-Kähler Ricci flow singularities that converge to Kähler–Ricci solitons

Abstract: In the joint work with Isenberg and Knopf we investigate Riemannian (non-Kähler) Ricci flow solutions that develop finite-time Type-I singularities with the property that parabolic rescalings at the singularities converge to singularity models taking the form of shrinking Kähler–Ricci solitons. More specifically, the singularity models for these solutions are given by the "blowdown soliton" discovered in [FIK03]. Our results support the conjecture that the blowdown soliton is stable under Ricci flow. This work also provides the first set of rigorous examples of non-Kähler solutions of Ricci flow that become asymptotically Kähler, in suitable space-time neighborhoods of developing singularities, at rates that break scaling invariance. These results support the conjectured stability of the subspace of Kähler metrics under Ricci flow.

Speaker: Leon Simon

Title: Nonlinear elliptic problems which admit solutions with cylindrical tangents at isolated singular points

Abstract: We discuss some nonlinear elliptic equations which admit solutions with isolated singularities at 0, yet which have cylindrical asymptotic behavior at 0

Speaker: Valentino Tosatti

Title: Estimates for collapsing Ricci-flat metrics

Abstract: I will discuss a priori estimates for families of Ricci-flat metrics on a compact Calabi-Yau manifold which fibers over a lower-dimensional base, as the size of the fibers shrinks to zero. These metrics are obtained by solving a family of complex Monge-Ampère

equations with ellipticity degenerating in the fiber directions. I will present C^k a priori estimates for all positive k when the fibers are isomorphic to each other, and C^α Holder estimates in general. The new technical tools are sharp new Schauder estimates for the Laplacian on cylinders, and nonlinear Liouville theorems on cylinders. This is joint work with H.-J. Hein.

Speaker: Dongrui Wan

Title: An introduction to quaternionic Monge-Ampre equation and viscosity approach

Abstract: In this talk, we will discuss some potential results for the quaternionic Monge-Ampre equations. We generalize the pluripotential theory for the complex Monge-Ampre operator to that for the quaternionic Monge-Ampre operator, and develop the theory of quaternionic closed positive currents on quaternionic space. Moreover, we present the viscosity approach to the quaternionic Monge-Ampre equations. We prove a viscosity comparison principle and a solvability theorem. The equivalence between viscosity and pluripotential solutions is also present.

Speaker: Feng Wang

Title: The existence of Kahler-Einstein metrics on K-polystable Q-Fano varieties with non-positive discrepancies

Abstract: I will talk out the recent work with Professor Tian and Chi Li. At first, we extend Tian's work to the log smooth case. Then for a K-polystable Q-Fano varieties X with non-positive discrepancies, we show that there exists conic KE metrics on the resolution and these metrics converges to the singular KE metric on X.

Speaker: Micah Warren

Title: Continuum Nash Bargaining Solutions

Abstract: Nash's classical bargaining solution suggests that n players in a non-cooperative bargaining situation should find a solution that maximizes the product of each player's utility functions. We consider a special case: Suppose that the players are chosen from a continuum distribution, and suppose they are to divide up a resource, that is also on a continuum. The utility to each player is determined by the exponential of a distance type function. The maximization problem becomes an optimal transport type problem, where the target density is the minimizer to the functional which is that sum of an entropy term and a Wasserstein distance term, similar to the minimization problem solved in the Jordan-Kinderlehrer-Otto scheme. Thanks to optimal transport theory, the solution may be described by a potential that solves a fourth order nonlinear elliptic PDE, similar to Abreu's equation. Using the PDE, we prove solutions are smooth when the measures have smooth positive densities.

Speaker: Changwei Xiong

Title: Convexity of nonnegatively curved hypersurfaces with free boundary on a sphere

Abstract: In 1970, do Carmo and Warner proved that any immersed closed hypersurface with sectional curvatures no less than one in the sphere must be globally convex. Recently, especially after Fraser and Schoen's work in 2011, geometric objects in the sphere and those in the Euclidean ball with free boundary are showing certain similarities. So it is natural to expect that any immersed hypersurface in the Euclidean ball with nonnegative sectional curvatures and with free boundary must be globally convex. This is indeed the case, as shown by my joint work with Mohammad Ghomi. In this talk we will review relevant works and outline the proof of our result.

Speaker: Gaoyong Zhang

Title: Minkowski Problems of Geometric Measures,

Abstract: The classical Minkowski problem asks for necessary and sufficient conditions in order to construct a convex body in the Euclidean space whose surface area measure (or Gauss curvature in the smooth case) is equal to a given measure on the unit sphere. The partial differential equation associated with the Minkowski problem is a Monge-Ampère equation with measure data. We explain unsolved Minkowski problems for other important geometric measures in convex geometry, and discuss recent work in solving Minkowski problems via measure concentration conditions.

Speaker: Zhenlei Zhang

Title: On the limit of Kahler metrics under La Nave-Tian continuity method

Abstract: La Nave and Tian introduced a new approach to the Analytic Minimal Model Program. It is a continuity method of complex Monge-Ampere equations. If the continuity method develops singularity in finite time and the class at that time is big, La Nave and Tian showed that there is a unique current limit. I will talk about the geometric structure of the limit current. An algebraic condition is assumed. It is a joint work with La Nave and Tian.

Speaker: Bin Zhou

Title: Properness of energy functionals on polarized compactifications of reductive Lie groups

Abstract: In this talk, I will first give an introduction on Tian's properness conjecture concerning on an analytic charactarization of the existence of canonical metrics in Kahler geometry. Then I will focus on compactifications of reductive Lie groups. The main results are criterion theorems of the properness of two important functionals: Ding functional and Mabuchi's K-energy on these manifolds. In particular, the existence of Kahler-Einstein metrics, Kahler-Ricci solitons and Mabuchi's generalized Kahler-Einstein metrics on Fano compactifications of reductive Lie groups can be established.