TOROIDAL ALFVÉN EIGENMODES

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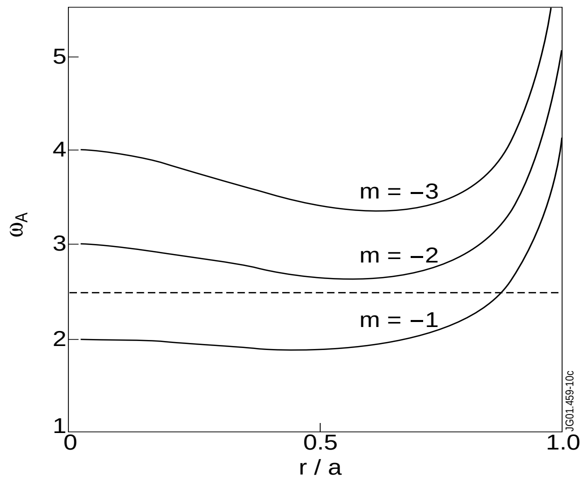
OUTLINE OF LECTURE 4

- Toroidicity induced frequency gaps and Toroidal Alfvén Eigenmodes
- TAE observations on JET
- Doppler shift
- n-dependence of the TAE fast ion drive
- Bursting Alfvén instabilities on MAST
- Summary

GLOBAL ALFVÉN EIGENMODE IN CYLINDRICAL PLASMA

- **The existence of weakly-damped GAE in cylindrical plasma is determined by the extremum point of $\omega_A(r) = k_\parallel(r)V_A(r)$:**

$$\left.\frac{d\omega_A(r)}{dr}\right|_{r=r_0} = 0, \text{ i.e. } \frac{1}{k_\parallel}\frac{dk_\parallel}{dr} = -\frac{1}{V_A}\frac{dV_A}{dr}$$



- **How the function $\omega_A(r) = k_\parallel(r)V_A(r)$ looks like in toroidal geometry? Could some extremum points exist there?**

THE PARALLEL WAVE-VECTOR IN TORUS

In a torus, the wave solutions are quantized in toroidal and poloidal directions:

$$\phi(r,\vartheta,\zeta,t) = \exp(-i\omega\, t + in\zeta)\sum_m \phi_m(r)\ \exp(-im\vartheta) + c.c.$$

n **is the number of wavelengths in toroidal direction and**
m **is the number of wavelengths in poloidal direction**

- **The parallel wave-vector for the** m**-th harmonic of a mode with toroidal mode number** n is

$$k_{\|m}(r) = \frac{1}{R}\left(n - \frac{m}{q(r)}\right).$$

- **This function is determined by the safety factor** $q(r) = rB_T / RB_P$ **.**

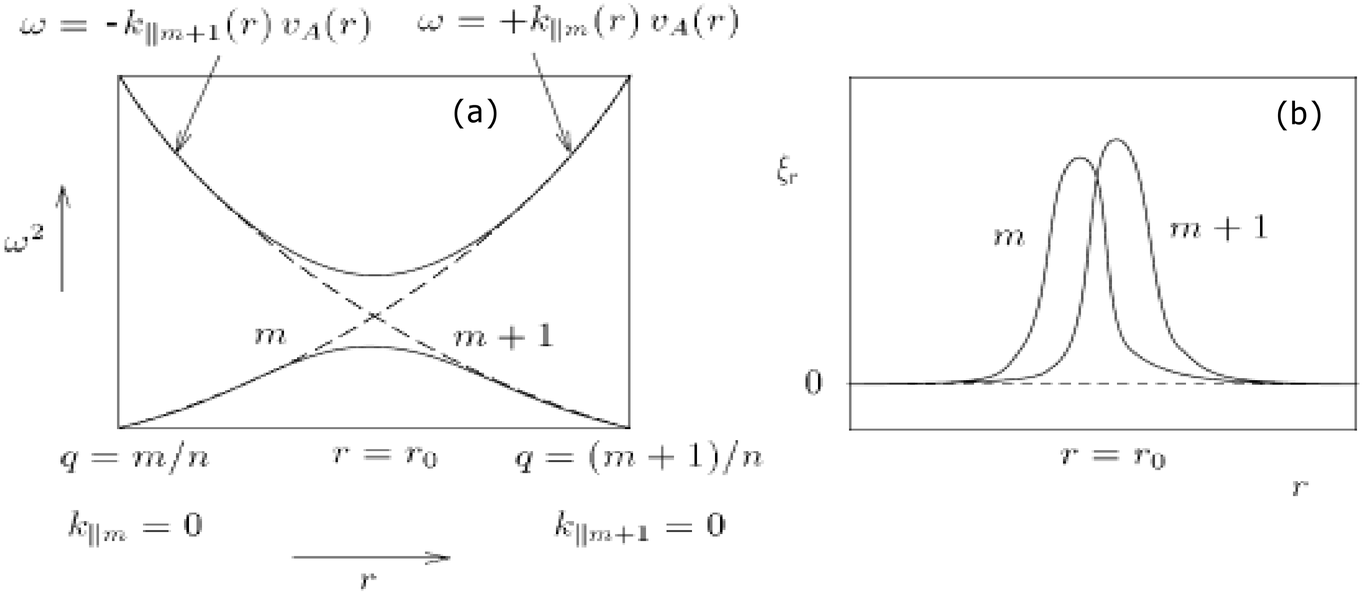
- **Depending on the value of** $q(r)$ **with respect to rational** m/n**, the parallel wave-vector can be zero, positive, or negative**

TOROIDICITY INDUCED COUPLING - 1

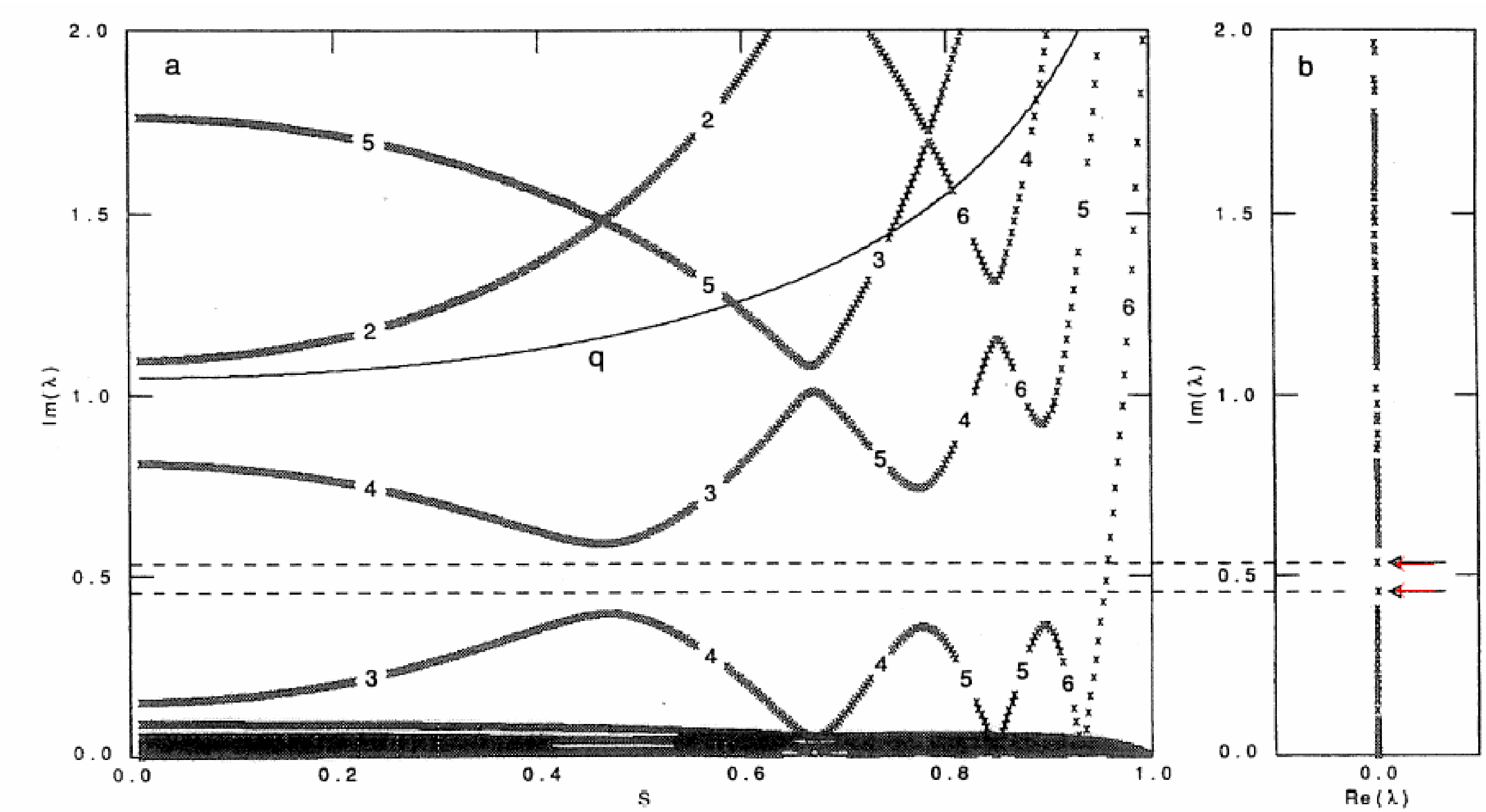
- In torus, two cylindrical SA modes with m and m+1 mode numbers (same n number) have the same frequency at radial point satisfying

$$\omega = k_{\|m}(r)V_A(r) = -k_{\|m+1}(r)V_A(r)$$

- Toroidicity induced "gap" is formed in the SA continuous spectrum at this frequency
- This extremum point in the continuum may cause Toroidal Alfvén Eigenmode to exist



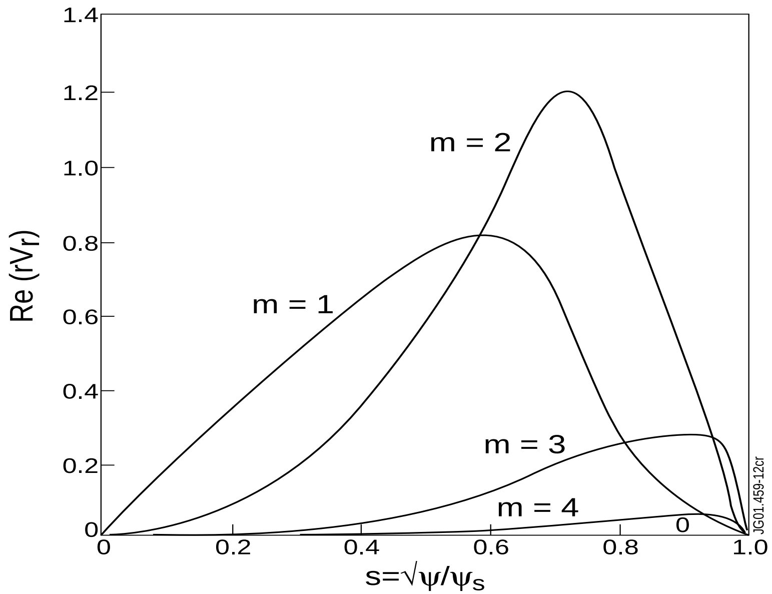
TOROIDICITY INDUCED COUPLING - 2



Typical structure of Alfvén continuum (and SM continuum) in toroidal geometry. Two discrete TAE eigenfrequencies exist inside the TAE gap.

PROPERTIES OF TOROIDAL ALFVÉN EIGENMODES - 1

- **In contrast to GAE, Toroidal Alfén Eigenmode consists of two poloidal harmonics minimum**
- **Similarly to GAE in cylinder, TAE frequency does not satisfy local Alfvén resonance condition in the region of TAE localization, $\omega_{TAE} \neq \omega_A(r) \rightarrow$ TAE does not experience strong continuum damping**



Radial dependence of the Fourier harmonics of TAE

PROPERTIES OF TOROIDAL ALFVÉN EIGENMODES - 2

- **Start from the TAE-gap condition** $\omega = k_{\|m}(r)V_A(r) = -k_{\|m+1}(r)V_A(r)$ **(*)**

$$\downarrow$$

$$k_{\|m}(r) = -k_{\|m+1}(r)$$

$$\downarrow$$

$$\frac{1}{R_0}\left(n - \frac{m}{q(r)}\right) = -\frac{1}{R_0}\left(n - \frac{m+1}{q(r)}\right)$$

$$\downarrow$$

$$q(r) = \frac{m+1/2}{n}$$

- **TAE gap is localised at** $q(r_0) = (m+1/2)/n$ **determined by the SA values of** n **and** m
- **Substitute this value of safety factor in the starting equation (*) to obtain**

$$\omega_0 = \frac{V_A(r_0)}{2R_0 q(r_0)}$$

PROPERTIES OF TOROIDAL ALFVÉN EIGENMODES - 3

- **Typical JET parameters are:**

$$B_0 \cong 3\ T\ ;\ n_i = 5\times 10^{19}\ m^{-3}\ ;\ m_i = m_D$$

$$\downarrow$$

$$V_A \cong 6.6\times 10^{6}\ m/s$$

- **For typical value of the safety factor q=1, the frequency of TAE mode should be**

$$\omega_0 \cong 10^{6}\ \sec^{-1}$$

$$f_0 \equiv \omega_0 / 2\pi \cong 160\ kHz$$

- **Add fast ions to plasma and see…**

HOW TO DETECT SHEAR ALFVÉN WAVES?

- ***Shear Alfvén waves*** **have the following perturbed electric and magnetic fields:**

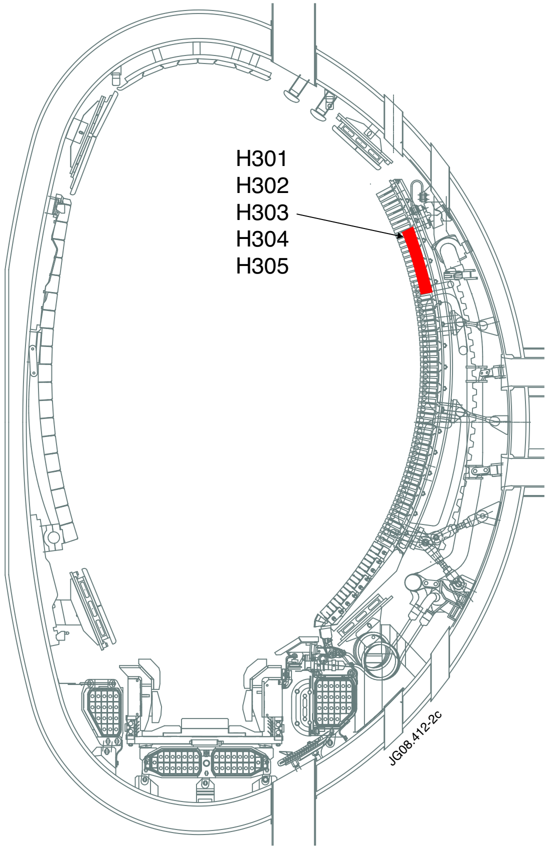
$$\delta B_{\parallel} = 0 \,,\; \delta E_{\parallel} = 0$$

$$\delta E_r = -\frac{\partial \phi_m}{\partial r} \exp\left(i\left[n\zeta - m\vartheta - \omega t\right]\right) + c.c. \quad \delta E_\vartheta = \frac{im}{r} \phi_m \exp\left(i\left[n\zeta - m\vartheta - \omega t\right]\right) + c.c.$$

$$\delta B_r = -\frac{k_\parallel c}{\omega} \cdot \frac{im}{r} \phi_m \exp\left(i\left[n\zeta - m\vartheta - \omega t\right]\right) + c.c. \quad \delta B_\vartheta = -\frac{k_\parallel c}{\omega} \cdot \frac{\partial \phi_m}{\partial r} \exp\left(i\left[n\zeta - m\vartheta - \omega t\right]\right) + c.c.$$

- **Oscillatory perturbed poloidal magnetic field, δB_ϑ, is the easiest to detect with magnetic pick-up coils (Mirnov coils) just outside the plasma**

MAGNETIC PICK-UP COILS



JET cross-section showing the position and directivity of five Mirnov coils separated toroidally

- **The coils measure**

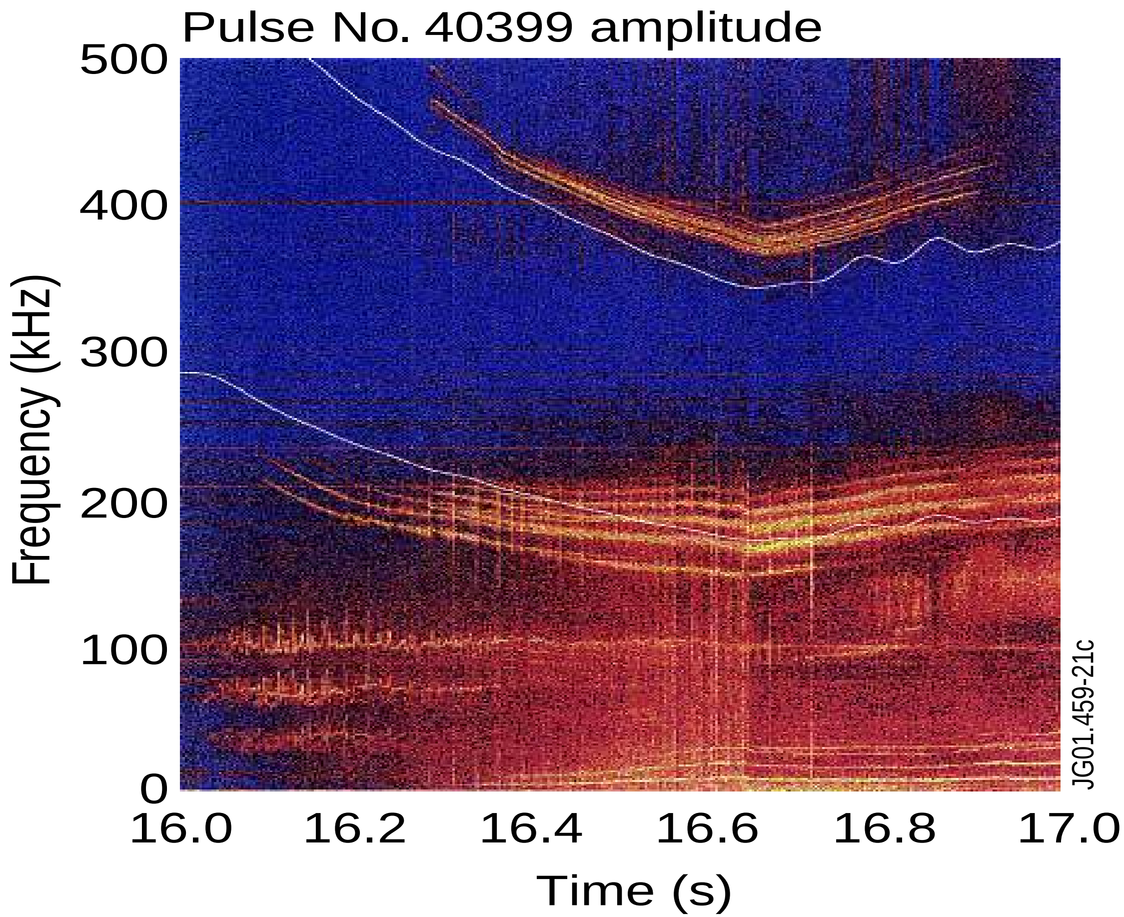
$$\frac{\partial}{\partial t}\delta B_{\vartheta}^{edge} \cong \omega \cdot \delta B_{\vartheta}^{edge}$$

and are VERY sensitive! Thanks to high values of $\omega \cong 10^{6} \ \sec^{-1}$ **perturbed fields** $\left|\delta B_{\vartheta}^{edge} / B_{0}\right| \cong 10^{-8}$ **are measured**

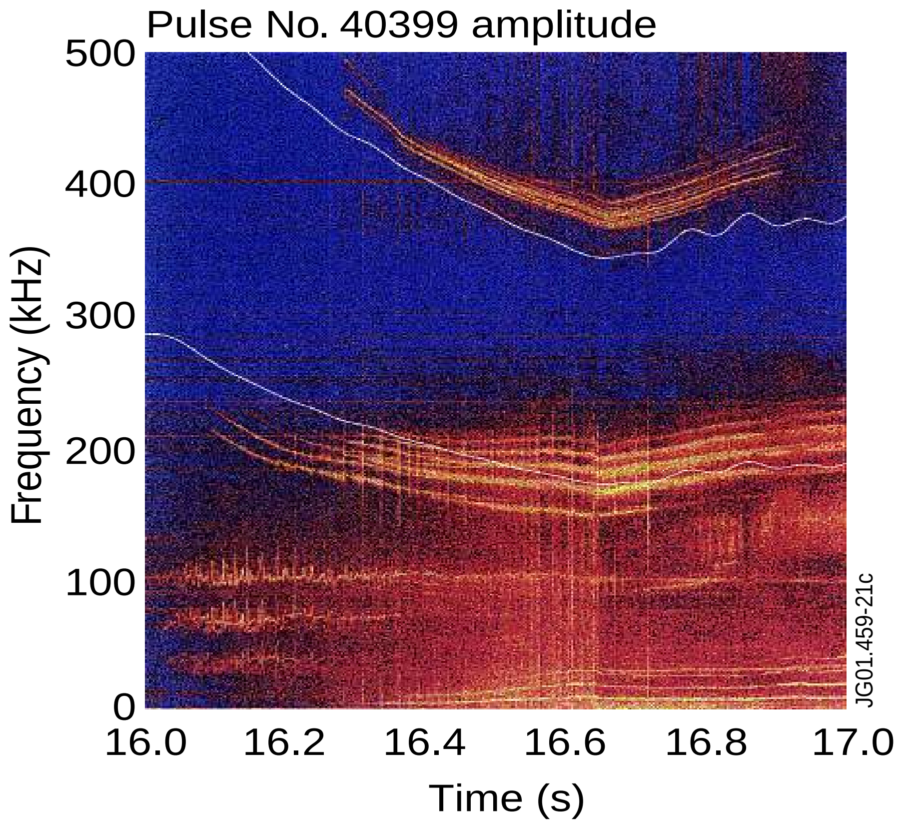
- **Sampling rate 1 MHz allows measurements of AE up to 500 kHz to be made**

- **The coils are calibrated, i.e. give the same amplitude and phase response to the same test signal**

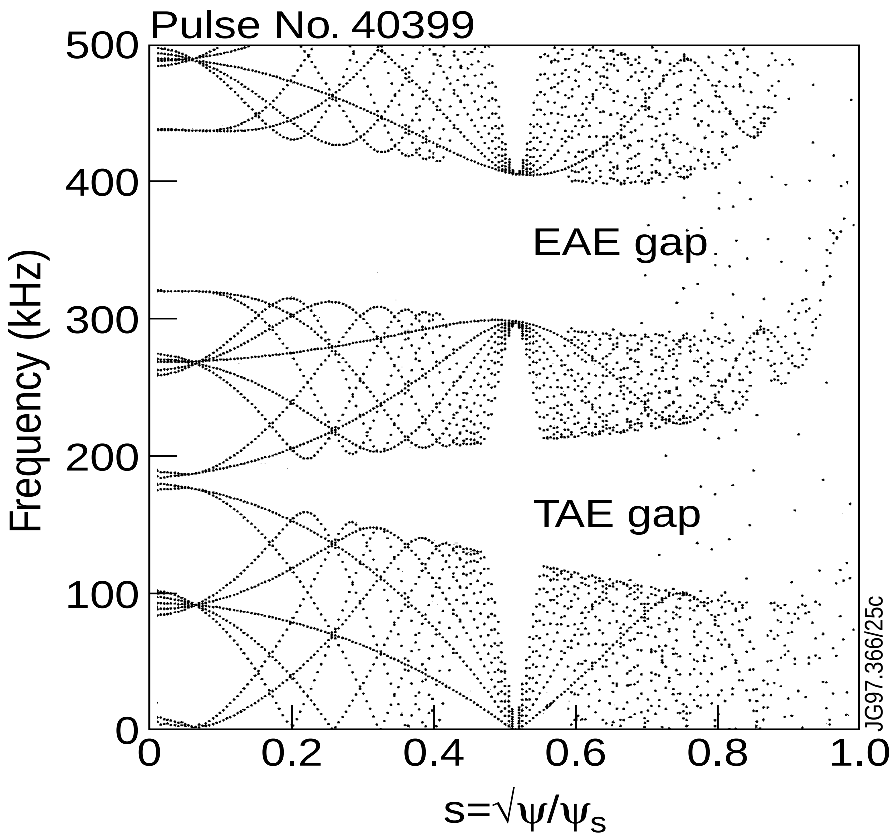
TAE IN JET DISCHARGE WITH ICRH-ACCELERATED FAST IONS



FREQUENCIES OF ALFVÉN EIGENMODES AGREE WITH THEORY

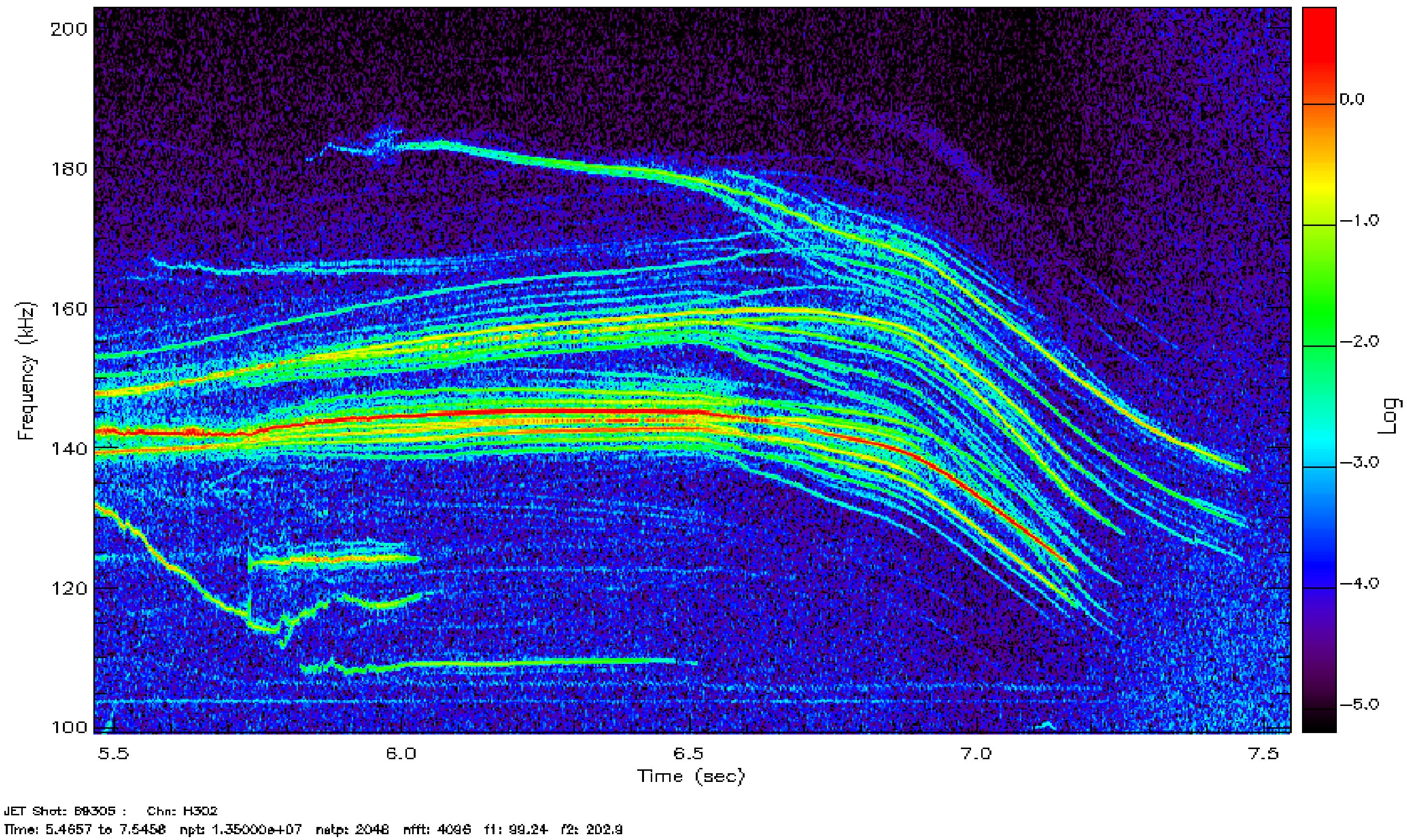


Spectrogram of AE activity excited by ICRH ions

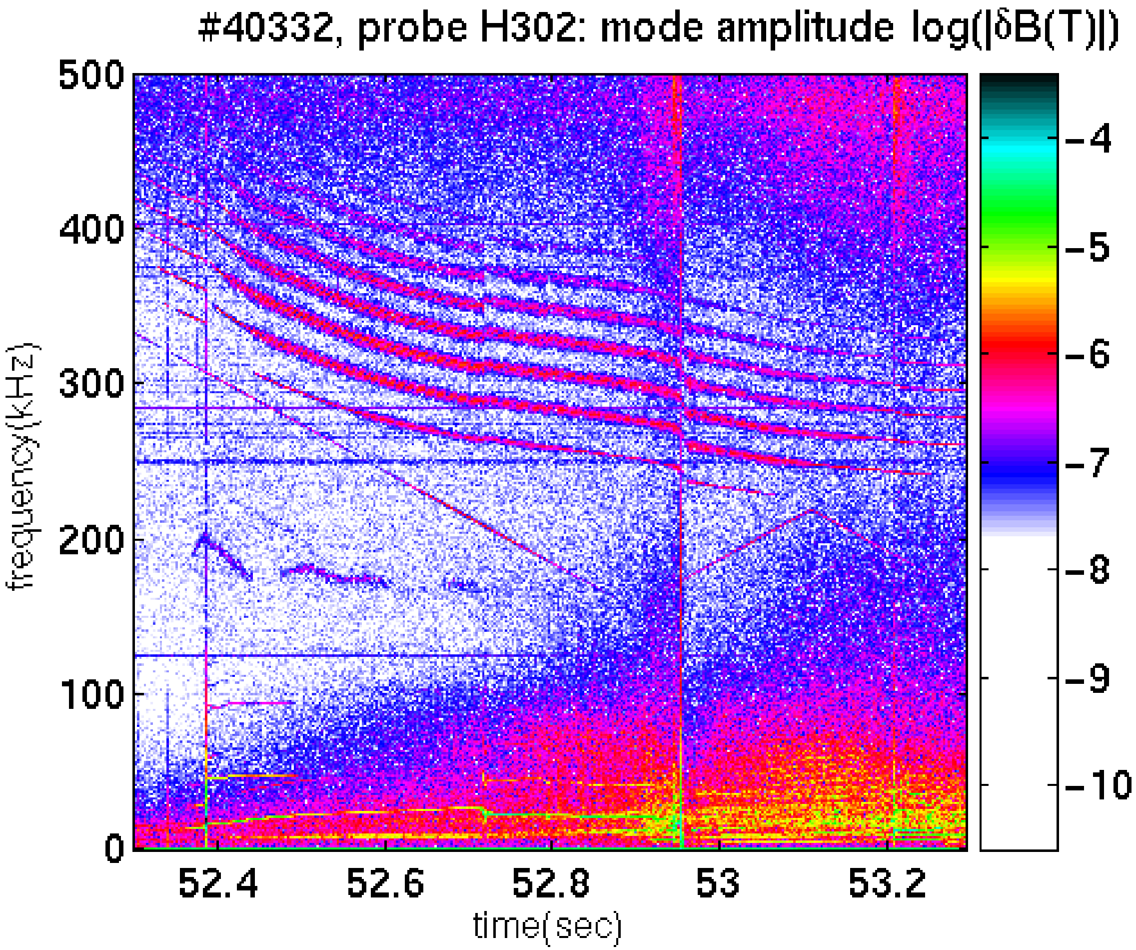


Continuous spectrum gaps for n=1…6 as functions of radius

MANY TAEs CAN BE EXCITED AT ONCE

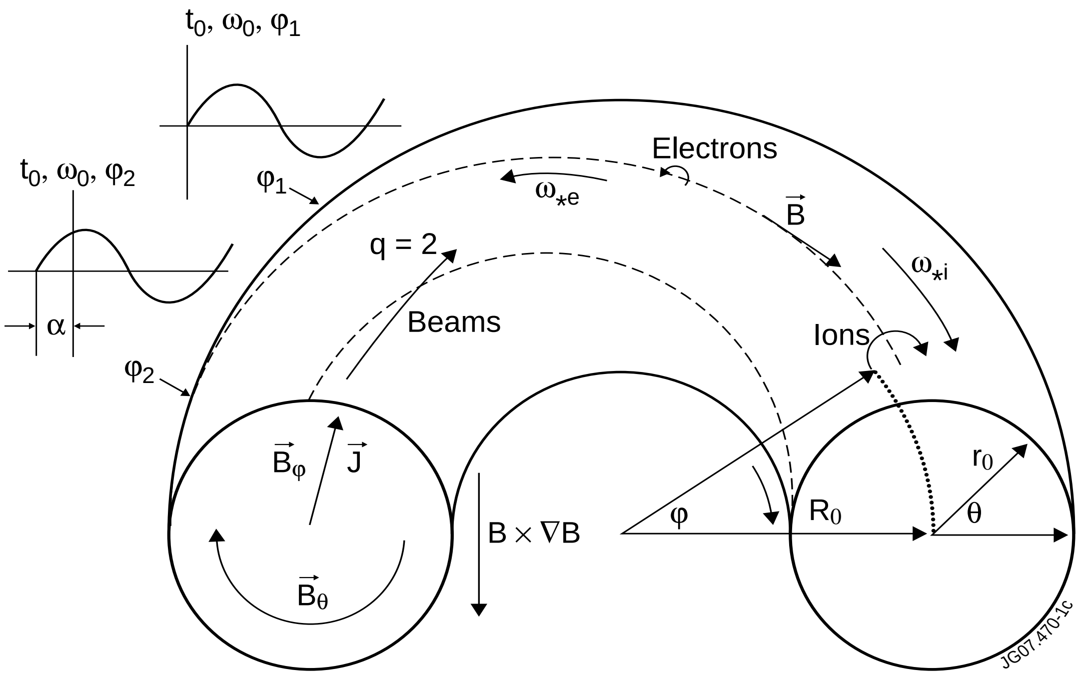


TAE ARE SEEN AT HIGHER FREQUENCY IF PLASMA ROTATES



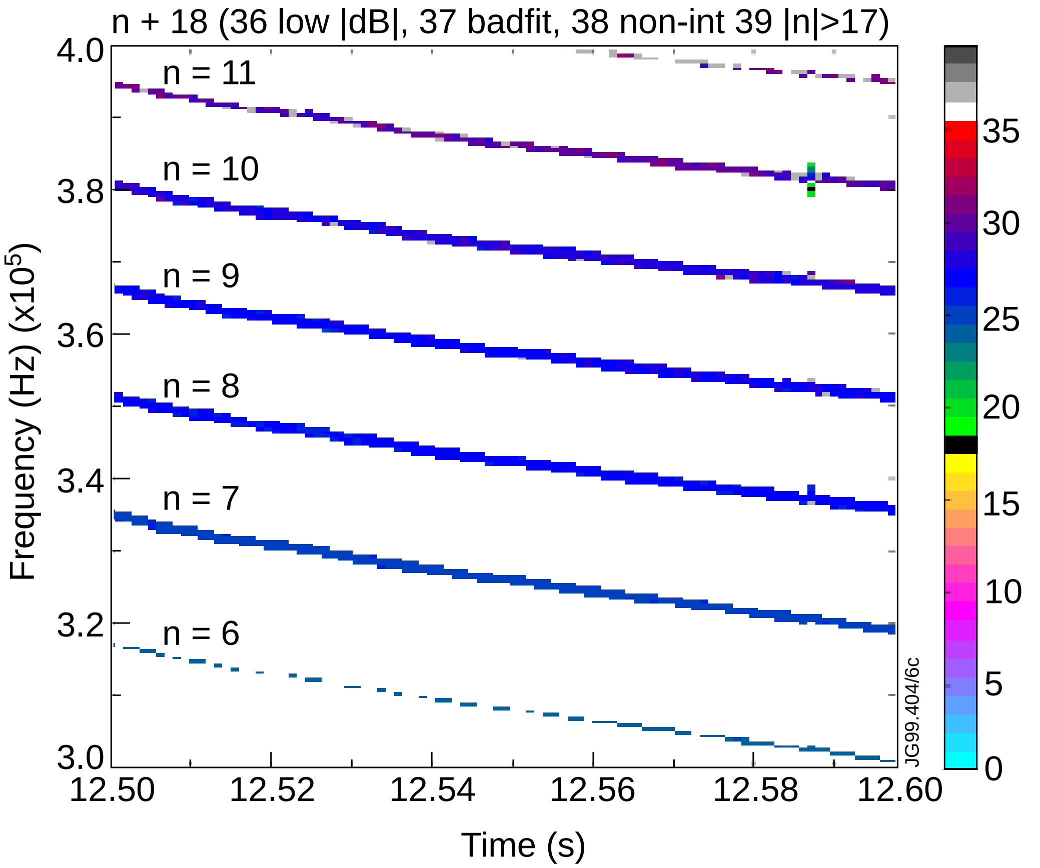
TOROIDAL MODE NUMBER MAY BE FOUND FROM *PHASE* DATA

- **In tokamak, toroidal mode number can be found from the phase shift measured between two (or more) Mirnov coils at different toroidal angles**

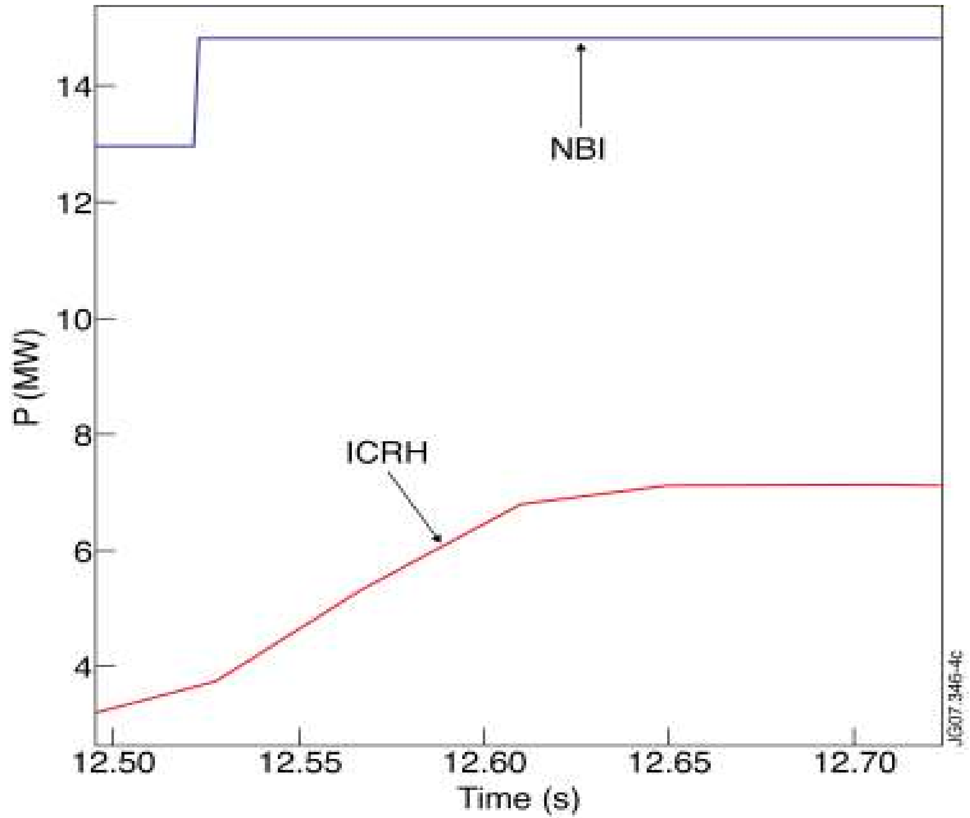


Sinusoidal signals measured at different toroidal angles φ_1, φ_2 at the same time and same frequency are shifted in phase by α.

TOROIDAL MODE NUMBERS FROM PHASE SPECTROGRAM

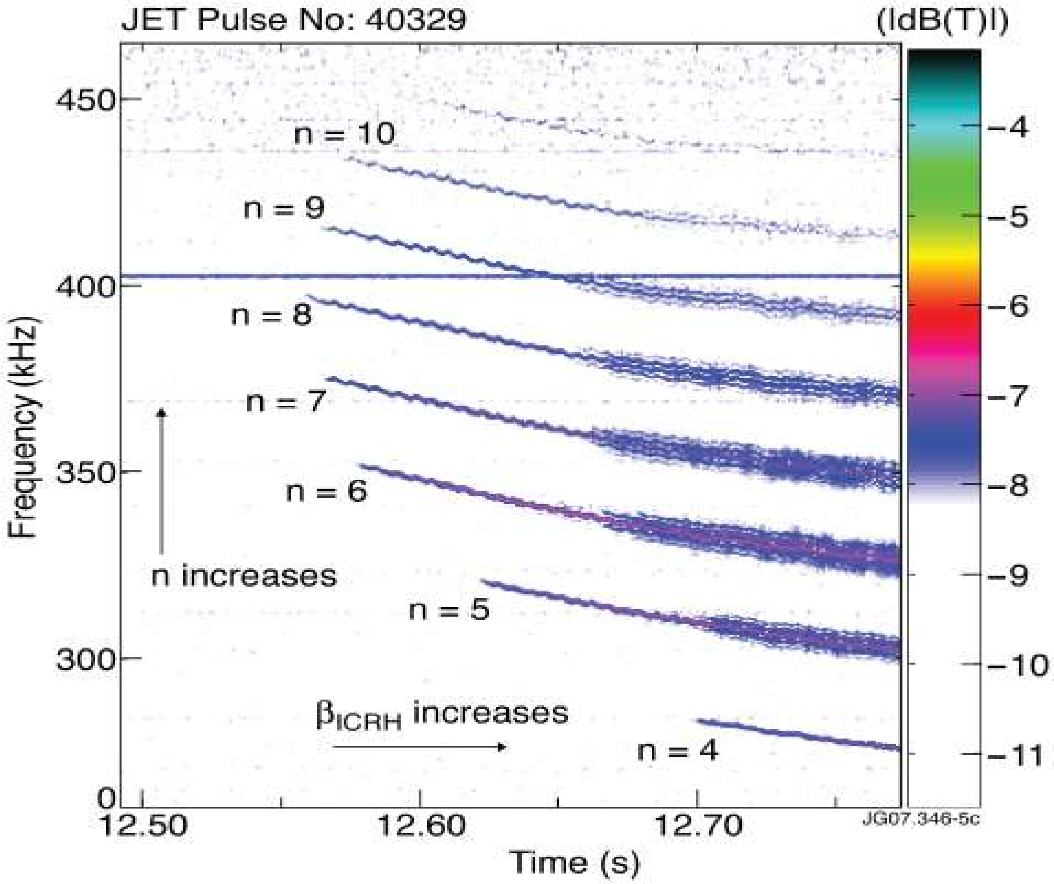


GRADUAL INCREASE OF ICRH POWER



- **Power waveforms of ICRH driving TAE and NBI (NBI provides damping)**

ICRH-DRIVEN TAE DURING THE INCREASE OF ICRH POWER

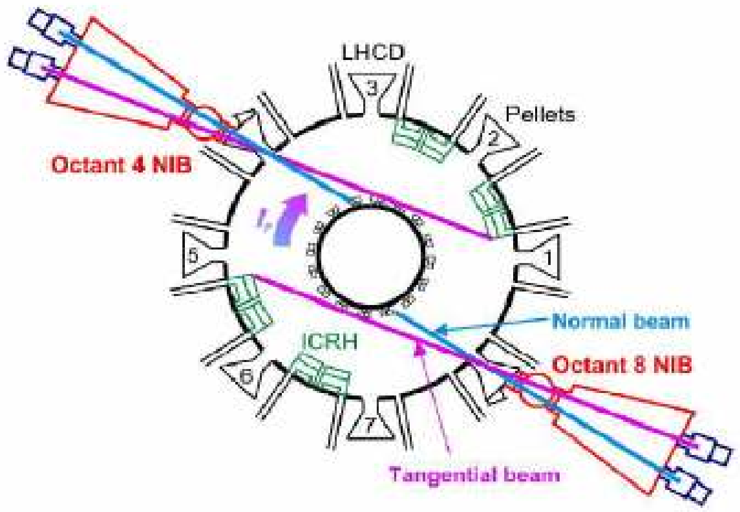


EXPERIMENTALLY OBSERVED TAE – THE QUESTIONS TO ASK

- **TAE frequencies are well above the expected 160 kHz. Why? Doppler shift**
- **TAE modes with $n = 6 - 9$ are excited easily, while TAE modes with lower or higher n's need a higher pressure of fast ions. Why? Stability depends on n**
- **The TAE spectral lines broaden and split after some time. Why? Nonlinear amplitude modulation becomes important (Lecture 5A)**

DOPPLER SHIFT OF THE MODE FREQUENCY

- **Uni-directional NBI on JET drives a significant toroidal plasma rotation (up to 40 kHz)**



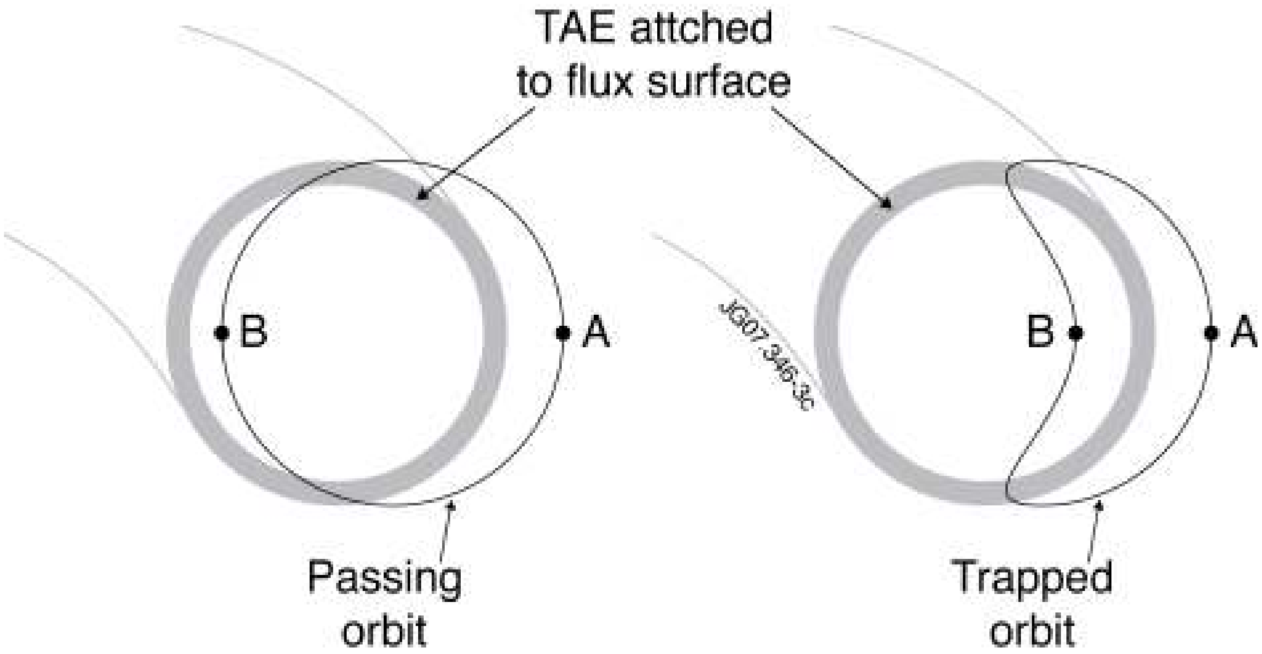
Geometry of NBI injection system on JET (view from the top of the machine)

- **Frequencies of waves with mode number n in laboratory reference frame, f_n^{LAB}, and in the plasma, f_n^0, are related through the Doppler shift $nf_{rot}(r)$:**

$$f_n^{LAB} = f_n^0 + nf_{rot}(r)$$

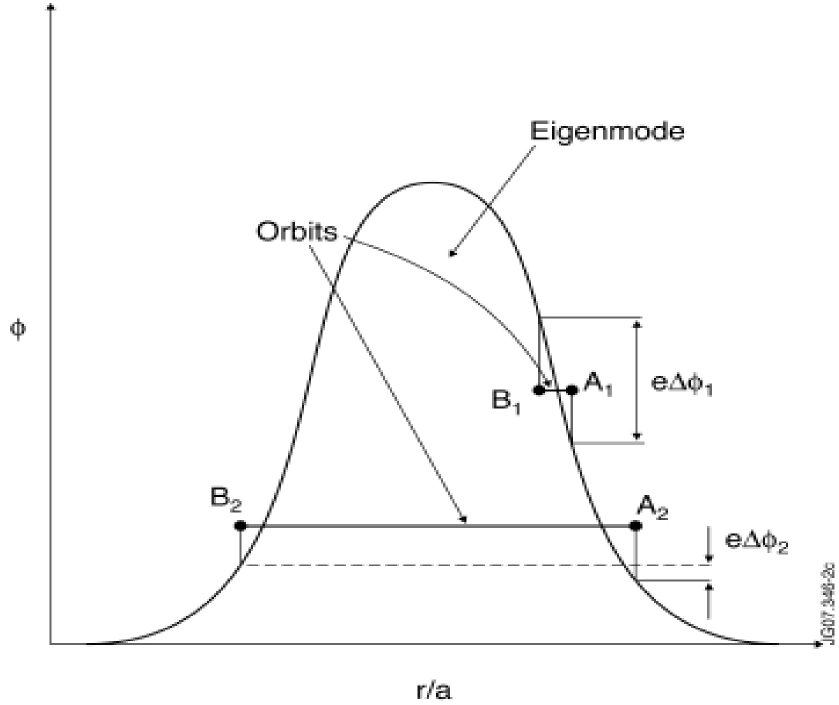
TAE DRIVE BY FAST IONS – 1

- **For TAE, parallel electric field is zero. How fast ions transfer their power to TAE via perpendicular electric field?**



- **TAE is attached to magnetic flux surface, while the resonant ions experience drift *across* the flux surfaces and the TAE mode structure**

TAE DRIVE BY FAST IONS - 2



- When a resonant ion moves radially across TAE from point A to point B, the mode and the ion exchange energy $e\Delta\phi$ as Figure shows.
- The maximum of $e\Delta\phi$ corresponds to the orbit size, Δ_{orbit}, equal to the mode width, $\Delta_{TAE} \approx r_{TAE} / m$. This is why energetic ion drive has a maximum at $m \approx nq \approx r_{TAE} / \Delta_{orbit}$.

MAIN TAE DAMPING EFFECTS

- **Thermal ion Landau damping due to $V_{\parallel i} = V_A / 3$ resonance with thermal (or beam) ions does not depend on n:**

$$\frac{\gamma_i}{\omega} \cong -\frac{\pi^{1/2}}{4} \beta_i q^2 \lambda_3 \left(1 + \left(1 + 2\lambda_3^2\right)^2\right) \exp\left(-\lambda_3^2\right), \quad \lambda_3 = 1/9\beta_i$$

- **Electron Landau damping and electron “collisional” damping for ideal MHD TAE does not depend on n either**

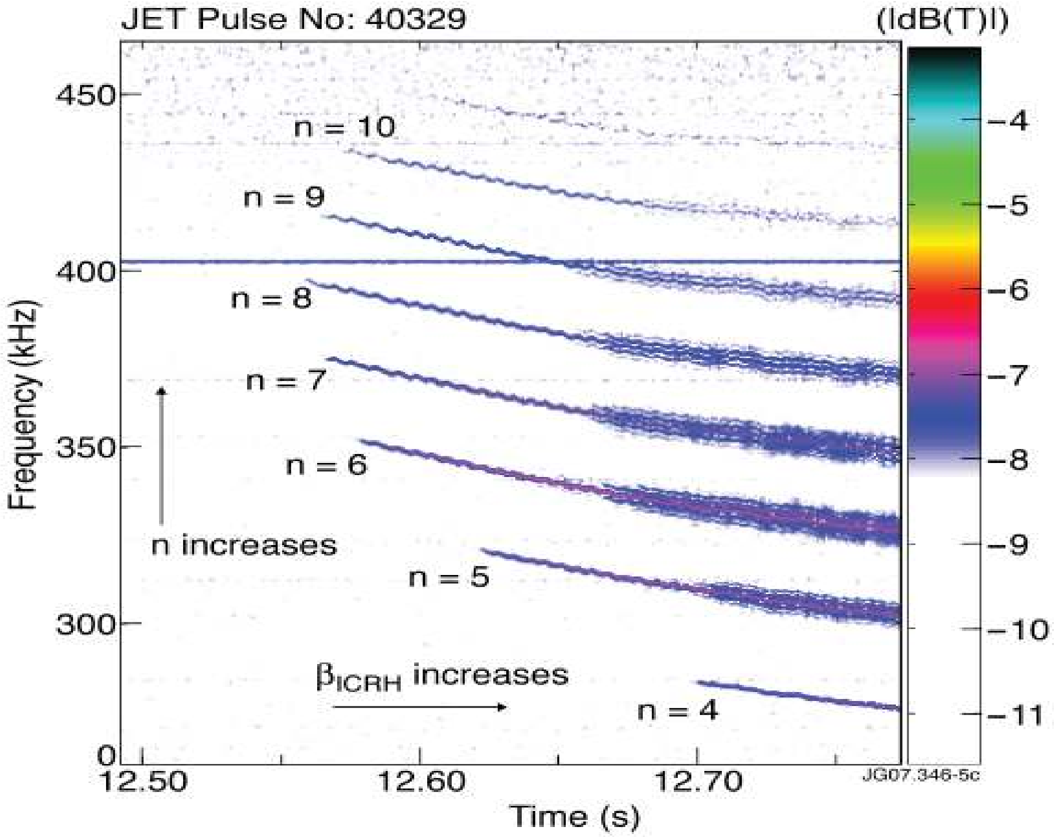
$$\downarrow$$

- **Threshold condition for TAE excitation,**

$$\gamma_{fast} > \left|\gamma_i + \gamma_e\right|$$

depends on n via fast ion drive alone

THE OBSERVED TAE MAY BE CONSIDERED AS THE TAE STABILITY DIAGRAM



ALFVÉN NBI-DRIVEN INSTABILITIES ON START AND MAST LOOK VERY DIFFERENT THAN ICRH-DRIVEN MODES ON JET

- Tight aspect ratio ($R_0 / a \sim 1.2 \div 1.8$) limits the value of magnetic field at level $B_T \sim 0.15 \div 0.6$ in present-day STs $\Rightarrow$ Alfvén velocity in ST is very low
-

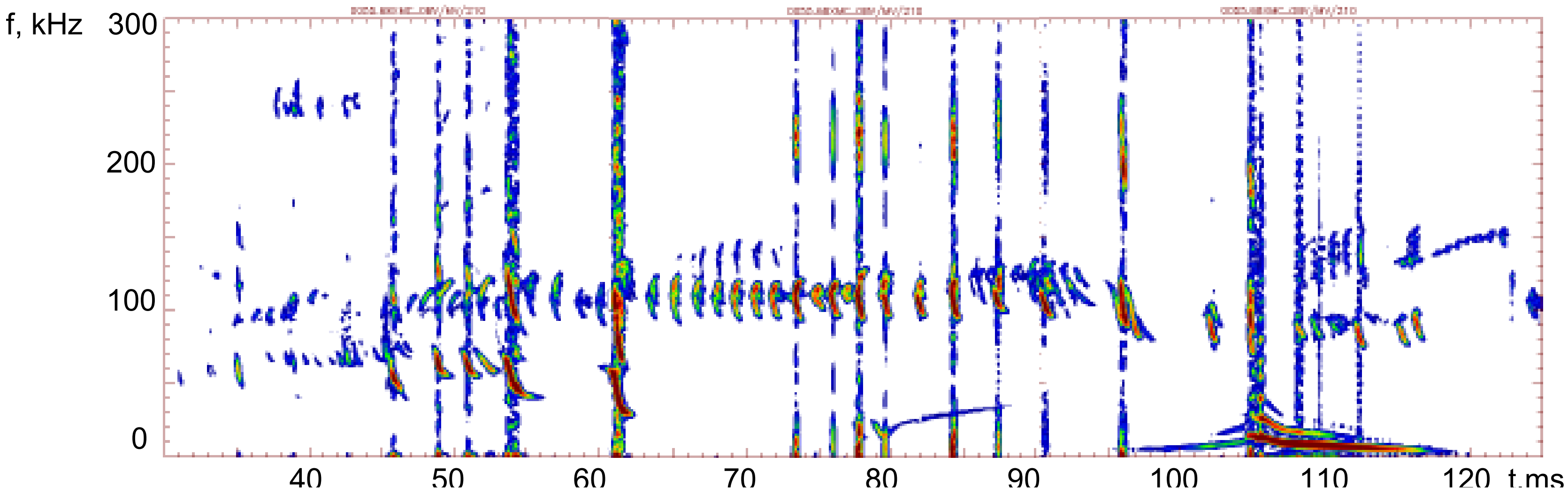
$$V_A = B_T / (4\pi n_i m_i)^{1/2} \cong 10^6 \text{ ms}^{-1} \text{ (MAST)}$$

- Even a relatively low-energy NBI, e.g. 30 keV hydrogen NBI is super-Alfvénic

$$V_{NBI} \cong 2.4 \times 10^6 \text{ ms}^{-1} > V_A ,$$

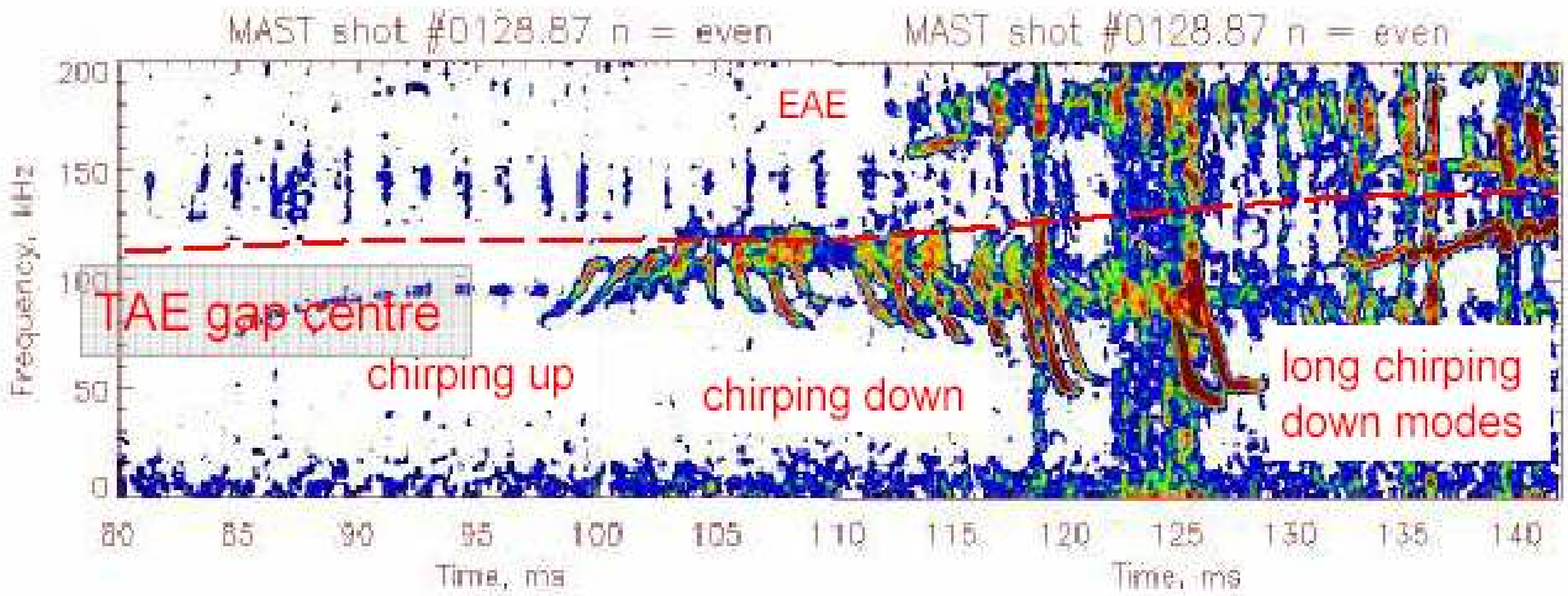
The super-Alfvénic NBI can excite Alfvén waves via the fundamental resonance $V_{||NBI} = V_A$ as in the case of fusion alphas.

NBI-DRIVEN AEs ARE DIFFERENT THAN ICRH-DRIVEN

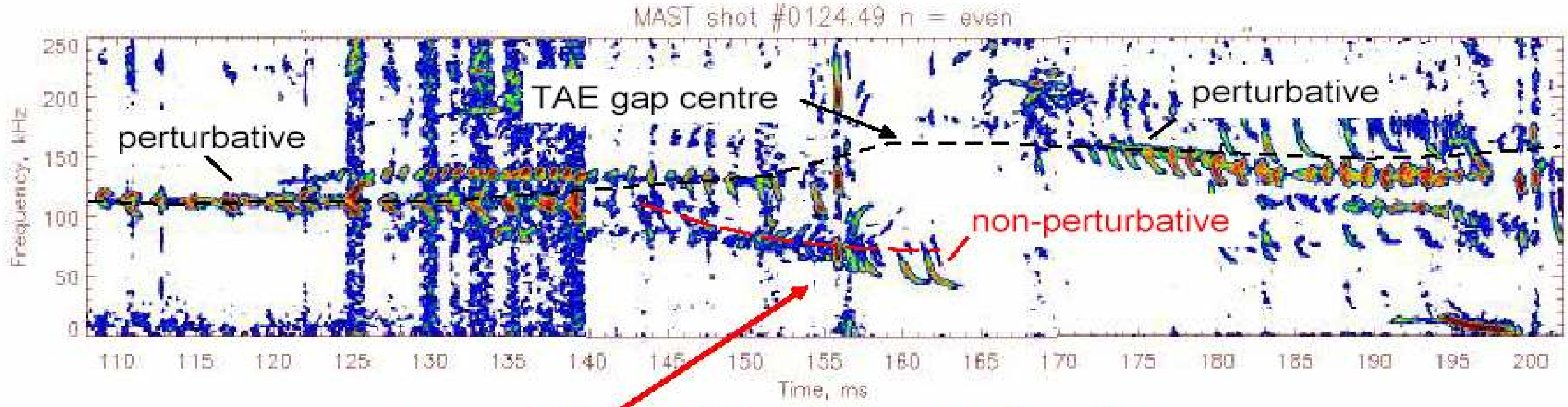


- **AEs are seen in bursts not as steady-state modes**

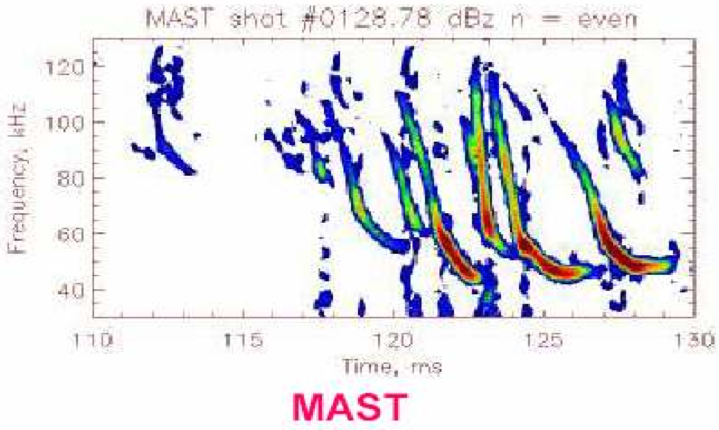
NBI-DRIVEN AEs on MAST

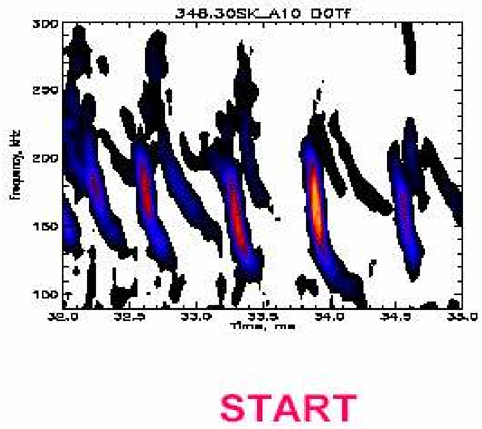


Examples of chirping **non-perturbative EP modes**:



Non-perturbative EP modes don't follow TAE frequency





- frequency sweep has characteristic **linear phase**

COMPARING TAE DRIVEN BY ICRH AND NBI

- **Why there is a dramatic difference in TAEs excited by ICRH and by NBI?**

- **Which regime of TAE excitation, the steady-state (JET-like) or the bursting (MAST-like) will be relevant for alpha-driven TAEs on ITER?**

SUMMARY

- **TAEs are localised at** $q(r_0) = (m+1/2)/n$ **and their frequency range is given by**

$$\omega_0 = \frac{V_A(r_0)}{2R_0 q(r_0)}$$

- **For typical JET parameters** $B_0 \cong 3\ T$ **;** $n_i = 5\times10^{19}\ m^{-3}$ **;** $m_i = m_D$ **, JET has**

$$V_A \cong 6.6\times10^6\ m/s\ ; \qquad f_0 \equiv \omega_0/2\pi \cong 160\ kHz$$

- **Mirnov coils are good for both amplitude and phase measurements of TAE with frequency below half of the sampling rate**
- **ICRH-driven TAEs are common on JET. These are seen as steady-state modes with frequencies satisfying**

$$f_n^{LAB} = f_n^0 + nf_{rot}(r)$$

- **NBI-driven TAEs are common on MAST. These are seen as bursting modes with sweeping frequency**