2012 ANU Maths Day Group Contest

The questions are arranged in increasing order of marks.

However, this is not necessarily the order of difficulty, which very much depends on your own strengths and preferences. You may find that some of the later questions will take a little longer to do, but are otherwise no harder than the earlier ones.

Remember there are more questions here than any team can be expected to do justice to. We hope that you'll find some amongst them that will pique your curiosity.

Enjoy!

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Question 1. Chocolate girls   15 marks

Emily and Elisabeth are sisters. Each sister buys three of her own favourite chocolate bar and two of her sister's favourite. Emily paid $7.75 and Elisabeth paid $7.25.

What is the price of one of Emily's favourite chocolate bars?

Question 2. Strings of letters   15 marks

A string of letters, such as a word, is 'coded' by replacing each of the letters with its number in the alphabet: 1 for A, 2 for B, and so on. For example DAG becomes 417 and LCFV becomes 123622.

Which of the following strings of digits arise from a unique string of letters?

      (a)   45106271   (b)   28310401   (c)   10923114

Question 3. Cubik's cube   20 marks

Mrs Cubik likes to construct cubes by stacking together a number of $1\,\text{cm} \times 1\,\text{cm} \times 1\,\text{cm}$ building blocks. The *skin* of 'Cubik's cube' consists of those building blocks which lie in at least one face of the constructed cube.

How many building blocks are in the skin of a $4\,\text{cm} \times 4\,\text{cm} \times 4\,\text{cm}$ cube?

Question 4. Cross number puzzle   20 marks

Sketch a 3×3 square grid and shade in the central square. Enter a single-digit number in each of the eight vacant cells. What is the three-digit number in the bottom row if the following conditions apply?

  a. the top row forms a sixth power;

  b. the right column forms a palindrome (i.e. a number that reads the same from the left and from the right);

  c. the left column forms a palindrome;

  d. the bottom row forms a multiple of 11.

2012 ANU Maths Day Group Contest

Question 5. How many sevens?   25 marks

The pages of a book are labelled by consecutive integers, beginning at 1. The digit 7 occurs among the page numbers 33 times.

How many pages does the book have?

Question 6. How many squares   25 marks

How many square numbers occur in the range one hundred (100) to ten thousand (10 000) inclusive?

Question 7. A curvy triangle   30 marks

Three identical, circular coins of radius 1 cm lie on a hard, flat surface. Each just touches the other two.

How many square centimetres are there in the curvy area that the coins surround? [Give exact answer, not an approximation]

Question 8. Tim finds fountain height   30 marks

Timothy would like to know the height of a fountain. He finds a sign which is 20 m from the fountain and 2 m tall. The top of the sign, the top of the fountain and his eye are in a straight line. In this position he is 2 m from the sign.

If Tim's eye is 1 m from the ground how tall is the fountain?

Question 9. Jim's special points   35 marks

Jim thinks of as 'special' a point inside a rectangle whose distances from three corners of the rectangle are 6, 7 and 13. Find all possibilities for the distance of a special point from the remaining fourth corner.

[Give exact answers, not an approximation]

Question 10. Even and Odd   35 marks

A function $E(x)$ is called *even* if $E(-x) = E(x)$; for example, the function x^2 is even. A function $O(x)$ is called *odd* if $O(-x) = -O(x)$; for example, the function x is odd.

Show that a function $f(x)$ that is neither even nor odd is nevertheless the sum of an even function $E(x)$ and an odd function $O(x)$: i.e. $f(x) = E(x) + O(x)$.

[For full marks: give correct answer with reasoning on the associated answer sheet.]