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NEW RESULTS ON NONRESONANT STELLARATOR DIVERTOR

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1. Introduction and background

2. New results:
    2.1 A new type of nonresonant divertor –
    The hybrid divertor
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3. Future plans

1. INTRODUCTION AND BACKGROUND

►Nonresonant divertors are important for development of stellarator concept as a Fusion Power Plant (FPP). Nonresonant divertor may be an alternative to island divertor.

►Nonresonant divertor is under-studied.

►Island divertor:
Rational surface + resonant perturbation → Chain of islands

►Nonresonant divertor:
Irrational surface → nonresonant perturbation → Cantorous → magnetic turnstiles

►In nonresonant divertor, field lines exiting the LGCS collimate into flux tubes which strike the wall and make footprints. This collimation into tubes is at the heart of nonresonant divertor. The locations of the footprints are fixed on the wall.

►A turnstile always comes in a pair of tubes, one outgoing tube, and one incoming tube. Flux is preserved.

►2018: Boozer and Punjabi developed a method for simulation of stellarator divertors [1]. This method uses maps to represent the magnetic system of the stellarator divertors. The method is related to 1984 study of MacKay *et al* [2] on the loss of the last confining surface of the standard map as the map parameter is increased. In this study, MacKay *et al* introduced the concepts of cantori and turnstiles.

►2020: Punjabi and Boozer used this method to study the nonresonant stellarator divertor [3]. It was found that the diffusive field lines exit and enter the outermost surface through flux tubes.

►2022: Punjabi and Boozer developed a method to calculate the full 3D structure of the magnetic turnstiles in the non-axisymmetric topology of stellarators [4]. This method was applied to the nonresonant stellarator divertor. It was found that the outgoing and incoming tubes of a turnstile can start at not only adjacent locations but also at separate locations outside the outermost surface, and that pseudo-turnstiles can also exist. Both results were surprising. Some lessons were learnt.

►Lesson 1: The final wall must be placed sufficiently far from the LGCS, otherwise one cannot distinguish between a true turnstile and a pseudo-turnstile. A pseudo-turnstile looks like a true turnstile, but it has a limited radial excursion. A pseudo-turnstile does not intersect the wall.

►Lesson 2: One cannot presume that the outgoing tube and the incoming tube of a turnstile start at the adjacent locations outside the LGCS. A turnstile whose outgoing and incoming tubes start at adjacent locations are called an adjoining turnstile. On the other hand, a turnstile whose tubes start at separate locations are called a separated turnstile.

►The model Hamiltonian for field lines in nonresonant stellarator divertor is

$$\psi_p = \left[\iota_0 + \frac{\varepsilon_0}{4}\Big((2\iota_0 - 1)cos(2\theta - \zeta) + 2\iota_0 cos(2\theta)\Big)\right]\psi_t + \frac{\varepsilon_t}{6}[(3\iota_0 - 1)cos(3\theta - \zeta) - 3\iota_0 cos(3\theta)]\psi_t{}^{3/2} + \frac{\varepsilon_x}{8}[(4\iota_0 - 1)cos(4\theta - \zeta) - 4\iota_0 cos(4\theta)]\psi_t{}^2.$$

►$\psi_p(\psi_t,\theta,\zeta)$ = the normalized poloidal flux,
ψ_t = the normalized toroidal flux,
φ = the toroidal angle of stellarator,
θ = the poloidal angle.

►$\psi_p(\psi_t,\theta,\zeta)$ = the Hamiltonian,
θ = the canonical position,
ψ_t = the canonical momentum, and
ζ = the canonical time.

ψ_t is conjugate to θ for evolution in ζ.

►ζ = the toroidal angle of the single period, $\zeta = n_P\varphi$,
n_P = number of periods of the stellarator, and
ι_0 = the rotational transform per period on the magnetic axis.

►There are three parameters $(\varepsilon_0, \varepsilon_t, \varepsilon_x)$ in the Hamiltonian. They are called the shape parameters. They control the magnetic configuration of nonresonant stellarator divertor.

ε_0 controls the elongation,
ε_t controls the triangularity, and
ε_x controls the sharpness of edges on the LGCS.

►In our 2018, 2020, and 2022 studies, the shape parameters were set as:

$\varepsilon_0 = \varepsilon_t = ½$, $\iota_0 = 0.15$, $n_P = 5$, $r_{WALL} = 4b, b =$ minor radius.

Step-size was $\delta\zeta = 2\pi/3600$.

Only the shape parameter ε_x is changed.

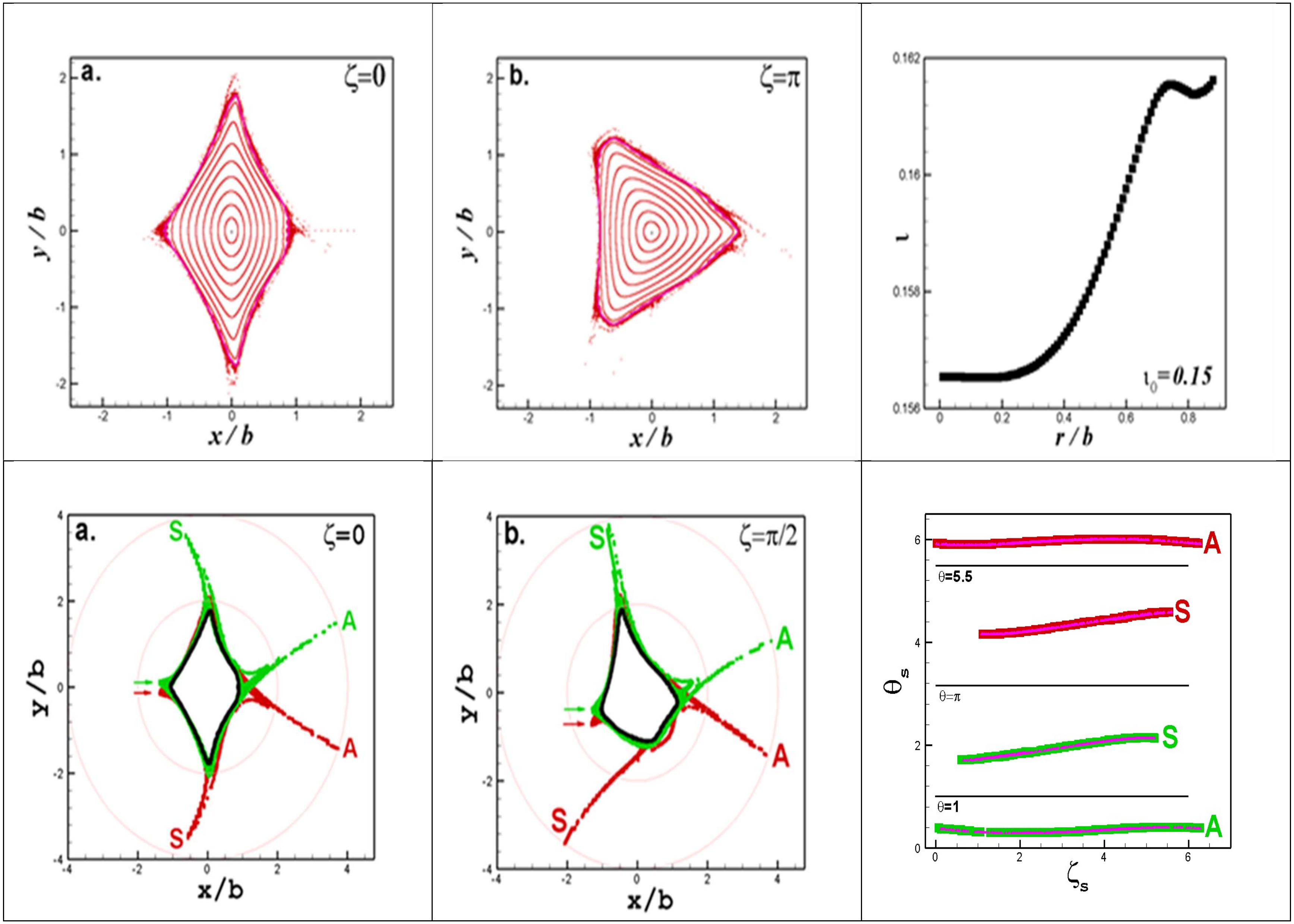
► Old results (2018-2022)

Nonresonant stellarator divertor

- $\varepsilon_x = -0.31$
- No large islands outside the LGCS
- Footprints have fixed locations on the wall

- Has 3 turnstiles
- 2 turnstiles are true turnstiles and 1 turnstile is a pseudo-turnstile
- 1 of the 2 true turnstiles is an adjoining turnstile and 1 is a separated turnstile
- Probability exponents are $d_A = 9/4$ are $d_S = 9/5$

- Notation: A = Adjoining turnstile, S = Separated turnstile
- Color code: ■ = outgoing tube, ■ = incoming tube, ■ = footprint from simulation, ■ = the wall



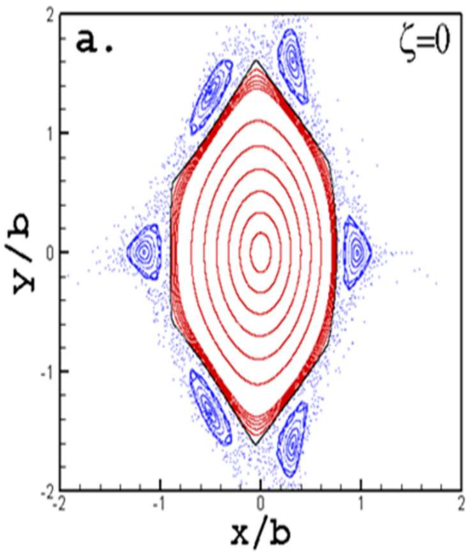
a.
ζ=0
y / b
x / b
b.
ζ=π
ι
r / b
ι0=0.15
a.
ζ=0
S
A
A
S
y/b
x/b
b.
ζ=π/2
S
A
A
S
y/b
x/b
A
θ=5.5
S
θ=π
S
θ=1
A
Θs
ζs

2. NEW RESULTS (not published yet)

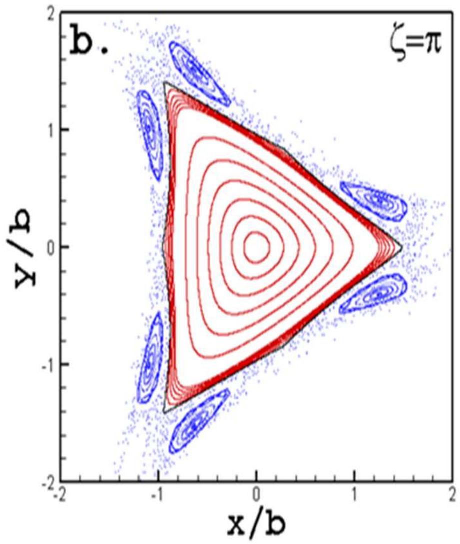
2.1 A NEW TYPE OF NONRESONANT DIVERTOR – THE HYBRID DIVERTOR

►The shape parameter ε_x is changed from -0.31 to -0.1, keeping all the other parameters as before, we get a new type of nonresonant divertor.
►This new divertor has properties of both the island divertor and the nonresonant divertor. It has 6 large islands outside the LGCS and also magnetic turnstiles. The LGCS has sharp edges. So, we have called it The Hybrid Divertor.

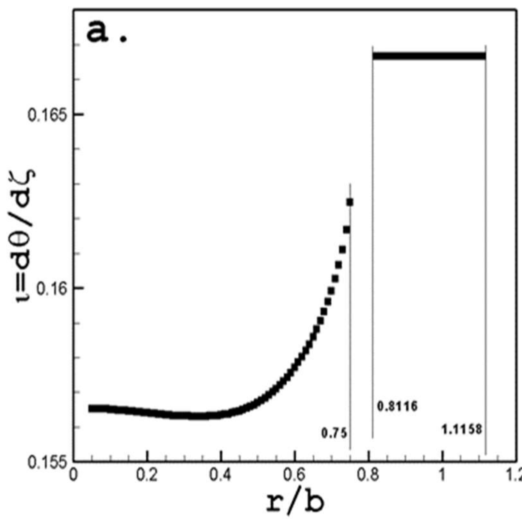
►$\varepsilon_x = -0.1$
►Footprints have fixed locations on the wall
►Has 3 turnstiles
►All 3 are true turnstiles
►2 turnstiles are adjoining turnstiles and 1 is a separated turnstile
►Probability exponents are $d_{A1} = 9/4$, $d_{A2} = 11/5$, and $d_S = 22/5$



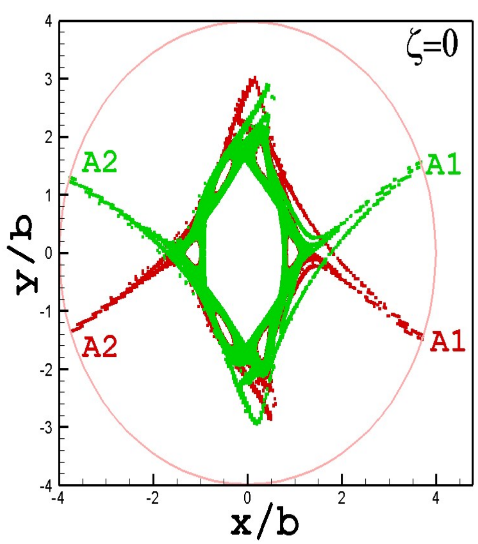
a.
ζ=0
x/b
y/b



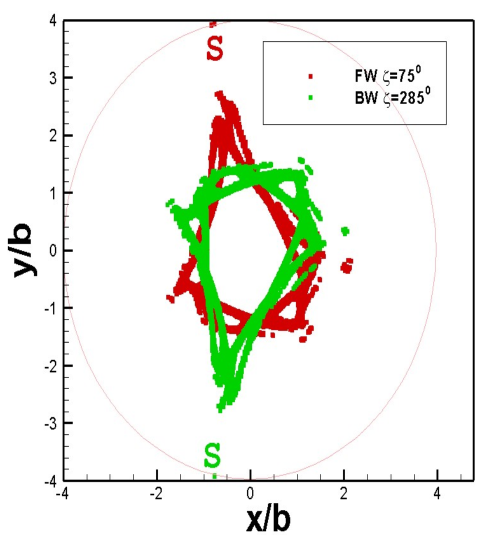
b.
ζ=π
x/b
y/b



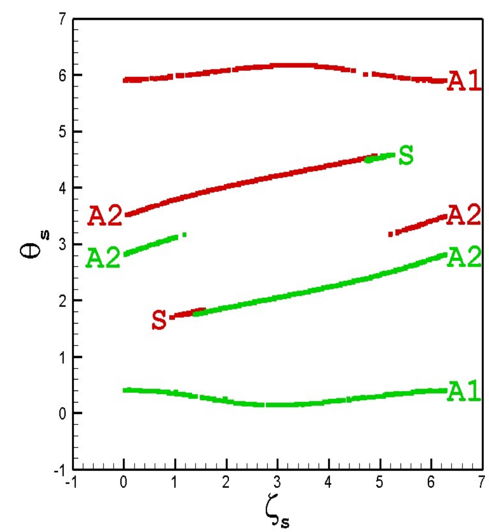
a.
ι=dθ/dζ
r/b
0.75
0.8116
1.1158



ζ=0
A2
A1
A2
A1
x/b
y/b



S
FW ζ=75°
BW ζ=285°
S
x/b
y/b



A1
S
A2
A2
A2
A2
S
A1
θs
ζs

2.2 ANOTHER NEW TYPE OF NONRESONANT DIVERTOR – THE TWO-MODE DIVERTOR

►The shape parameter is set at $\varepsilon_x = 0$, keeping all the other parameters as before, we get a new type of nonresonant divertor. We have called this The Two-Mode Divertor.

►This new type of divertor looks very simple but is very subtle.

►$\varepsilon_x = 0$

►Footprints have fixed locations on the wall

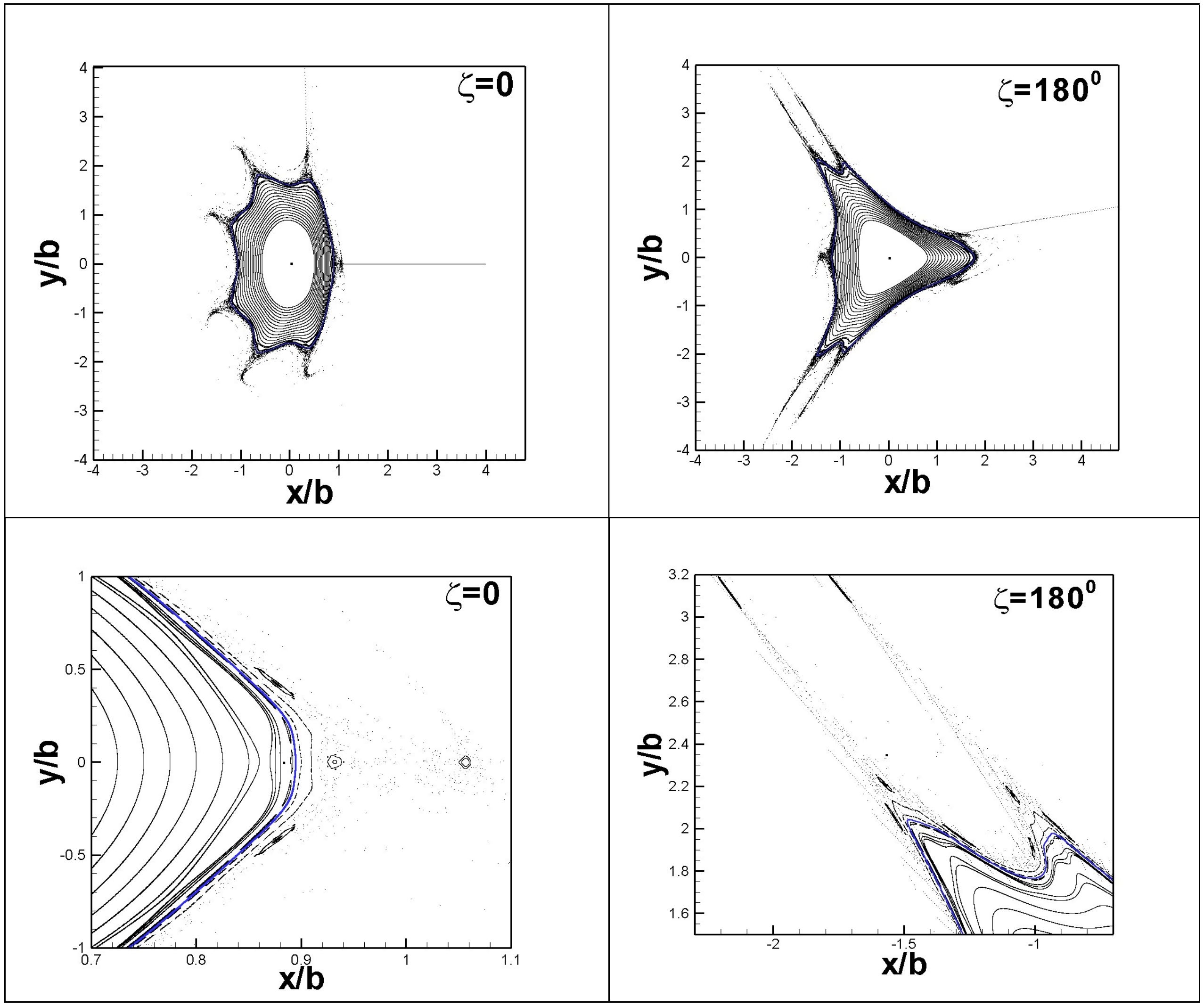
►Has 4 turnstiles

►2 turnstiles are true turnstiles and 2 are pseudo-turnstiles

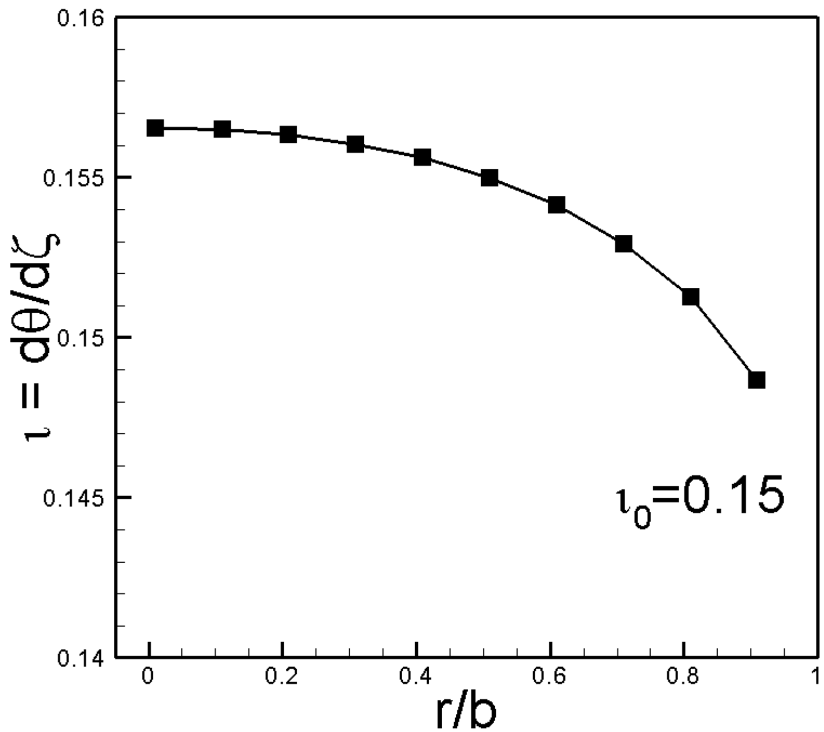
►All 4 turnstiles are separated turnstiles

►The two-mode divertor has some very interesting and important properties.

Phase portraits of the two-mode divertor



Rotational transform



►Magnetic shear is negative for all *r*.

Calculation of probability exponents

►1000 lines are started on the good surface located midway between the magnetic axis and the outermost surface

►Lines are advanced forward and backward for 20 K toroidal circuits of the period

►For the nonresonant divertor and the hybrid divertor, the lines were advanced for 10 K circuits. For two-mode divertor, the number of toroidal circuits is doubled.

►The field lines are given a radial velocity u_ψ in the ψ_t-space. u_ψ is varied from 1E-2, 9E-3, 8E-3, … , 2E-5; in all 27 values of velocity. Footprints and loss-times are calculated.

►For $u_\psi < 2 \times 10^{-5}$, none of the outgoing or incoming field lines strike the wall for 20 K toroidal circuits.

►From the footprints, we find that there are in all *4 turnstiles; two of them are true turnstiles* and the remaining *two are pseudo-turnstiles*.

►The locations of these footprints on the wall are fixed for all values of the velocity.

►The loss-times ζ_l are at least twice as long as in nonresonant and hybrid divertors.

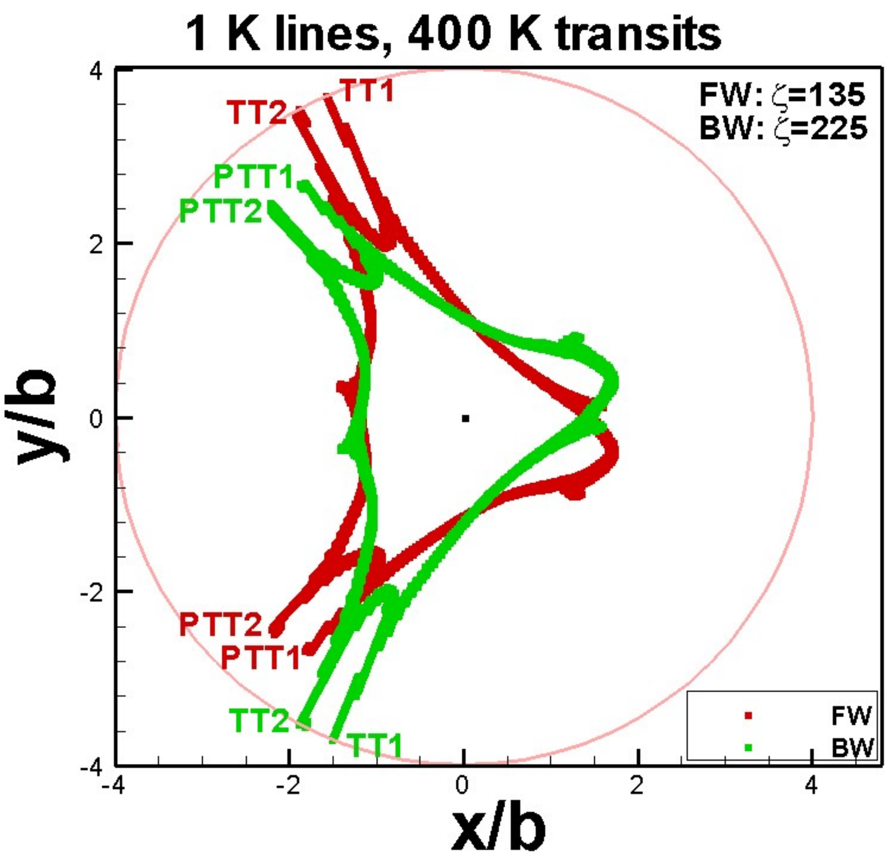
►Probability exponents

Turnstile	d
1st true turnstile	$3.2435 \cong 3.25 = 3\frac{1}{4}$
2nd true turnstile	$3.1299 \cong 3.25 = 3\frac{1}{4}$

►Probability exponents are universal because both exponents are equal. This is not the case for nonresonant and hybrid divertors.

Calculation of turnstiles

►1000 points on the LGCS are shifted radially outwards through a distance $0 < \Delta r/b \leq 3 \times 10^{-2}$. The lines are advanced forward and backward for 400 K circuits. In nonresonant and hybrid divertors, $\Delta r/b = 10^{-2}$ and circuits = 200 K.

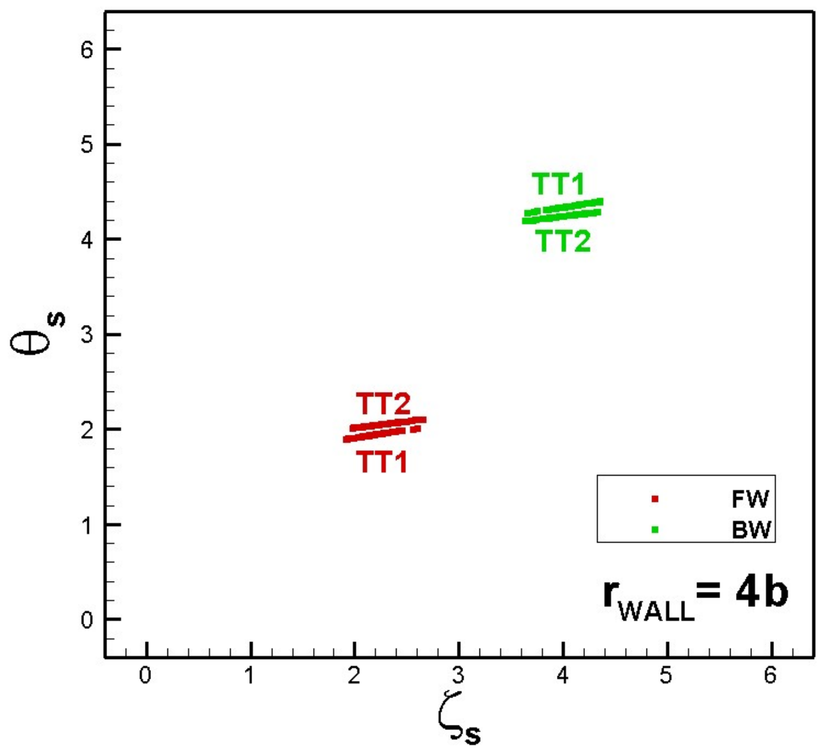


►TT1, TT2, PTT1, and PTT2 = the 1st true turnstile, the 2nd true turnstile, the 1st pseudo-turnstile, and the 2nd pseudo-turnstile

►Color code:

- = outgoing tube,
- = incoming tube,
- = the wall

Magnetic footprints



►Magnetic footprint occupy a very small fraction, $f_s \cong 10^{-3}$, of the area of the wall compared to the nonresonant and hybrid divertors.

►A very small fraction of lines goes into turnstiles compared to nonresonant and hybrid divertors.

	True turnstile 1	True turnstile 2	Circulating lines
Outgoing lines	70	84	846
Incoming lines	73	83	844

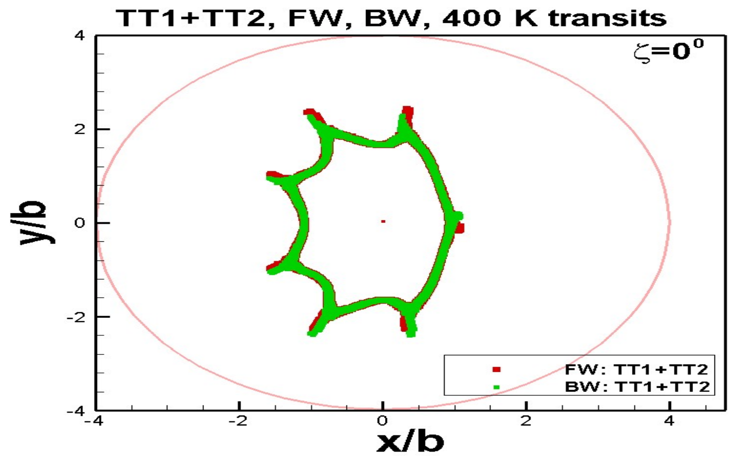
Connection transits M_d and connection lengths L_c

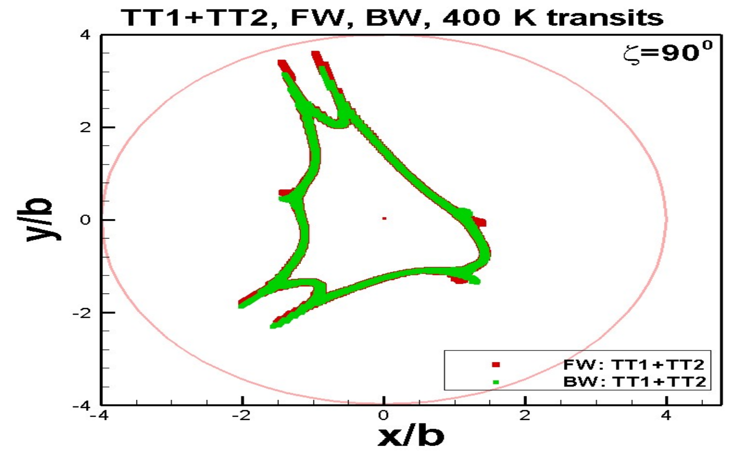
►Weighted average connection transits for 1st turnstile is $M_d = 16{,}523$, and the for the 2nd turnstile is $M_d = 18{,}859$.

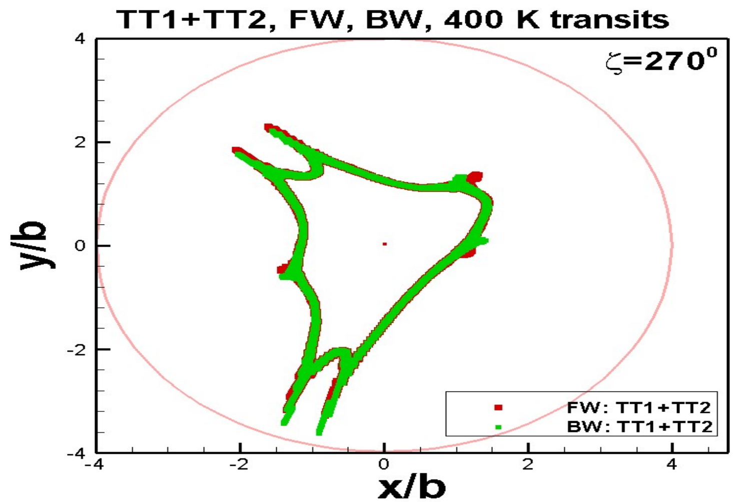
►Loss-times are much larger than in nonresonant and hybrid divertor.

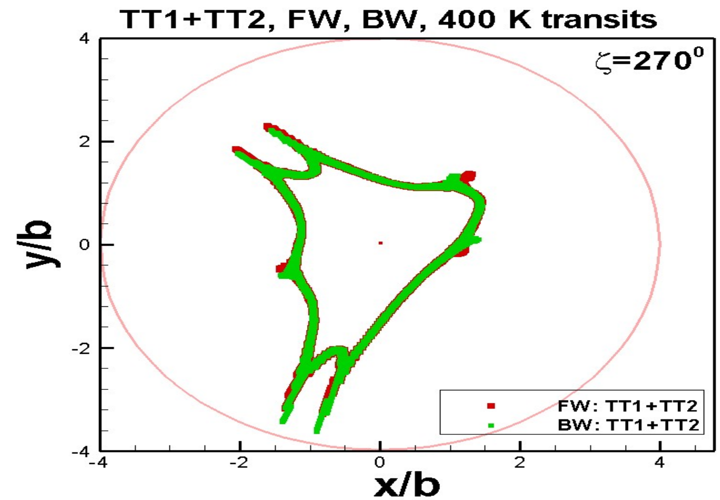
►Connection length $L_c = 2\pi R_0 M_d$. For W7-X, $R_0 = 5.5\ m$, $L_c \cong 571\ km$.

In the two-mode divertor, all turnstiles form a single flux tube in which the outgoing and the incoming lines counter-stream in opposite directions.









Key results on the two-mode divertor

►Two-mode divertor has two true turnstiles and two pseudo-turnstiles. All turnstiles are separated.

►All four turnstiles form a single flux tube in which outgoing and incoming lines counter-stream.

►A very large fraction of lines, ~ 84 %, circulate around the outermost surface and do not go into the turnstiles. Plasma particles flowing these lines will radiate.

►A small fraction, ~ 16 %, go into turnstiles.

►Connection times for true turnstiles are long, ~ 16 K to 19 K toroidal transits.

►Footprints have fixed locations on walls and cover a small fraction, $f_s = 10^{-3}$, of the wall area.

3. FUTURE PLANS

►Make a comparative analysis of the three distinct types of the nonresonant stellarator divertors: the nonresonant divertor; the hybrid divertor; and the two-mode divertor.

►Calculate the critical parameters for all three types.

►The critical parameters: Connection lengths L_C, the loss-times ζ_L, the fraction of the area of the wall covered by the magnetic footprints on the wall f_s, the width of magnetic footprints on the wall w_f, the width of the layer of the circulating field lines around the LGCS δ_c, and the dwelling times of the field lines inside the circulating layer τ_C, width of the turnstiles w_T, fractions of lines going in turnstiles f_T, and fraction going in circulating layer f_C.

►Compare the critical parameters for three types, and assess relative merits and demerits of the three types.

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References

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[2] A. H. Boozer and A. Punjabi, Simulation of stellarator divertors, Phys. Plasmas **25** (2018) 092505, 1-13.

[3] A. Punjabi and A. H. Boozer, Nonresonant stellarator divertor, Phys. Plasmas **27** (2020) 012503, 1-10.

[4] A. Punjabi and A. H. Boozer, Magnetic turnstiles in nonresonant stellarator divertor, Phys. Plasmas **29** (2022) 012502, 1-9.

[5] R. S. MacKay, J. D. Meiss, and I. C. Percival, Transport in Hamiltonian systems, Physica D **13** (1984), 55-81.

SUPPLEMENTARY MATERIAL

1. THE HAMILTONIAN FOR THE FIELD LINES

► The magnetic field B in stellarators in generalized contravariant representation is [A. H. Boozer, Evaluation of the structure of ergodic fields, Phys Fluids 26 (1983) 1288-1291] is

$$\boldsymbol{B}(\psi_t,\theta,\phi) = \boldsymbol{\nabla}\psi_t\times\boldsymbol{\nabla}\theta+\boldsymbol{\nabla}\phi\times\boldsymbol{\nabla}\psi_p(\psi_\mathrm{t},\theta,\phi). \qquad (1)$$

ψ_p = the normalized poloidal flux,
ψ_t = the normalized toroidal flux,
φ = the toroidal angle of stellarator,
θ = the poloidal angle.
$\psi_p(\psi_t,\theta,\zeta)$ = the Hamiltonian,
θ = the canonical position,
ψ_t = the canonical momentum, and
ζ = the canonical time.

Here,

ζ = the toroidal angle of the single period, $\zeta = n_P\varphi$,
n_P = number of periods of the stellarator, and
ι_0 = the rotational transform per period on the magnetic axis.

►The magnetic field lines are given by the equations

$d\psi_t/d\varphi = \boldsymbol{B}\cdot\boldsymbol{\nabla}\psi_t/\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi$ and
$d\theta/d\varphi = \boldsymbol{B}\cdot\boldsymbol{\nabla}\theta/\boldsymbol{B}\cdot\boldsymbol{\nabla}\phi$. (2)

►Using the canonical representation, these equations can be rewritten as

$d\psi_t/d\varphi = -\partial\psi_p/\partial\varphi$ and
$d\theta/d\varphi = \partial\psi_p/\partial\psi_t$,

►These equations are mathematically identical to Hamiltonian equations with

$\psi_p(\psi_t,\theta,\zeta)$ = the Hamiltonian,
θ = the canonical position,

ψ_t = the canonical momentum, and
ζ = the canonical time.

Here,

ζ = the toroidal angle of the single period, $\zeta = n_P\varphi$,
n_P = number of periods of the stellarator, and
ι_0 = the rotational transform per period on the magnetic axis.

►The model Hamiltonian for the trajectories of magnetic field lines in a single period of the nonresonant stellarator divertor is given by [A. H. Boozer and A. Punjabi, Simulation of stellarator divertors, Phys. Plasmas 25 (2018) 092505, 1-13]

$$\begin{aligned}\psi_p = &\left[\iota_0 + \frac{\varepsilon_0}{4}\left((2\iota_0 - 1)cos(2\theta - \zeta) + 2\iota_0 cos(2\theta)\right)\right]\psi_t \\ &+\frac{\varepsilon_t}{6}[(3\iota_0 - 1)cos(3\theta - \zeta) - 3\iota_0 cos(3\theta)]{\psi_t}^{3/2} \\ &+\frac{\varepsilon_x}{8}[(4\iota_0 - 1)cos(4\theta - \zeta) - 4\iota_0 cos(4\theta)]{\psi_t}^2. \qquad (3)\end{aligned}$$

►The Hamiltonian is related to the 1984 study of MacKay *et al* [R. S. MacKay, J. D. Meiss, and I. C. Percival, Physica D 13, 55 (1984)] on the loss of confining surfaces in the Standard Map as the map parameter k is increased. In this study, MacKay *et al* introduced the concepts of cantori and turnstiles.

►STELLARATOR MAP EQUATIONS

[A. Punjabi and A.H. Boozer, Nonresonant stellarator divertor, Phys. Plasmas 27 (2020) 012503, 1-10]

$$\psi_t^{(j+1)} = \psi_t^{(j)} - \frac{\partial\left(\psi_t^{(j+1)}, \theta^{(j)}, \zeta^{(j)},\right)}{\partial \theta^{(j)}}, \tag{4}$$

$$\theta^{(j+1)} = \theta^{(j)} + \frac{\partial\left(\psi_t^{(j+1)}, \theta^{(j)}, \zeta^{(j)},\right)}{\partial \psi_t^{(j+1)}}, \tag{5}$$

$$\zeta^{(j+1)} = \zeta^{(j)} + \delta\zeta. \tag{6}$$

Step-size $\delta\zeta = \frac{2\pi}{3600.}$

►The map preserves the symplectic invariant

$$\frac{\partial\left(\psi_t^{(j+1)},\theta^{(j+1)}\right)}{\partial\left(\psi_t^{(j)},\theta^{(j)}\right)} = +1. \qquad (7)$$

j denotes the iteration number.

► THE SHAPE PARAMETERS $(\varepsilon_0, \varepsilon_t, \ \varepsilon_x)$

There are three shape parameters $(\varepsilon_0, \varepsilon_t, \ \varepsilon_x)$ in the Hamiltonian which control the magnetic configuration:

ε_0 controls the elongation,
ε_t controls the triangularity, and
ε_x controls the sharpness of edges on the LGCS.

2. CALCULATION OF PROBABILITY EXPONENTS d'S

► An efficient simulation method for stellarator divertors was developed in 2018 by Boozer and Punjabi. In this method:

► Field lines are started on a good surface well inside the outermost surface

► Field lines are given an artificial constant velocity, u_ψ, in ψ_t – space.

► These lines are integrated forwards and backwards and footprints on the wall and loss-times ζ_l are calculated as a function of the velocity u_ψ.

► The method allows us to calculate the exponents d_j for magnetic turnstiles.

► $P(\psi_t) = (d+1)c_p\left(\frac{\psi_t-\psi_0}{\psi_0}\right)^d; \quad \zeta_l = \frac{1}{c_p}\left(\frac{c_p}{u_\psi}\right)^{\frac{d}{d+1}}.$

► Simulation gives $\zeta_l(u_\psi)$.

► Linear fit to $log(\zeta_l)$ vs $log(u_\psi)$ gives $\zeta_l = c{u_\psi}^p$.

► Then, $d = -\frac{p}{p+1}$.

3. METHOD FOR CALCULATION OF TURNSTILES

► An axisymmetric wall is chosen.
► The torus formed by this wall is divided into 360×360×400 cells. Each cell is of the size $\Delta\zeta = 2\pi/360$, $\Delta\theta = 2\pi/360$, and $\Delta r/b = 0.01$. b is the minor radius of the torus.
► The array C_{ijk} is initialized to zero.
► 1 K Field lines on the outermost surface are given a radial outward random kick of about 1% of minor radius in the poloidal plane $\zeta = 0$.
► These lines are integrated forward and backward for 200 K toroidal circuits of the period. At the end of each step of integration the position of line is calculated. If the line is inside a cell, its occupancy count is raised by unity. This is done only for lines that hit the wall. This gives a 3 D picture of the magnetic turnstiles.