The DCON3D Code for the Ideal MHD Stability of DESC Stellarator Equilibria

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Ideal MHD Stability Criteria

- Bernstein, Frieman, Kruskal & Kulsrud (1958) derived an ideal MHD potential energy functional $\delta W[\boldsymbol{\xi}(\mathbf{x})]$. They showed that a static equilibrium is unstable iff there is a perturbation $\boldsymbol{\xi}(\mathbf{x})$ that makes $\delta W < 0$.
- Newcomb (1960) applied this to a cylindrical plasma. He derived a 2nd-order Euler-Lagrange equation, $(f\,\xi')' - g\,\xi = 0$, and showed that there is a $\xi(r)$ that makes $\delta W < 0$ iff the solution $\xi(r)$ changes sign between singular points, where $f = 0$. This criterion is limited to fixed-boundary modes.
- PEST, ERATO, GATO, CAS3D, TERPSICHORE (1975++): expansion in basis functions, large matrix eigenvalue problem for entire spectrum. Related method: Hessian matrix, negative eigenvalues.
- Glasser (2016) generalized Newcomb's criterion to tokamaks, with $\xi(r)$ replaced by a vector $\boldsymbol{\Xi}(\psi)$ of complex Fourier coefficients of coupled poloidal harmonics m and a $2M^{th}$ order ODE. Implemented in the DCON code.
- A simple procedure extends this to free-boundary modes, with a vacuum region surrounding the plasma. Free-boundary SPEC; Merkel Green's function code.
- The present work further generalizes this to non-axisymmetric plasmas with stellarator symmetry, coupling multiple toroidal harmonics $n = n_0 + k*l$, with l the number of stellarator field periods and k any integer. Formulated for DESC equilibria.

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Stellarator Equilibrium Codes

- **VMEC**
  - S.P. Hirshman and J.C. Whitson, "Steepest-descent moment method for 3D magneohydrodynamic equilibria," Phys. Fluids **26**, 3553 (1983).
  - Assumes nested toroidal flux surfaces, solves inverse equilibrium problem for R,Z(ρ,θ,ϕ).
  - Radial finite differences equally space in toroidal magnetic flux.
  - Problems: very coarse near magnetic axis, fails to ensure div **B** = 0. Show stoppers for DCON3D.
- **DESC**
  - D.W. Dudt and E. Kolemen, "DESC: A Stellarator Equilibrium Solver" Phys. Plasmas **27**, 10, 102513 (2020).
  - Assumes nested flux surfaces, solves inverse equilibrium problem for $R,Z,\lambda(\rho,\theta,\phi)$.
  - Radial discretization uses Zernike polynomials, resolves VMEC issue.
- **SPEC**
  - S.R. Hudson, R.L. Dewar *et al.*, "Computation of multi-region relaxed magnetohydrodynamic equilibria," Phys. Plasmas 19, 112502 (2012).
  - Infinitesimal interfaces with finite pressure discontinuities, [[p+B^2/2]] = 0, $B_{normal} = 0$.
  - Volumes between interfaces where **B** may be stochastic.
  - Solves direct equilibrium problem for vector potential **A** rather than magnetic field **B.**

Geometry

Coordinate Systems

$$\begin{aligned} \text{Cylindrical}: &\quad (R,\phi,Z) \\ \text{Native Flux}: &\quad (\rho,\theta,\phi) \\ \text{Straight Fieldline}: &\quad (\rho,\theta^*,\phi) \end{aligned}$$

$$\theta^* = \theta + \lambda(\rho,\theta,\phi)$$

$$\begin{aligned} \partial_{\theta^*} f|_{(\phi,\rho)} &= (1+\partial_\theta\lambda)^{-1}\,\partial_\theta f \\ \partial_\rho f|_{(\theta^*,\phi)} &= \partial_\rho f - (\partial_\rho\lambda)\,(1+\partial_\theta\lambda)^{-1}\,\partial_\theta f \\ \partial_\phi f|_{(\rho,\theta^*)} &= (1+\partial_\theta\lambda)^{-1}\,\partial_\theta f \end{aligned}$$

Hereafter suppress the * on θ^*.

Jacobian and Metric Tensors

$$d\xi^i = (d\rho, d\theta, d\phi), \quad dx^j = (dR, R d\phi, dZ)$$

$$J_i^j \equiv \frac{\partial x^j}{\partial \xi^i}, \quad \mathcal{J} \equiv \det \mathbf{J}, \quad \mathbf{g} \equiv \frac{\mathbf{J}\mathbf{J}^\dagger}{\mathcal{J}^2}$$

$$\begin{aligned} g_{11} &= |\nabla\theta\times\nabla\phi|^2, & g_{23} &= (\nabla\phi\times\nabla\rho)\cdot(\nabla\rho\times\nabla\theta) \\ g_{22} &= |\nabla\phi\times\nabla\rho|^2, & g_{31} &= (\nabla\rho\times\nabla\theta)\cdot(\nabla\theta\times\nabla\phi) \\ g_{33} &= |\nabla\rho\times\nabla\theta|^2, & g_{12} &= (\nabla\theta\times\nabla\phi)\cdot(\nabla\phi\times\nabla\rho) \end{aligned}$$

Fourier Transforms

$$\langle\mathcal{F}\rangle_{m,n;m',n'} \equiv \frac{1}{4\pi^2}\oint d\theta \oint d\phi\, \mathcal{J}\mathcal{F} \exp\left[i(m-m')\theta - i(n-n')\phi\right]$$

$$(G_{ij})_{m,n;m',n'} \equiv \langle g_{ij}\rangle_{m,n;m',n'}, \quad \mathbf{G}_{ij}^\dagger = \mathbf{G}_{ij}$$

$$M_{m,m'} \equiv m\delta_{m,m'}, \quad N_{n,n'} \equiv n\delta_{n,n'}$$

$$Q_{m,n;m',n'} \equiv (m\chi' - n\psi')\delta_{m,m'}\delta_{n,n'}$$

Equilibrium Magnetic Field and Current Density

Equilibrium Magnetic Field

$$\chi(\rho) \equiv \int_0^\rho d\rho' \oint d\theta \oint d\phi \mathcal{J}\mathbf{B} \cdot \nabla\theta$$
$$\psi(\rho) \equiv \int_0^\rho d\rho' \oint d\theta \oint d\phi \mathcal{J}\mathbf{B} \cdot \nabla\phi$$

$$\begin{aligned}\mathbf{B} =& \chi' \nabla\phi \times \nabla\rho + \psi' \nabla\rho \times \nabla\theta \\ =& B_\rho \nabla\rho + B_\theta \nabla\theta + B_\phi \nabla\phi\end{aligned}$$

$$q(\rho) \equiv \frac{1}{\iota(\rho)} \equiv \frac{\psi'(\rho)}{\chi'(\rho)}$$

$$\begin{aligned}B_\rho =& \mathcal{J}\left(g_{\rho\theta}\chi' + g_{\rho\phi}\psi'\right) \\ B_\theta =& \mathcal{J}\left(g_{\theta\theta}\chi' + g_{\theta\phi}\psi'\right) \\ B_\phi =& \mathcal{J}\left(g_{\phi\theta}\chi' + g_{\phi\phi}\psi'\right)\end{aligned}$$

$$\begin{aligned}B^2 =& B_\rho B^\rho + B_\theta B^\theta + B_\phi B^\phi \\ =& g_{\theta\theta}\chi'^2 + 2 g_{\theta\phi}\chi'\psi' + g_{\phi\phi}\psi'^2\end{aligned}$$

Equilibrium Current Densiity

$$\mathbf{J} = \mathcal{J} J^\theta \left(\nabla\phi \times \nabla\rho\right) + \mathcal{J} J^\phi \left(\nabla\rho \times \nabla\theta\right) = \nabla \times \mathbf{B}$$

$$\begin{aligned}\mathcal{J} J^\rho =& \partial_\theta B_\phi - \partial_\phi B_\theta = 0 \\ \mathcal{J} J^\theta =& \partial_\phi B_\rho - \partial_\rho B_\phi \\ \mathcal{J} J^\phi =& \partial_\rho B_\theta - \partial_\theta B_\rho\end{aligned}$$

$$\mathbf{J} \times \mathbf{B} = \nabla P, \quad J^\theta \psi' - J^\phi \chi' = P'$$

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Perturbations

Perturbed Displacement and Vector Potential

$$\boldsymbol{\xi} = \mathcal{J}\left(\xi^\rho \nabla\theta \times \nabla\phi + \xi^\theta \nabla\phi \times \nabla\rho + \xi^\phi \nabla\rho \times \nabla\theta\right)$$

$$\boldsymbol{\alpha} \equiv \boldsymbol{\xi} \times \mathbf{B} = \alpha_\rho \nabla\rho + \alpha_\theta \nabla\theta + \alpha_\phi \nabla\phi, \quad \xi_\parallel \equiv \boldsymbol{\xi} \cdot \mathbf{B}/B^2$$

$$\boldsymbol{\xi} = \frac{\mathbf{B}}{B^2} \times \left(\alpha_\rho \nabla\rho + \alpha_\theta \nabla\theta + \alpha_\phi \nabla\phi\right) + \xi_\parallel \mathbf{B}$$

$$\mathbf{B} \cdot (\boldsymbol{\xi} \times \mathbf{B}) = B^\rho \alpha_\rho + B^\theta \alpha_\theta + B^\phi \alpha_\phi = 0$$

$$\mathcal{J}\nabla \cdot \boldsymbol{\xi} = \partial_\rho\left(\mathcal{J}\xi^\rho\right) + \partial_\theta\left(\mathcal{J}\xi^\theta\right) + \partial_\phi\left(\mathcal{J}\xi^\phi\right)$$

$$\alpha_\rho = \psi' \xi^\theta - \chi' \xi^\phi, \quad \mathcal{J}\xi^\rho = \frac{1}{B^2}\left(B_\theta \alpha_\phi - B_\phi \alpha_\theta\right)$$

$$\alpha_\theta = -\psi' \xi^\rho, \qquad \mathcal{J}\xi^\theta = \frac{1}{B^2}\left(B_\phi \alpha_\rho - B_\rho \alpha_\phi\right) + \chi' \xi_\parallel$$

$$\alpha_\phi = \chi' \xi^\rho, \qquad \mathcal{J}\xi^\phi = \frac{1}{B^2}\left(B_\rho \alpha_\theta - B_\theta \alpha_\rho\right) + \psi' \xi_\parallel$$

Perturbed Magnetic Field

$$\begin{aligned}
\mathbf{Q} =& \nabla \times \boldsymbol{\alpha} \\
=& \mathcal{J}\left[Q^\rho \nabla\theta \times \nabla\phi + Q^\theta \nabla\phi \times \nabla\rho + Q^\theta \nabla\rho \times \nabla\theta\right] \\
\mathcal{J}Q^\rho =& \partial_\theta \alpha_\phi - \partial_\phi \alpha_\theta = \left(\chi' \partial_\theta + \psi' \partial_\phi\right) \xi^\rho \\
\mathcal{J}Q^\theta =& \partial_\phi \alpha_\rho - \partial_\rho \alpha_\phi = \partial_\phi \alpha_\rho - \partial_\rho\left(\chi' \xi^\rho\right) \\
\mathcal{J}Q^\phi =& \partial_\rho \alpha_\theta - \partial_\theta \alpha_\rho = -\partial_\theta \alpha_\rho - \partial_\rho\left(\psi' \xi^\rho\right)
\end{aligned}$$

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Fourier Representation of Perturbations

$$\boldsymbol{\alpha} = \sum_j \alpha_j \exp\left[i\left(m_j\theta - n_j\phi\right)\right]$$

$$\boldsymbol{\Xi} = \sum_j \xi_j \exp\left[i\left(m_j\theta - n_j\phi\right)\right]$$

$$\alpha_j = \frac{1}{(2\pi)^2} \oint d\theta \oint d\phi \left(\mathcal{J}\boldsymbol{\alpha} \cdot \nabla\theta \times \nabla\phi\right) \exp\left[-i\left(m_j\theta - n_j\phi\right)\right]$$

$$\xi_j = \frac{1}{(2\pi)^2} \oint d\theta \oint d\phi \left(\boldsymbol{\xi} \cdot \nabla\rho\right) \exp\left[-i\left(m_j\theta - n_j\phi\right)\right]$$

If the equilibrium has periodicity l, then only toroidal harmonics $n_k = n_0 + k\,l$ couple.

Choose finite set of harmonics (m_j, n_j) that converge most rapidly.

The Ideal MHD Energy Principle

$$
\begin{aligned}
\delta W =& \frac{1}{2}\int d\mathbf{x}\left[\mathbf{Q}^2 + \mathbf{J}\cdot\boldsymbol{\xi}\times\mathbf{Q} + (\boldsymbol{\xi}\cdot\nabla P)(\nabla\cdot\boldsymbol{\xi}) + \gamma P(\nabla\cdot\boldsymbol{\xi})^2\right] \\
=& \frac{1}{2}\int d\mathbf{x}\Big\{\Big[(\chi'\partial_\theta + \psi'\partial_\phi)\,\xi^\rho\nabla\theta\times\nabla\phi \\
& + \left[\partial_\phi\alpha_\rho - \partial_\rho\left(\chi'\xi^\rho\right)\right]\nabla\phi\times\nabla\rho \\
& + \left[-\partial_\theta\alpha_\rho - \partial_\rho\left(\psi'\xi^\rho\right)\right]\nabla\rho\times\nabla\theta\Big]^2 \\
& + \xi^\rho\left(J^\theta\partial_\theta + J^\phi\partial_\phi\right)\alpha_\rho - \alpha_\rho\left(J^\theta\partial_\theta + J^\phi\partial_\phi\right)\xi^\rho \\
& + 2P'\xi^\rho\partial_\rho\xi^\rho + \left(J^\theta\psi'' - J^\phi\chi'' + P'\mathcal{J}'/\mathcal{J}\right)\xi^{\rho 2}\Big\} \\
=& 2\pi^2\int_0^1 d\rho\Big[\boldsymbol{\alpha}^\dagger\mathsf{A}\boldsymbol{\alpha} + \boldsymbol{\alpha}^\dagger(\mathsf{B}\Xi' + \mathsf{C}\Xi) + (\Xi'^\dagger\mathsf{B}^\dagger + \Xi^\dagger\mathsf{C}^\dagger)\boldsymbol{\alpha} \\
& + \Xi'^\dagger\mathsf{D}\Xi' + \Xi'^\dagger\mathsf{E}\Xi + \Xi^\dagger\mathsf{E}^\dagger\Xi' + \Xi^\dagger\mathsf{H}\Xi\Big]
\end{aligned}
$$

The equilibrium is unstable if and only if a perturbation (α,ξ) exists that makes $\delta W < 0$.

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Coefficient Matrices

$$
\begin{aligned}
\mathbf{A} =& \mathbf{N}\mathbf{G}_{22}\mathbf{N} + \mathbf{N}\mathbf{G}_{23}\mathbf{M} + \mathbf{M}\mathbf{G}_{23}\mathbf{N} + \mathbf{M}\mathbf{G}_{33}\mathbf{M} \\
\mathbf{B} =& -i\chi' \left(\mathbf{N}\mathbf{G}_{22} + \mathbf{M}\mathbf{G}_{23}\right) - i\psi' \left(\mathbf{N}\mathbf{G}_{23} + \mathbf{M}\mathbf{G}_{33}\right) \\
\mathbf{D} =& \chi'^2\mathbf{G}_{22} + 2\chi'\psi'\mathbf{G}_{23} + \psi'^2\mathbf{G}_{33}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{C} =& -i\chi''(\mathbf{N}\mathbf{G}_{22} + \mathbf{M}\mathbf{G}_{23}) - i\psi''(\mathbf{N}\mathbf{G}_{23} + \mathbf{M}\mathbf{G}_{33}) \\
& - \chi' \left(\mathbf{N}\mathbf{G}_{12} + \mathbf{M}\mathbf{G}_{31}\right)\mathbf{Q} - i\left(\mathbf{M}\mathbf{J}^{\theta} - \mathbf{N}\mathbf{J}^{\phi}\right) \\
\mathbf{E} =& \chi' \left(\chi''\mathbf{G}_{22} + \psi''\mathbf{G}_{23}\right) + \psi' \left(\chi''\mathbf{G}_{23} + \psi''\mathbf{G}_{33}\right) \\
& - i\chi' \left(\chi'\mathbf{G}_{12} + \psi'\mathbf{G}_{31}\right)\mathbf{Q} + P'\mathbf{V}' \\
\mathbf{H} =& \chi''^2\mathbf{G}_{22} + 2\chi''\psi''\mathbf{G}_{23} + \psi''^2\mathbf{G}_{33} \\
& + i\chi' \left[\chi'' \left(\mathbf{Q}\mathbf{G}_{12} - \mathbf{G}_{12}\mathbf{Q}\right) + \psi'' \left(\mathbf{Q}\mathbf{G}_{31} - \mathbf{G}_{31}\mathbf{Q}\right)\right] \\
& + \chi'^2\mathbf{Q}\mathbf{G}_{11}\mathbf{Q} + \left(\psi''\mathbf{J}^{\theta} - \chi''\mathbf{J}^{\phi} + P'\mathbf{V}''\right)
\end{aligned}
$$

Minimization of δW

Eimination of α

$$\mathbf{A}\alpha + \mathbf{B}\Xi' + \mathbf{C}\Xi = 0$$

$$\alpha = -\mathbf{A}^{-1}(\mathbf{B}\Xi + \mathbf{C}\Xi)$$

Schur Complement Matrices

$$\mathbf{F} \equiv \mathbf{D} - \mathbf{B}^\dagger \mathbf{A}^{-1} \mathbf{B} = \mathbf{F}^\dagger$$

$$\mathbf{K} \equiv \mathbf{E} - \mathbf{B}^\dagger \mathbf{A}^{-1} \mathbf{C} \neq \mathbf{K}^\dagger$$

$$\mathbf{G} \equiv \mathbf{H} - \mathbf{C}^\dagger \mathbf{A}^{-1} \mathbf{C} = \mathbf{G}^\dagger$$

Reduced Energy Principle

$$\delta W = 2\pi^2 \int_0^1 d\rho \left(\Xi'^\dagger \mathbf{F} \Xi' + \Xi'^\dagger \mathbf{K} \Xi + \Xi^\dagger \mathbf{K}^\dagger \Xi' + \Xi^\dagger \mathbf{G} \Xi \right)$$

The Euler-Lagrange Equation

Reduced Energy Principle

$$\delta W = 2\pi^2 \int_0^1 d\rho \left(\Xi'^\dagger \mathsf{F} \Xi' + \Xi'^\dagger \mathsf{K} \Xi + \Xi^\dagger \mathsf{K}^\dagger \Xi' + \Xi^\dagger \mathsf{G} \Xi \right)$$

Euler-Lagrange Equation

$$(\mathsf{F}\Xi' + \mathsf{K}\Xi)' - (\mathsf{K}^\dagger \Xi' + \mathsf{G}\Xi) = 0$$

$$\mathrm{u} \equiv \begin{pmatrix} \Xi \\ \mathsf{F}\Xi' + \mathsf{K}\Xi \end{pmatrix}, \quad \mathsf{L} \equiv \begin{pmatrix} -\mathsf{F}^{-1}\mathsf{K} & \mathsf{F}^{-1} \\ \mathsf{G} - \mathsf{K}^\dagger \mathsf{F}^{-1} \mathsf{K} & \mathsf{K}^\dagger \mathsf{F}^{-1} \end{pmatrix}, \quad \mathrm{u}' = \mathsf{L}\mathrm{u}$$

Hamiltonian and Symplectic Symmetries

$$\mathsf{L}_{22}^\dagger = -\mathsf{L}_{11}, \quad \mathsf{L}_{12}^\dagger = \mathsf{L}_{12}, \quad \mathsf{L}_{21}^\dagger = \mathsf{L}_{21}$$

$$\mathsf{J} \equiv \begin{pmatrix} \mathbf{0} & \mathsf{I} \\ -\mathsf{I} & \mathbf{0} \end{pmatrix}, \quad \mathsf{JLJ} = \mathsf{L}^\dagger$$

$$\mathsf{U}' = \mathsf{LU}, \quad \mathsf{U}(0) = \mathsf{I}, \quad \left(\mathsf{U}^\dagger \mathsf{J} \mathsf{U}\right)' = 0, \quad \mathsf{U}^\dagger \mathsf{J} \mathsf{U} = \mathsf{J}$$

Singular Surfaces

- The Euler-Lagrange Equation has singular surfaces wherever $m_j - n_j q = 0$.
- In a non-axisymmetric system, there can be $M > 1$ singular points at a given rational value of q.
- Example: $q = 2, (m,n) = (2,1), (4,2), (6,3), \ldots$
- Presented at Simons Hidden Symmetries meeting in Princeton, March, 2023.
- A.H. Glasser, "Generalized Mercier stability criterion for stellarators," Phys. Plasma 30, 052502 (2023).
- At each singular point, the large resonant solutions are eliminated and new small resonant solutions are launched. This is required to maintain $\delta W < \infty$.

Generalized Newcomb Crossing Criterion

Energy Principle

$$\delta W = \frac{1}{2}\int_0^a ds\left(\Xi^{\dagger\prime}\mathbf{F}\Xi' + \Xi^{\dagger\prime}\mathbf{K}\Xi + \Xi^\dagger\mathbf{K}^\dagger\Xi' + \Xi^\dagger\mathbf{G}\Xi\right)$$

Perfect Derivative

For any $\mathbf{P} = \mathbf{P}^\dagger, \quad \mathbf{P}(0) = \mathbf{P}(a) = 0,$

$$\frac{V'}{2}\int_0^a ds\left(\Xi^\dagger\mathbf{P}\Xi\right)' = \frac{V'}{2}\int_0^a ds\left(\Xi'^\dagger\mathbf{P}\Xi' + \Xi^\dagger\mathbf{P}\Xi' + \Xi^\dagger\mathbf{P}'\Xi\right) = 0$$

$$\delta W = \frac{V'}{2}\int_0^a ds\left[\Xi^{\dagger\prime}\mathbf{F}\Xi' + \Xi^{\dagger\prime}\left(\mathbf{K}-\mathbf{P}\right)\Xi + \Xi^\dagger\left(\mathbf{K}^\dagger - \mathbf{P}\right)\Xi' + \Xi^\dagger\left(\mathbf{G}-\mathbf{P}'\right)\Xi\right]$$

Completing the Square

Choose $\mathbf{P} \equiv \mathbf{U}_{22}\mathbf{U}_{12}^{-1}, \quad D_C \equiv \det\mathbf{P}^{-1}$

$\mathbf{P}$ is self-adjoint by the symplectic symmetry of $\mathbf{U}$
It exists iff $D_C \neq 0$ for $s \in (0, a)$.

$$\mathbf{P}' = \mathbf{G} - \left(\mathbf{P}-\mathbf{K}\right)\mathbf{F}^{-1}\left(\mathbf{P}-\mathbf{K}^\dagger\right)$$

$$\delta W = \frac{V'}{2}\int_0^a \left[\Xi'^\dagger + \Xi^\dagger\left(\mathbf{K}^\dagger - \mathbf{P}\right)\mathbf{F}^{-1}\right]\mathbf{F}\left[\Xi' + \mathbf{F}^{-1}\left(\mathbf{K}-\mathbf{P}\right)\Xi\right] ds$$

δW is positive definite if $\mathbf{P}$ exists. Integrate the Euler-Lagrange equation from $\rho = 0$ to 1, monitor D_C. The equilibrium is unstable if and only if D_C changes sign between the magnetic axis and the plasma edge.

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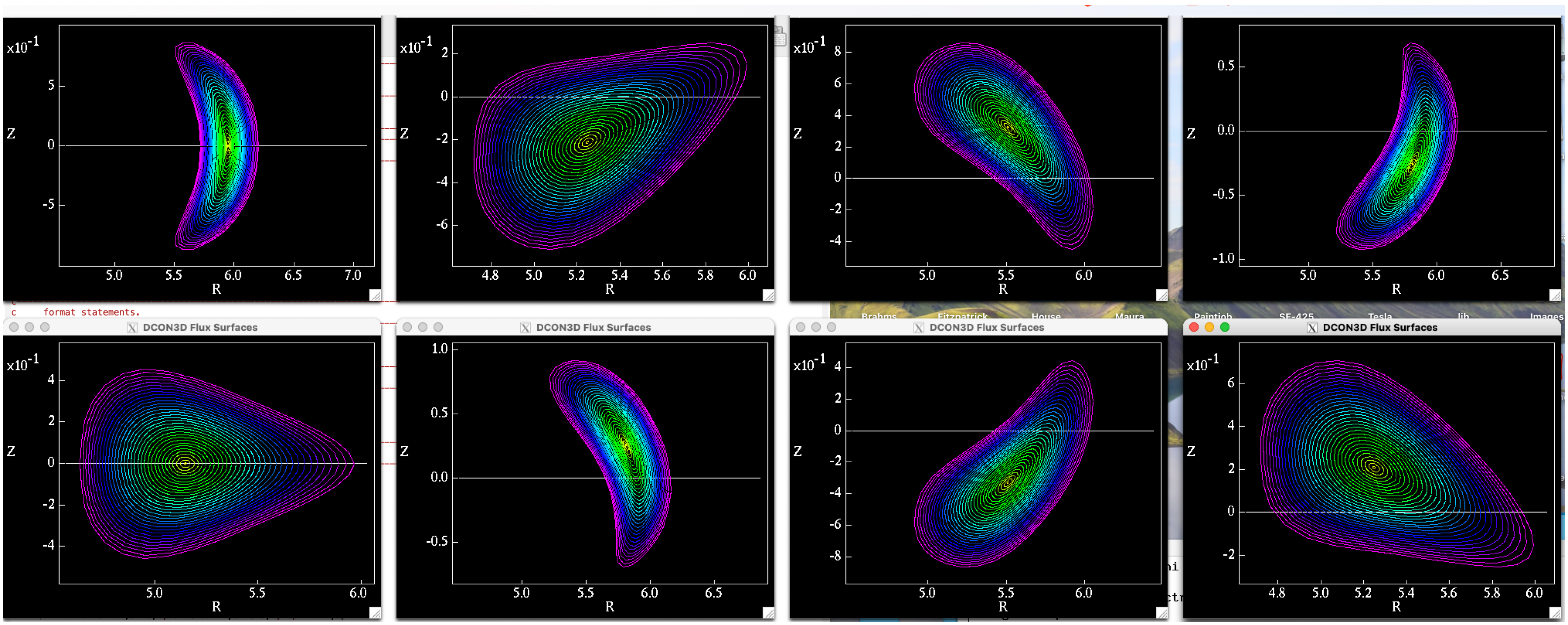
Computational Methods

- Equilibrium data file written by DESC is read by DCON3D.
- $R, Z, \lambda(\rho, \theta, \phi)$ computed as Zernike polynomials and complex exponentials and stored as real cubic splines in native coordinate system.
- Cubic splines allow rapid evaluation of function values and derivatives at nodes. Derivatives are transformed to straight-fieldline coordinates.
- Derived quantities, *e.g.* B^2, J^i, g_{ij} computed and stored as cubic splines.
- Fourier integrals of equilibrium quantities computed by analytical integration of cubic splines.
- Euler-Lagrange equation $d\mathbf{U}/d\rho = \mathbf{L}.\mathbf{U}$: $\mathbf{L}(\rho)$ interpolated from cubic splines; $\mathbf{L}.\mathbf{U}$ computed with BLAS3 routine ZGEMM; integrated with complex adaptive ODE solver ZVODE.
- After each step of ZVODE, the critical determinant $D_C = \det(\boldsymbol{U}_{11}\,\boldsymbol{U}_{21}^{-1})$ is computed, written to a file and graphed.
- An instability exists if and only if D_C vanishes for ρ in (0,1). If D_C vanishes, code halts.
- DCON3D runs in a cpu time < 1 minute on a 2023 MacBook Pro M2.

Example: Wendelstein 7-X
Flux Surfaces



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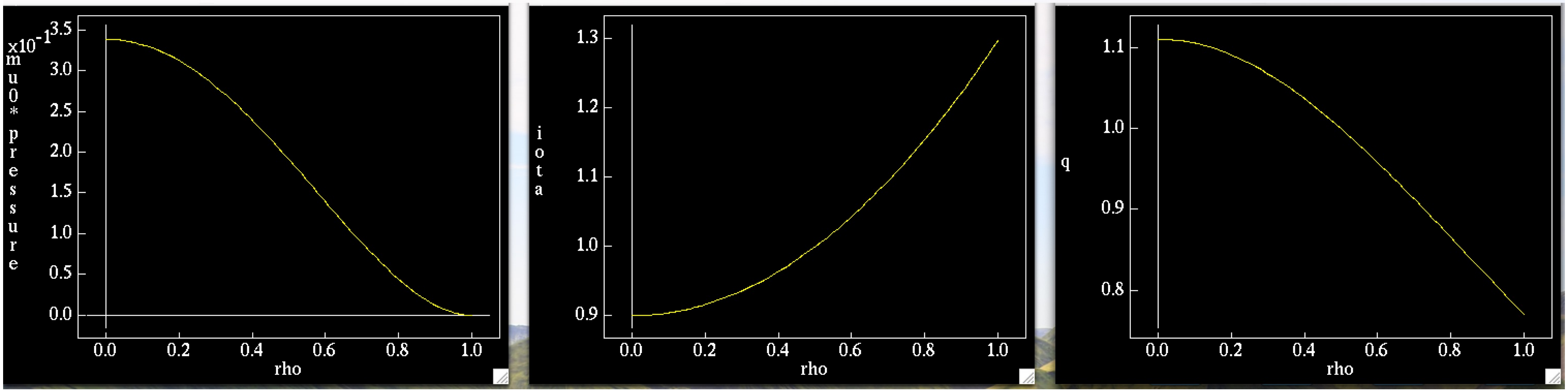
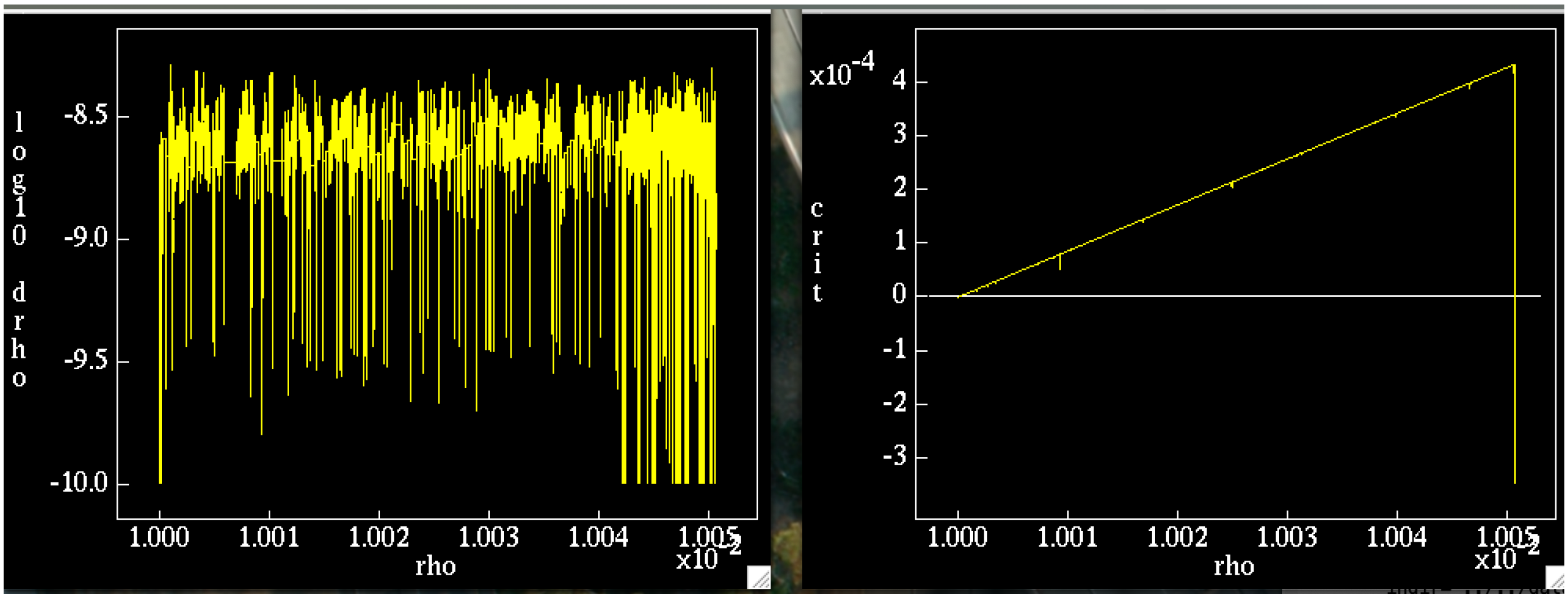
Example: Wendelstein 7-X Profiles



Example: Wendelstein 7-X Stability



Zero crossing at rho = 1.005e-2 indicates instability.
That took 20 seconds on a MacBook Pro M2.
47 Fourier harmonics.

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Conclusions and Status

- DCON3D for DESC equilibria is ready for prime time.
- Verification against axisymmetric DCON code on axisymmetric equilibria
- Agreement between two methods of computing Mercier criterion for non-axisymmetric equilibria.
- BLAS3 matrix-matrix multiplication and ZVODE complex adaptive integrator make the code very fast.xx
- The full code runs in < 1 minute on a 2023 MacBook Pro M2.
- Next step: verification against CAS3D and TERPSICHORE.
- Principal goal: incorporation into stellarator optimization codes, so they only produce ideal MHD stable configurations.
- Extension to free-boundary modes requires coupling to vacuum code, e.g. one ceveloped by Peter Merkel at IPP Garching.

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