Conference Schedule

	Monday	Tuesday	Wednesday	Thursday
8.30am	*Welcome*			
9am	Alina Ostafe	Fabien Mehdi Pazuki	Alexandru Ghitza	Lance Gurney
9.30am				
10am	*Tea*	*Tea*	*Tea*	*Tea*
10.30am	Florian Breuer	Nikita Shulga	Vandita Patel	Randell Heyman
11am	Kamil Bulinski	Chiara Bellotti	Muhammed Afifurrahman	Anthony Henderson
11.30am	Ethan Lee	Sidney Morris	Mumtaz Hussain	James Borger
12pm	*Lunch*	*Lunch*	*Lunch*	*Lunch*
12.30pm				
1pm				
1.30pm	Felipe Voloch	Johannes Schleischitz	Owen Patashnick	
2pm				
2.30pm	*Tea*	*Tea*	*Tea*	
3pm	Igor Shparlinski	Chandler Corrigan	Christian Bagshaw	
3.30pm	Bryce Kerr	Ben Ward	Zhenlin Ran	
4pm	Daniel Johnston	*Free afternoon + hike*	Dion Nikolic	
4.30pm	Tim Trudgian		Gerardo González Robert	
5pm	*Welcome Reception*			
5.30pm				
6pm			*Conference Dinner*	

Welcome Reception: There will be a one hour welcome reception at 5pm after the first day. This will be held in the foyer of the Hanna Neumann building (directly outside the room in which the talks are held).
Hike: After the conclusion of talks on Tuesday there will be a hike up Black Mountan, which is adjacent to ANU. It should be no more than 90 minutes - 2 hours as a round trip, and we hope to be back before sunset.
Conference Dinner: The conference dinner will be held in the Mosaic Room at Ovolo Nishi, from 6-9pm on Wednesday. Look forward to a trivia event run by Tim Trudgian!

Plenary Talks

Alexandru Ghitza
University of Melbourne
Title: A tour of modular forms and quaternions
Abstract: There is a beautiful relation between the action of Hecke operators on spaces of modular forms mod p and spaces of functions on quaternion algebras, first elucidated via geometric means by Jean-Pierre Serre almost 50 years ago. I aim to describe Serre's approach, explain why the quaternionic viewpoint is profitable for the study of modular forms, touch upon the incredibly wide generalisations of this relation proved over the last couple of decades, and point at some challenges that may inform future developments.

Lance Gurney
University of Melbourne
Title: q-de Rham cohomology
Abstract: de Rham cohomology is one of the fundamental invariants of a variety X, giving a collection of $\mathbf{Z}$-modules $H^i_{\mathrm{dR}}(X)$ which are functorial in X. If X admits a certain special type of coordinates, then, following Aomoto (and Scholze), one also has q-de Rham cohomology, giving a collection of $\mathbf{Z}[q]$-modules $H^i_{q\text{-dR}}(X)$ which upon specialisation $q \to 1$ recover the usual de Rham cohomology groups. However, q-de Rham cohomology is no longer (obviously) functorial in the variety X, owing to its dependence on coordinates. I will explain how functoriality can be recovered using an integral version of Bhatt–Scholze's p-adic prismatic cohomology.

Alina Ostafe
UNSW Sydney
Title: On some arithmetic statistics for integer matrices

Abstract: We consider the set $\mathcal{M}_n(\mathbb{Z}; H))$ of $n \times n$-matrices with integer elements of size at most H and obtain a new upper bound on the number of matrices from $\mathcal{M}_n(\mathbb{Z}; H)$ with a given characteristic polynomial $f \in \mathbb{Z}[X]$, which is uniform with respect to f. This complements the asymptotic formula of A. Eskin, S. Mozes and N. Shah (1996) in which f has to be fixed and irreducible. We use our result to address various other questions of arithmetic statistics for matrices from $\mathcal{M}_n(\mathbb{Z}; H)$, eg satisfying certain multiplicative relations. Some of these problems generalise those studied in the scalar case $n = 1$ by F. Pappalardi, M. Sha, I. E. Shparlinski and C. L. Stewart (2018) with an obvious distinction due to the non-commutativity of matrices.
Joint works with Igor Shparlinski.

Owen Patashnick
King's College London
Title: E-motive-ating formal periods via the special values $L(Sym^n(E), n+m)$
Abstract: In this talk, we will trick the audience into thinking we are talking about special values of L-functions, but really we will use these values as a trojan horse to explore the motivic periods that underlie geometric content associated to these L-values. In particular, we will "motivate" an explicit construction of classes built out of algebraic cycles associated with the L-values $L(Sym^n(E), n+m)$ and muse on the consequences. We will try to make the talk as accessible as possible, and hopefully keep discussion of machinery to a minimum. The audience is invited to help explore with the speaker the number theoretic information supported by these classes.

Fabien Mehdi Pazuki
University of Copenhagen
Title: Northcott numbers and applications
Abstract: A set of algebraic numbers with bounded degree and bounded height is a finite set, by Northcott's theorem. The set of roots of unity is of height zero, but is infinite. What about other sets of algebraic numbers? When is a set of bounded height still infinite? A way to approach this question is through the Northcott number of these sets. We will study some of their properties, discuss links to Julia Robinson's work on undecidability, and explain other applications towards height controls in Bertini statements. The talk is based on joint work with Technau and Widmer.

Johannes Schleischitz
Middle East Technical University
Title: Sum and product sets of classes of sets relevant in Diophantine approximation
Abstract: Liouville numbers are real numbers that admit very good approximation by rationals. Paul Erdos proved that every real number can be written as the sum (and product) of two Liouville numbers. We extend this result in several directions, in particular considering sum sets of classes of numbers with prescribed or bounded irrationality exponent (the case of infinite exponent recovers Erdos result). This will also admit conclusions on metrical properties of Cartesian products of these sets.

Felipe Voloch
University of Canterbury
Title: Random Diophantine Equations
Abstract: Diophantine equations are polynomial equations in several variables and integer coefficients where the solutions are sought among integer or rational values. It is notoriously difficult to decide whether such equations have solutions. In this talk we will discuss an old conjecture of B. Poonen and the speaker about what happens for a random such equation and recent progress made on this conjecture.

Contributed Talks

Muhammad Afifurrahman
UNSW Sydney
Title: Counting products of integer matrices with bounded height
Abstract: How many numbers can be written as a product of m integers whose (absolute values) are at most H? Erdős popularized this problem for $m = 2$, which is asymptotically solved by Ford, and later generalized for $m > 2$ by Koukoulopoulos.
I will talk about some analogous problems when we replace "numbers" with "matrices" and give some related bounds. Related to these results, I will also talk about bounding the number of solutions to the related equations

over integer matrices with bounded entries, such as $A_1 \dots A_m = B_1 \dots B_m$ and $A_1 \dots A_m = C$. for a fixed matrix C.

Christian Bagshaw
UNSW Sydney
Title: Exponential sums in function fields
Abstract: Number theorists have long noticed similarities between the theory of number fields and the theory of function fields over finite fields. One notable aspect of this connection is the stark similarity between the theories of exponential sums in these two spaces. In this talk, we will give an introduction to working with exponential sums in rational function fields over finite fields, by using and building upon our intuition for analogous sums over the real numbers. We will also discuss some recent results and applications, building upon recent work of Sawin and Shusterman.

Chiara Bellotti
UNSW Canberra
Title: On the generalised Dirichlet divisor problem
Abstract: In this talk we present new unconditional estimates on $\Delta_k(x)$, the remainder term of the generalised divisor function, for large k. By combining new estimates of exponential sums and Carlson's exponent, we show that $\Delta_k(x) \ll x^{1-1.224(k-8.37)^{-2/3}}$ for $k \geq 30$ and $\Delta_k(x) \ll x^{1-1.889k^{-2/3}}$ for all sufficiently large fixed k. This is a joint work with Andrew Yang.

James Borger
Australian National University
Title: Narrow class groups and reflexive Picard groups of semirings
Abstract: Much of the purely algebraic part of basic 19th-century algebraic number theory was subsumed in the mid 20-th century by commutative algebra and scheme theory. For example, the class group of a number field is the Picard group of its subring of algebraic integers. This places the class group in its true home, as a very special instance of a much broader and even more natural construction.
But the infinite prime, the ever-so-slightly analytic ingredient in algebraic number theory, has never really mixed naturally with scheme theory. Any way of incorporating it has felt ad hoc. A basic instance of this is that the narrow class group of a number field has had no scheme-theoretic description which is as satisfying as that of the ordinary class group.
In this talk, I'll explain how it's possible to build a commutative algebra and scheme theory out of not just rings but all semirings ("rings possibly without subtraction"). So just as usual scheme theory extended algebraic geometry from base fields to base rings, thus incorporating integrality phenomena, this extends scheme theory from base rings to base semirings, thus incorporating positivity phenomena. And then the narrow class group of a number field has a satisfying description: it is just the reflexive Picard group of the subsemiring of elements which are non-negative under all the real embeddings in question.
The purpose of the talk is to explain this. It is based on forthcoming work with Jaiung Jun.

Florian Breuer
University of Newcastle
Title: Parity of fundamental units
Abstract: Suppose $p \equiv 5 \bmod 8$ is a prime, and consider the fundamental unit u of the real quadratic field $\mathbb{Q}(\sqrt{p})$. There are three possibilities for u modulo the prime above 2. How often does each occur? Stated another way, when does the Pellian equation $x^2 - py^2 = -4$ have odd solutions? I will present numerical data suggesting a possible link to Shintani zeta functions, and invite audience members to join me in these investigations.

Kamil Bulinski
UNSW Sydney
Title: Counting embeddings of free subgroups in $\mathrm{SL}_2(\mathbb{Z})$.
Abstract: A classical result states that if one randomly chooses s elements from a connected non-solvable lie group G (e.g., $G = \mathrm{SL}_d(\mathbb{R})$) then Haar almost surely these elements are free (generate the free group of rank s). We show that an analogous statement holds for $G = \mathrm{SL}_2(\mathbb{Z})$: If one selects uniformly i.i.d matrices $A_1, \dots, A_s \in \mathrm{SL}_2(\mathbb{Z})$ from a ball of large radius X then with probability at least $1 - X^{-1+o(1)}$ the matrices $A_1, \dots, A_s$ are free generators for a free subgroup of $\mathrm{SL}_2(\mathbb{Z})$. This improves a claim of E. Fuchs and I. Rivin (2017) which states that this probability

converges to 1 as $X \to \infty$. We also disprove a Lemma in their work that they used to deduce their claim. Based on a joint work with Ostafe and Shparlinski.

Chandler Corrigan
UNSW Sydney
Title: Zero-density estimates for L-functions associated to fixed-order Dirichlet characters
Abstract: An average bound on the second moment of L-functions associated to families of fixed-order Dirichlet characters is presented, from which a collection of zero-density estimates are derived. These results improve on previous bounds in certain regions.

Anthony Henderson
Defence Science and Technology Group
Title: Parametrizing Heron triangles symmetrically
Abstract: A Heron triangle is one whose side-lengths and area are all rational numbers. Up to scaling, these are given by rational points on the projective surface S defined by the homogeneous equation $xyz = (x + y + z)w^2$. It is trivial to find rational parametrizations which break the symmetry between x, y, z. However, to address the unsolved problem of whether there exist Heron triangles whose median-lengths are also rational, it would be better to have a symmetric parametrization. I will explain how to find one using the fact that the minimal resolution of S can be obtained from the projective plane by a sequence of six blow-ups.

Randell Heyman
UNSW Sydney
Title: Sparse sets that satisfy the PNT
Abstract: There has been much research on Piatetski-Shapiro and Beatty sequences/sets, both of which use the floor function. Recently there has been research interest in a family of sets based the floor of x/n. Interestingly, some of the sets mentioned above, though sparse, satisfy the Prime Number Theory. We ask what is the sparsest of these sets?
This is joint work with Olivier Bordellès and Dion Nikolic.

Mumtaz Hussain
La Trobe University
Title: Hausdorff measure for limsup sets
Abstract: In this talk, I will discuss a general principle for studying the Hausdorff measure of limsup sets. A consequence of this principle is the well-known Mass Transference Principle of Beresnevich and Velani (2006). To highlight the breadth of this principle, I will list two applications of this theorem, (1) for the sets of Dirichlet non-improvable numbers, and (2) for recurrent sets.

Daniel Johnston
UNSW Canberra
Title: The error term in the explicit Riemann-von Mangoldt formula
Abstract: One of the main focuses of analytic number theory is obtaining good estimates for the number of primes less than any given number x. The primary tool for obtaining such estimates is the explicit Riemann-von Mangoldt formula. In this talk we will discuss recent estimates of the error term in this formula, in both an asymptotic and explicit sense. Moreover, we will discuss several applications of these estimates to other number-theoretic problems.

Bryce Kerr
UNSW Canberra
Title: Statistical inverse theorems for power sums
Abstract: This talk is focused on problems which aim to extract structure from sequences of complex numbers which are close to extremal in Turán's power sum problems. We give some motivation for such problems, sketch some basic results in this direction and conclude with open problems.

Ethan Lee
University of Bristol
Title: The distribution of primes in arithmetic progressions

Abstract: Using the theory of Dirichlet L-functions and orthogonality relations, we have established analogues of the prime number theorem and Mertens' theorems for primes in arithmetic progressions, which enable us to study the distribution of primes in an arithmetic progression. In 1976, Norton gave an asymptotic (but ineffective) description of the constant in Mertens' second theorem for primes in arithmetic progressions. In this talk, I will describe how to prove that if the Generalised Riemann Hypothesis is true, then an asymptotic refinement to Norton's observation is available; this was joint work with Daniel Keliher.

Sidney Morris
La Trobe University
Title: Topology Meets Number Theory
Abstract: This talk reports on research by Taboka Prince Chalebgwa and the speaker. When point-set topology meets transcendental number theory awesome results pop out. These results are inspired by those of Paul Erdos, Kurt Mahler, and Yann Bugeaud.

Dion Nikolic
UNSW Canberra
Title: Counting the Number and Dimension of Classes of Matrix Solutions for a Given Polynomial
Abstract: In this talk we extend the Fundamental Theorem of Algebra to matrix polynomials by finding a formula and the asymptotic behaviour for the number of equivalence classes of matrix solutions for any given polynomial. We also study these equivalence classes as Lie groups and find a formula and the asymptotic behaviour for the average dimension amongst equivalence classes solving a polynomial.

Vandita Patel
University of Manchester
Title: Shifted powers in Lucas-Lehmer sequences
Abstract: The explicit determination of perfect powers in (shifted) non-degenerate, integer, binary linear recurrence sequences has only been achieved in a handful of cases. In this talk, we combine bounds for linear forms in logarithms with results from the modularity of elliptic curves defined over totally real fields to explicitly determine all shifted powers by two in the Fibonacci sequence. This is joint work with Mike Bennett (UBC) and Samir Siksek (Warwick).

Zhenlin Ran
University of Newcastle
Title: Heights of Drinfeld modules
Abstract: Drinfeld module is the analogue of elliptic curve in function fields. The theory of heights for elliptic curves also works for Drinfeld modules. In this talk, we briefly review the background of Drinfeld modules and Weil heights. Also, we introduce two modular heights of Drinfeld modules: Taguchi heights and graded heights. The former could be regarded as the analogue of Faltings heights of abelian varieties and the latter could be regarded as the generalization of the Weil height of j-invariant. We present some results about the variation of the two modular heights under isogenies. In particular, we can obtain for Drinfeld modules of rank 2 an analogous result of Nakkajima and Taguchi's formula for the variation of Faltings heights for elliptic curves. Based on this, we can get a lower bound for the Weil height of a singular modulus of Drinfeld modules.

Gerardo González Robert
La Trobe University
Title: Symbolic dynamics and complex Diophantine approximation
Abstract: In 1887, A. Hurwitz introduced a continued fraction expansion for complex numbers. Hurwitz continued fractions associate an infinite sequence of Gaussian integers $\mathbb{Z}[i]$ to each complex number which is not a Gaussian rational. The resulting space of sequences Ω is known to be complicated. In particular, it is not closed a closed subset of the space of sequences in Gaussian integers. In this talk, we propose a (closed) sub-shift of $\mathbb{Z}[i]^{\mathbb{N}}$ which allows us to study Hurwitz continued fractions. Under this perspective, we show that the set of normal numbers (with respect to a natural measure) belong to the third level of the Borel hierarchy. This is joint work with Felipe García-Ramos and Mumtaz Hussain.

Igor Shparlinski
UNSW Sydney

Title: Square-free values of random polynomials
Abstract: The question of whether or not a given integral polynomial takes infinitely many square-free values has only been addressed unconditionally for polynomials of degree at most 3. We address this question, on average, for polynomials of arbitrary degree.
Joint work with Tim Browning (IST, Austria).

Nikita Shulga
La Trobe University
Title: Metrical properties of exponentially growing partial quotients.
Abstract: Over the past years, there was a significant progress in metrical theory of continued fractions. Wang and Wu in 2008 completely determined the Hausdorff dimension of the set

$$E(\phi) = \{x \in [0, 1) : a_n(x) \geq \phi(n) \text{ for i.m. } n\}.$$

Since then, the Hausdorff dimension was calculated in many set-ups, where instead of the growth rate of individual partial quotient, authors have considered the growth rate of products of partial quotients, maximum partial quotient, weighted products of partial quotients and others.
Usually, the upper bound for the Hausdroff dimension follows easily by considering a natural cover of a given set. The lower bound, however, is often tricky to deal with.
In this talk I construct a special set of continued fractions and find the Hausdroff dimenstion of it. This set for a suitable choice of parameters, becomes a subset of a set under consideration for all of the set-ups mentioned above, providing an optimal lower bound of the Hausdorff dimension. Some new applications are also provided.
This is a joint work with M. Hussain.

Tim Trudgian
UNSW Canberra
Title: Seagull C ρ's force mall queue.
Abstract: Not only will the deciphering of the title put readers in good stead for NTDU trivia, but it may lead them to the rich topic of exceptional zeroes for Dirichlet L-functions. I shall give a brief history of this problem and outline some new work joint with Dave Platt (Bristol).

Ben Ward
La Trobe University
Title: Hausdorff dimension of certain Badly approximable sets
Abstract: In this talk I will discuss joint work with Henna Koivusalo, Jason Levesley, and Xintian Zhang on the set of ψ-badly approximable points. ψ-badly approximable points are those which are ψ-well approximable, but not $c\psi$-well approximable for arbitrary small constant $c > 0$. In 2003 Bugeaud proved in the one dimensional setting that the Hausdorff dimension of ψ-badly approximable points is the same as the Hausdorff dimension of ψ-well approximable points. Our main result provides a partial d-dimensional analogue of Bugeaud's result. In order to do this we construct a Cantor set that simultaneously captures the well approximable and badly approximable nature of ψ-badly approximable points.