EPOS Optimization Progress And Perturbation Modelling

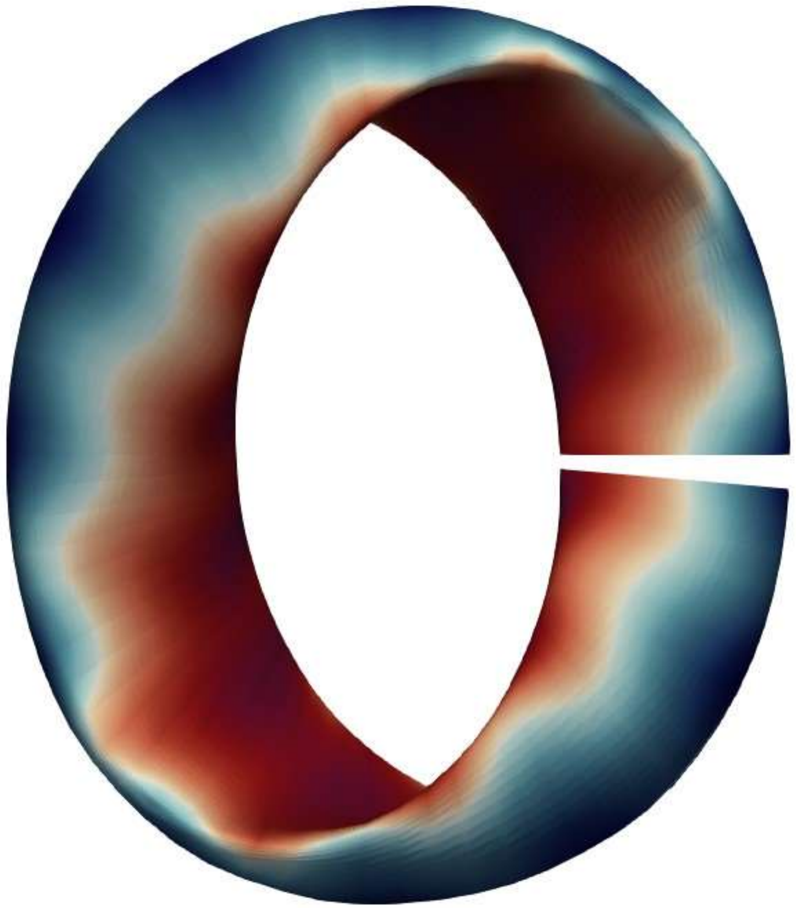
- Pedro Gil

MOTIVATIONS

- **Stellarator with good confinement/quasisymmetry for e- and e+.**
- **Achieve low debye length compared to the minor radius.**
- **Respect HTS strain limits.**
- **Achieve a robust configuration to perturbations of the coils.**
- **Allows injection of positrons through ExB drift.**

WHAT HAS BEEN ACHIEVED SO FAR?

- **CLASSICAL PROCEDURE:**
  - Stage I: Generate the plasma equilbrium

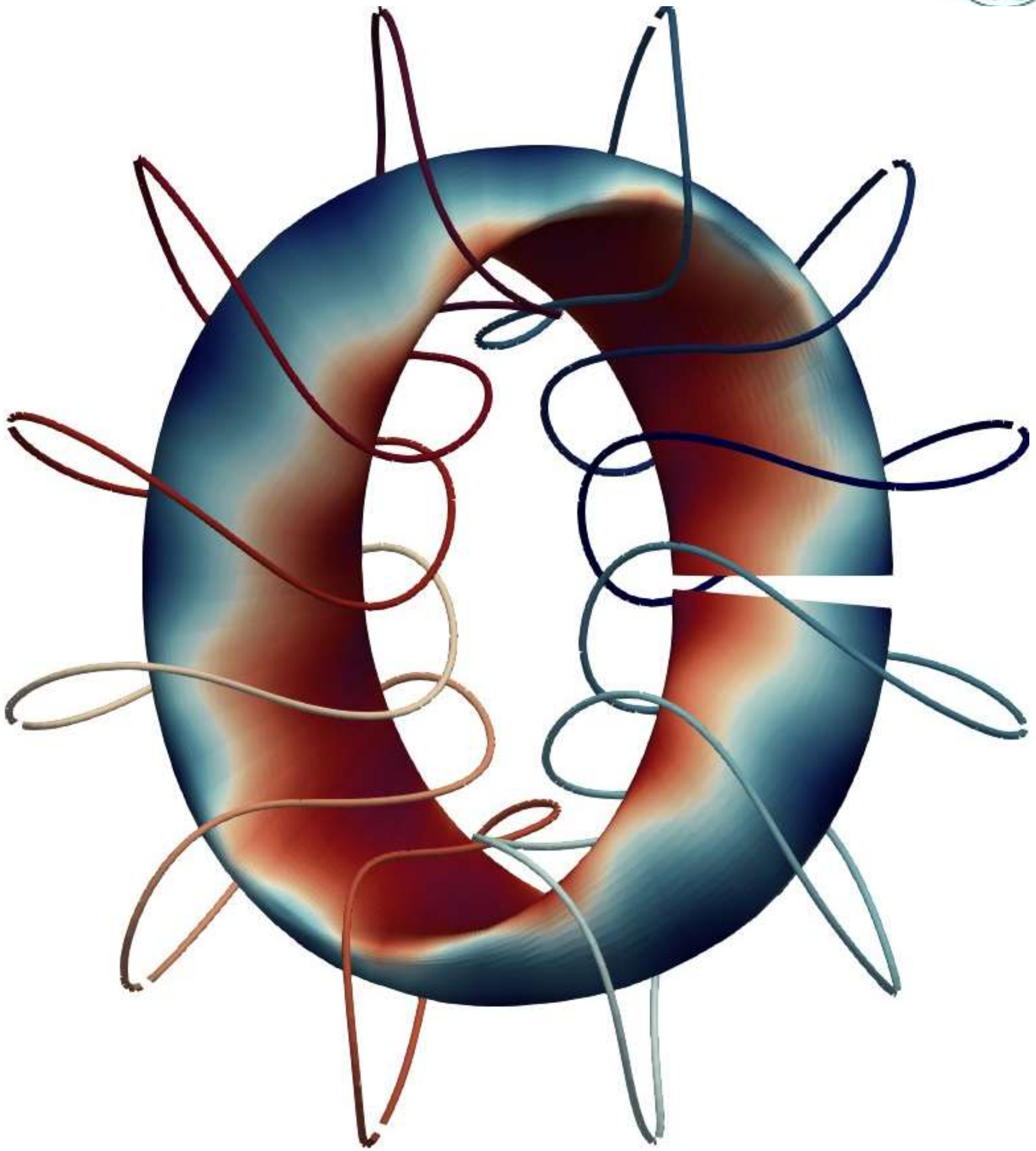


- **METHODS:**
  - Stage I: Least Squares Optimization (Non-Linear)

WHAT HAS BEEN ACHIEVED SO FAR?

- **CLASSICAL PROCEDURE:**
  - Stage I: Generate the plasma equilbrium
  - Stage II: Generate the coils

- **METHODS:**
  - Stage I: Least Squares Optimization (Non-Linear)
  - Stage II: Quasi-Newton Non-Linear Optimization



ONE PROPOSAL TO IMPROVE IT: SINGLE STAGE OPTIMIZATION*

- **Stage I + Stage II → Single Stage = PLASMA and COILS optimization all at once.**
- **COST FUNCTION:**

$$\min_{x_{\text{coils}},\, x_{\text{surface}}} J(x_{\text{coils}}, x_{\text{surface}}) = J_1 + \omega_{\text{coils}} J_2, \qquad (27a)$$

$$\text{subject to } \psi = \psi_0,\ R_{\text{major}} = R_0, \qquad (27b)$$

* Work developped in *Single-Stage Stellarator Optimization: Combining Coils with Fixed Boundary Equilibria*, R Jorge *et al* 2023 *Plasma Phys. Control. Fusion* **65** 074003

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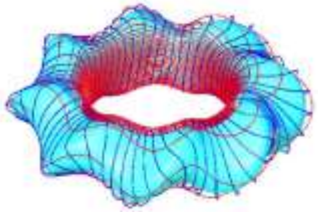

Scientific Visualization of 3-dimensional Optimized Stellarator Configurations
Donald A. Spong
Oak Ridge National Laboratory

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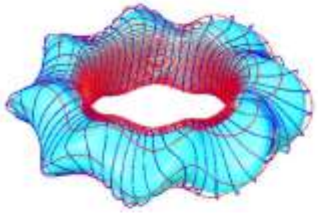

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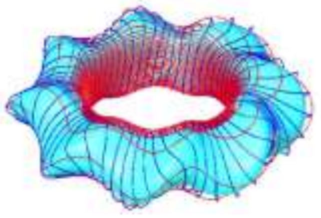

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Long-Poe Ku, Allen H. Boozer; Stellarator coil design and plasma sensitivity. *Physics of Plasmas* 1 December 2010; 17 (12): 122503. https://doi.org/10.1063/1.3527994

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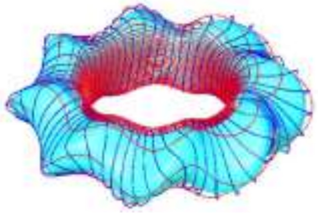

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$$J = \frac{1}{2}\int_S (\mathbf{B}\cdot\mathbf{n} - B_T)^2 ds$$

Ensures consistency!

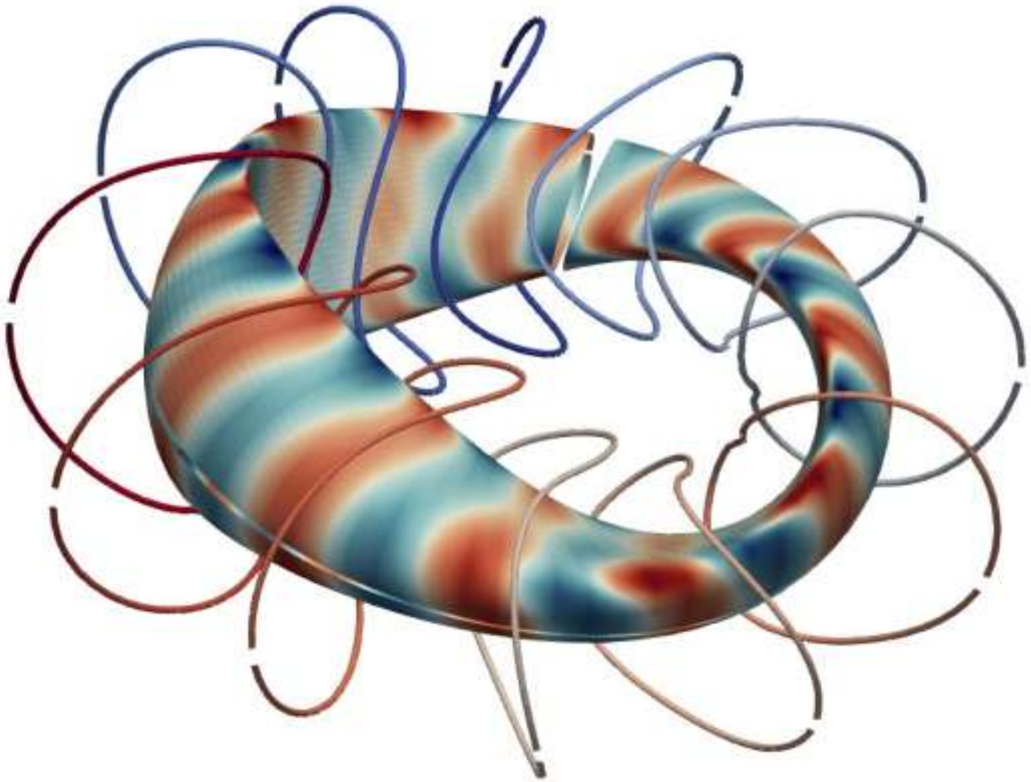
$$x_{\text{coils}} = \left[c_{j,l}^{(i)}, s_{j,l}^{(i)}, I_i\right].$$

Long-Poe Ku, Allen H. Boozer; Stellarator coil design and plasma sensitivity. *Physics of Plasmas* 1 December 2010; 17 (12): 122503. https://doi.org/10.1063/1.3527994

NEW UPDATES ON OPTIMIZATION

OPTIMIZATION REQUIREMENTS:

- Single Stage (Rogério Jorge) ✓
- Stochastic ✓
- Flexible to vary major radius ✓
- Coil dynamic resolution ✓
- Equilibrium dynamic resolution ✓
- Good Quasisymmetry ?
- Finite Build ✓
- HTS Strain (Now merged to SIMSOPT) ✓
- Weave-Lane Coils ✓
- Resilient to pertubations ?



WHY STOCHASTIC OPTIMIZATION ?

For field errors less than 2e-4 of the magnetic field amplitude.	MANUFACTURING TOLERANCE (MM)	TILT ANGLE TOLERANCE (DEG)
W7X	**2**	**0,1**
EPOS	**0,1**	**~ 0,2**

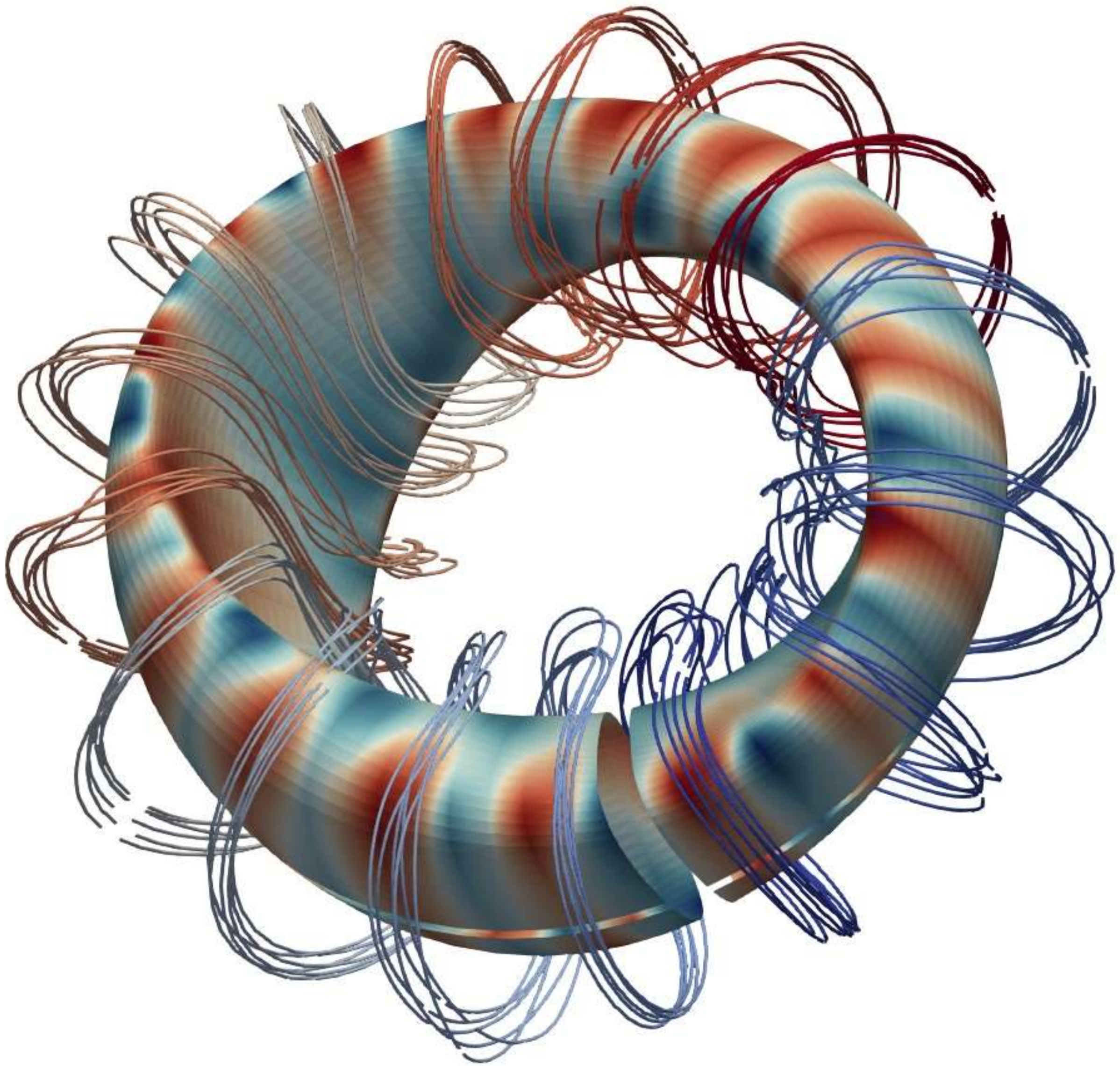
- **Reach "flatter" minima → More robust configurations.**

  → EPOS appears to have strict building tolerances.

- **Avoid sharp minima → avoid getting stuck in local mínima.**

PERTURBATION MODELLING

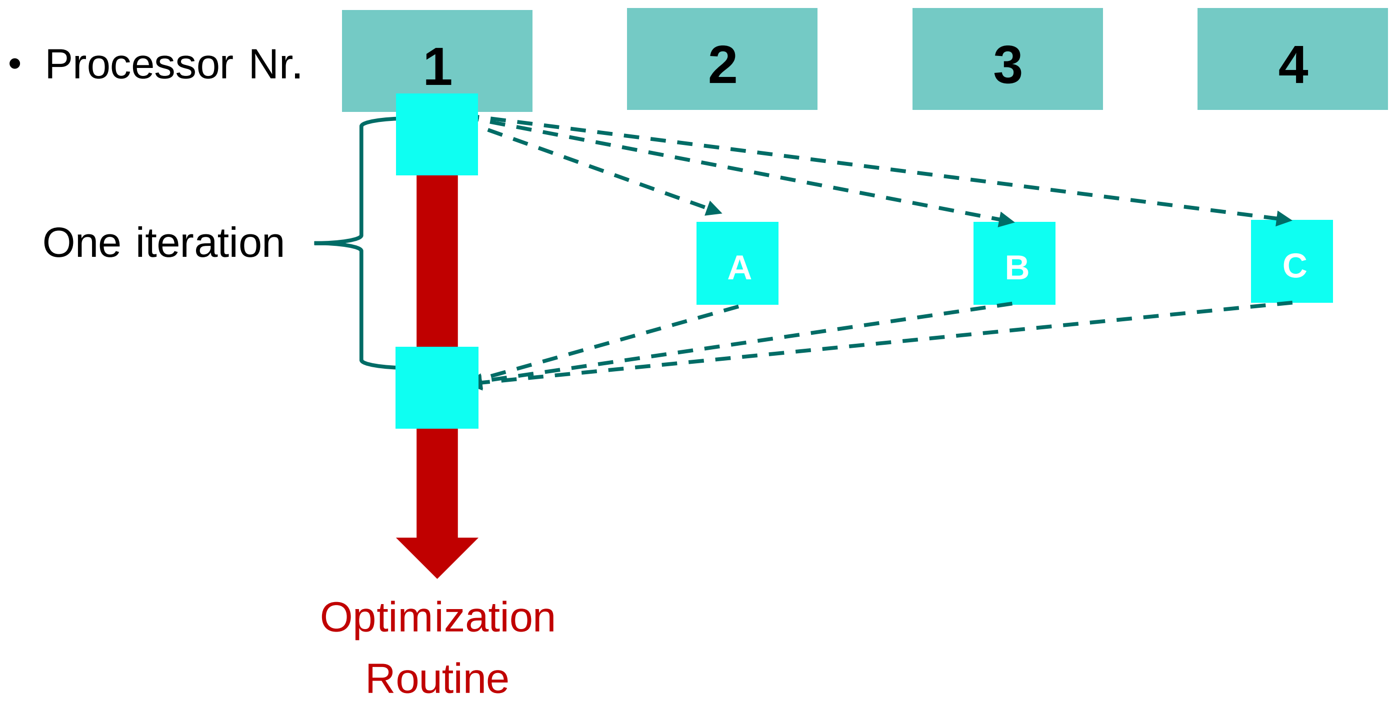
- **So far manufacturing errors have been modeled on SIMSOPT:**

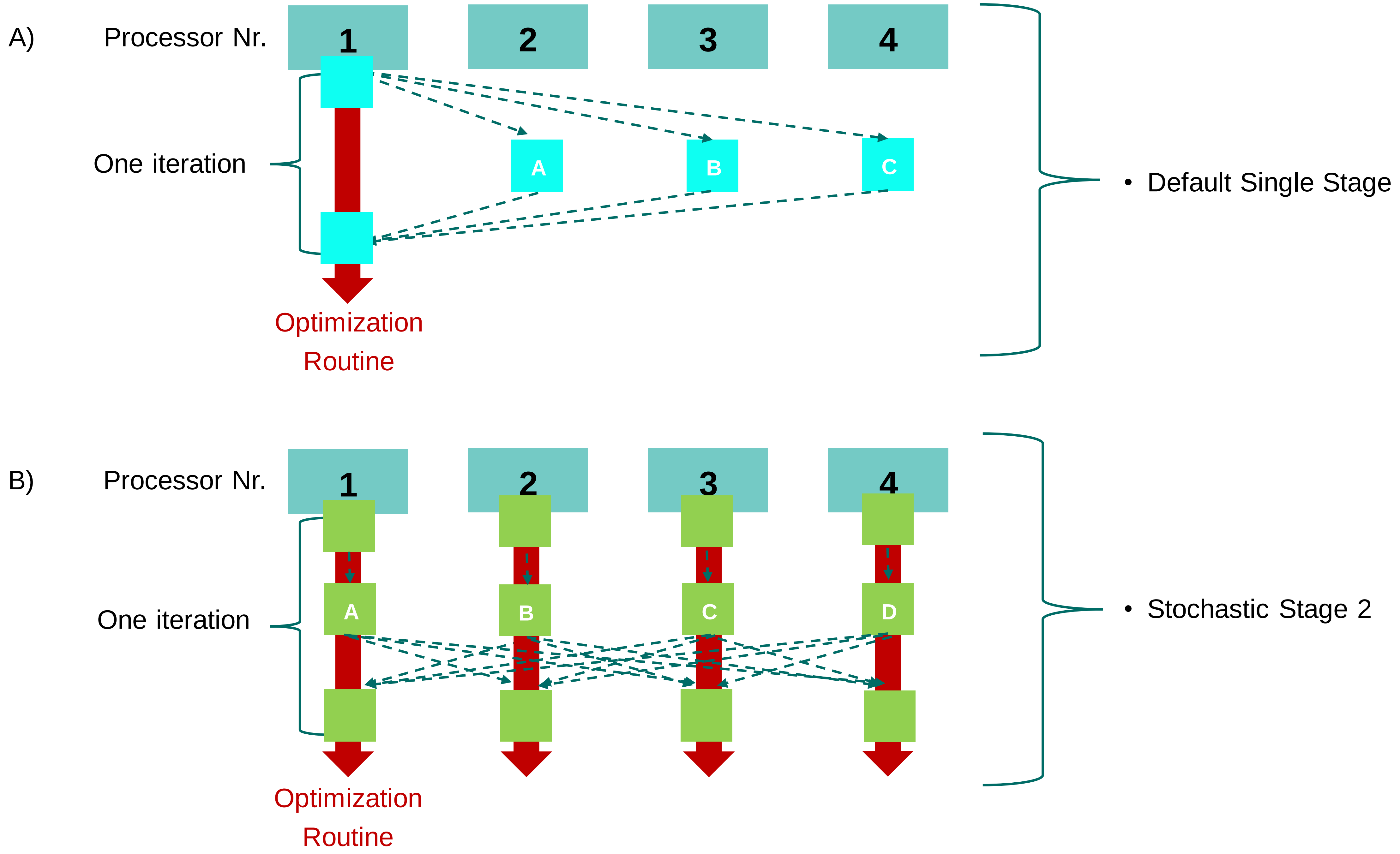


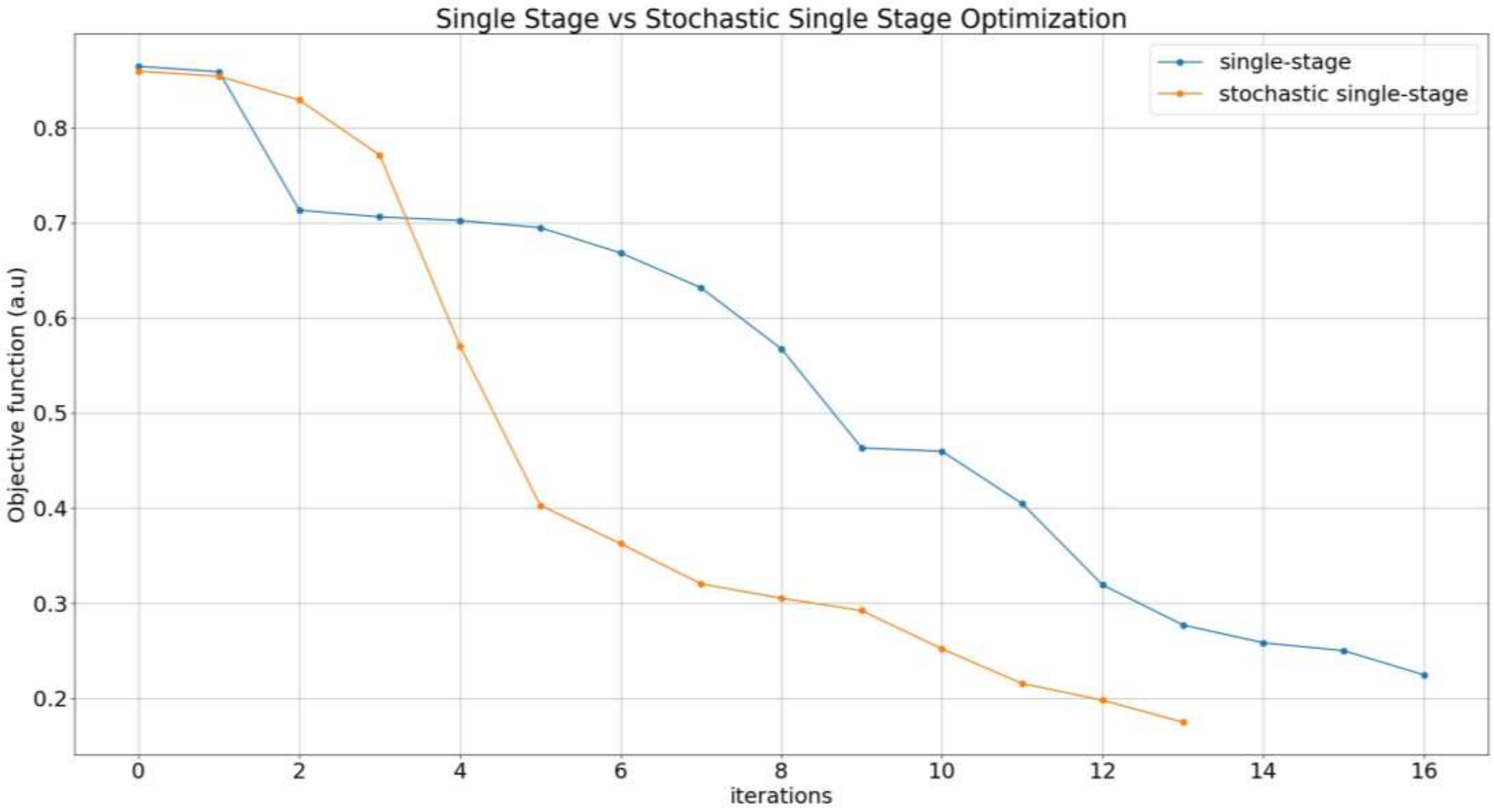
Florian Wechsung *et al* 2022 *Nucl. Fusion* **62** 07

LATEST WORK:

- **MAIN GOAL: GET STOCHASTIC SINGLE STAGE GOING**
  - **First Problem Found:** The machine cannot handle parallel calculations for of the squared flux and perform MPIFiniteDifference estimations of the gradient.




A)
Processor Nr.
1
2
3
4
One iteration
A
B
C
Optimization Routine
Default Single Stage
B)
Processor Nr.
1
2
3
4
One iteration
A
B
C
D
Optimization Routine
Stochastic Stage 2



Single Stage vs Stochastic Single Stage Optimization
single-stage
stochastic single-stage
Objective function (a.u)
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0
2
4
6
8
10
12
14
16
iterations

EXAMPLE OF HOW IT CAN GO WRONG

DYNAMIC COIL AND SURFACE RESOLUTION

- Targeted optimization, reiterated with an increasing number of targeted modes:

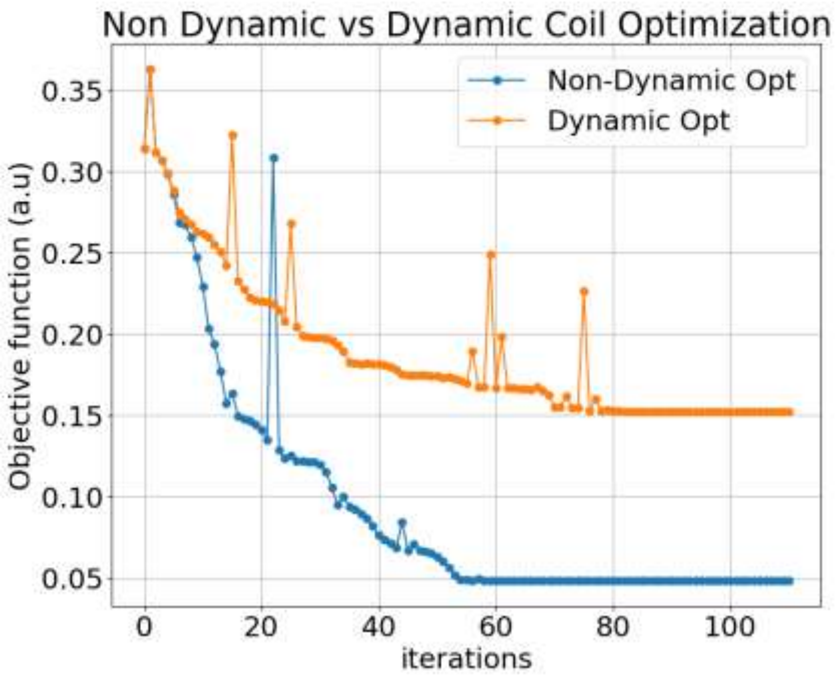
$$R(\theta,\phi) = \sum_{m=0}^{m_{pol}} \sum_{n=-n_{tor}}^{n_{tor}} r_{c,m,n} cos(m\theta - n_{fp} n\phi) + r_{s,m,n} sin(m\theta - n_{fp} n\phi)$$

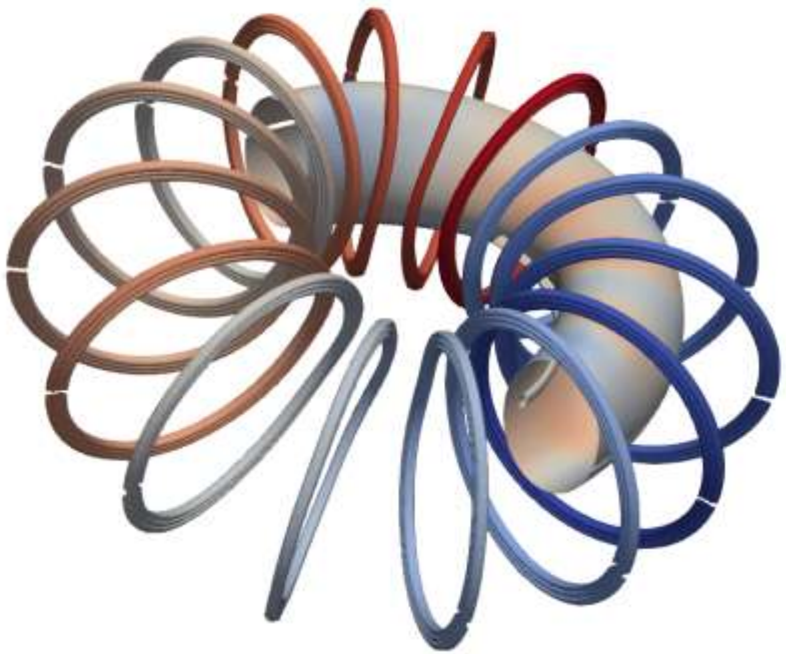
$$x(\theta) = \sum_{m=0}^{order} x_{c,m} cos(m\theta) + \sum_{n=0}^{order} x_{s,n} sin(n\theta)$$

COMPARATIVE RESULTS ON DYNAMIC COIL RESOLUTION

Squared Flux on Non-Dynamic Routine:
SF = 7.366290890536663e-05

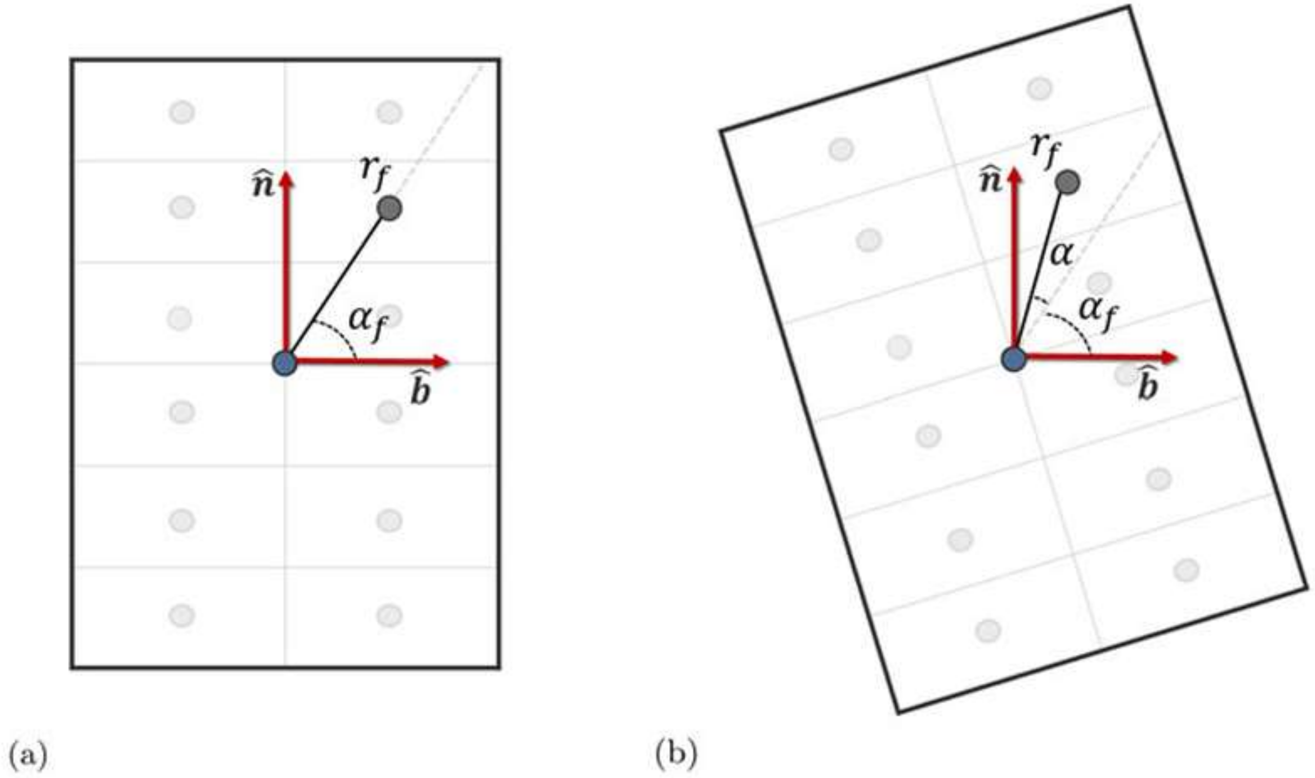
Squared Flux on Dynamic Routine:
SF = 8.759861049042492e-05





FINITE BUILD

- **What is Finite Built?**
  - Turning single curves into real coils with a multi-filament coil model.



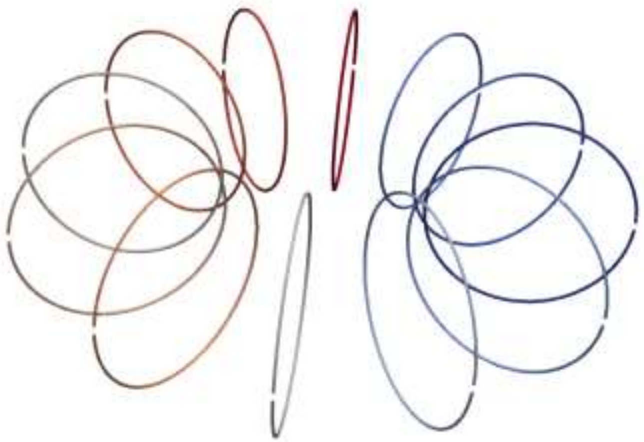
- Here the angle α is optimized to replicate as well as possible the plasma boundaries.

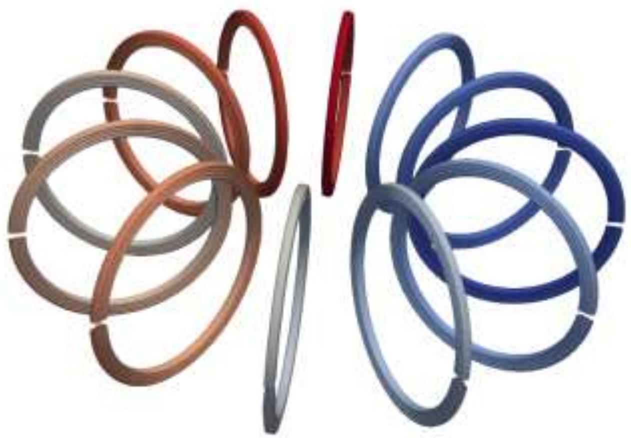
$$\alpha(\phi) = \alpha_{c,0} + \sum_{n=1}^{N_\alpha} \left[\alpha_{c,n}\cos(n\phi) + \alpha_{s,n}\sin(n\phi)\right]$$

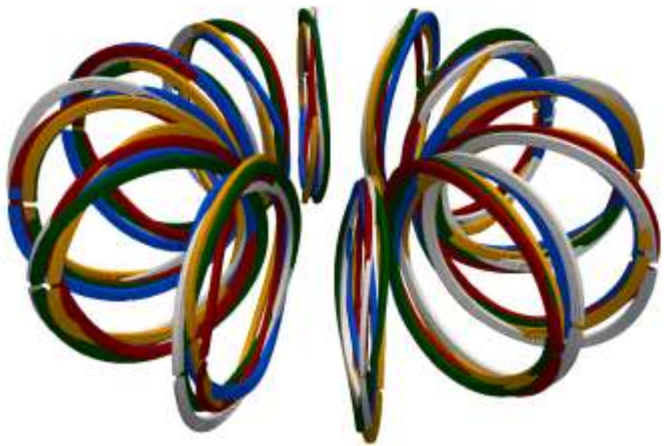
- From Singh, L., Kruger, T., Bader, A., Zhu, C., Hudson, S., & Anderson, D. (2020). Optimization of finite-build stellarator coils. *Journal of Plasma Physics, 86*(4), 905860404. doi:10.1017/S0022377820000756

FINITE BUILD AND COHERENCE WITH PERTURBED COILS

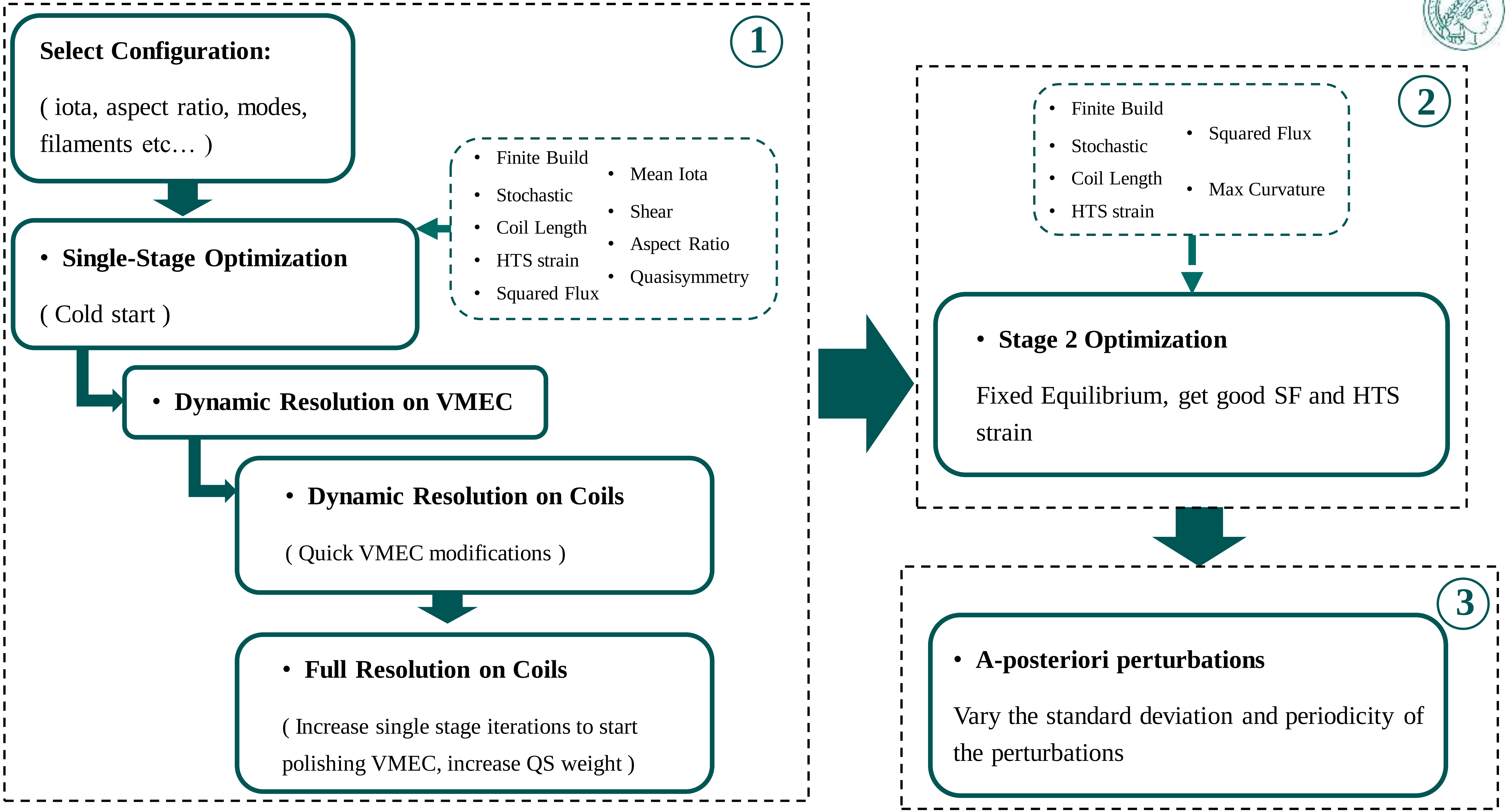
- Next step: Stochastic Finite Build.



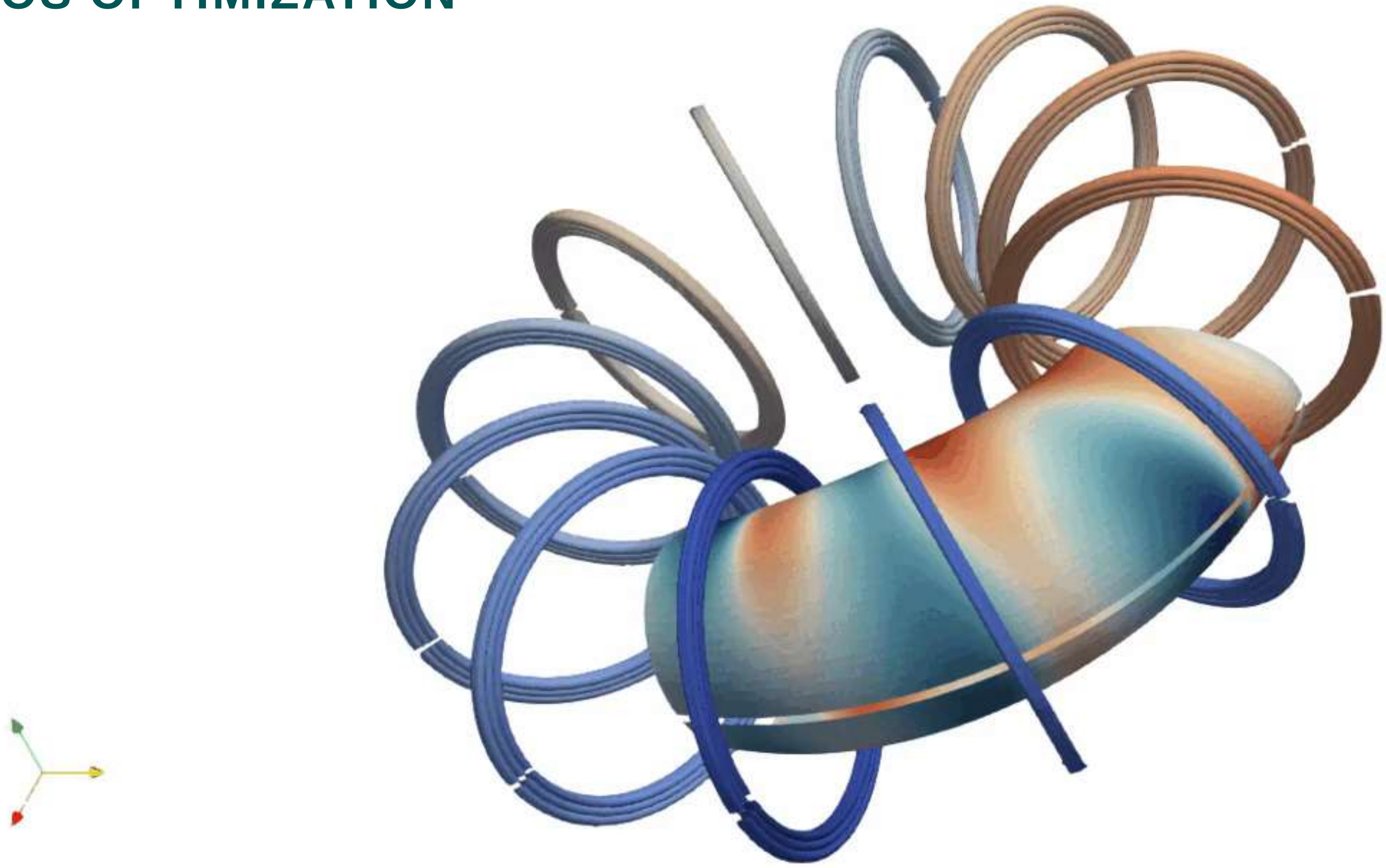




OPTIMISATION ROUTINE

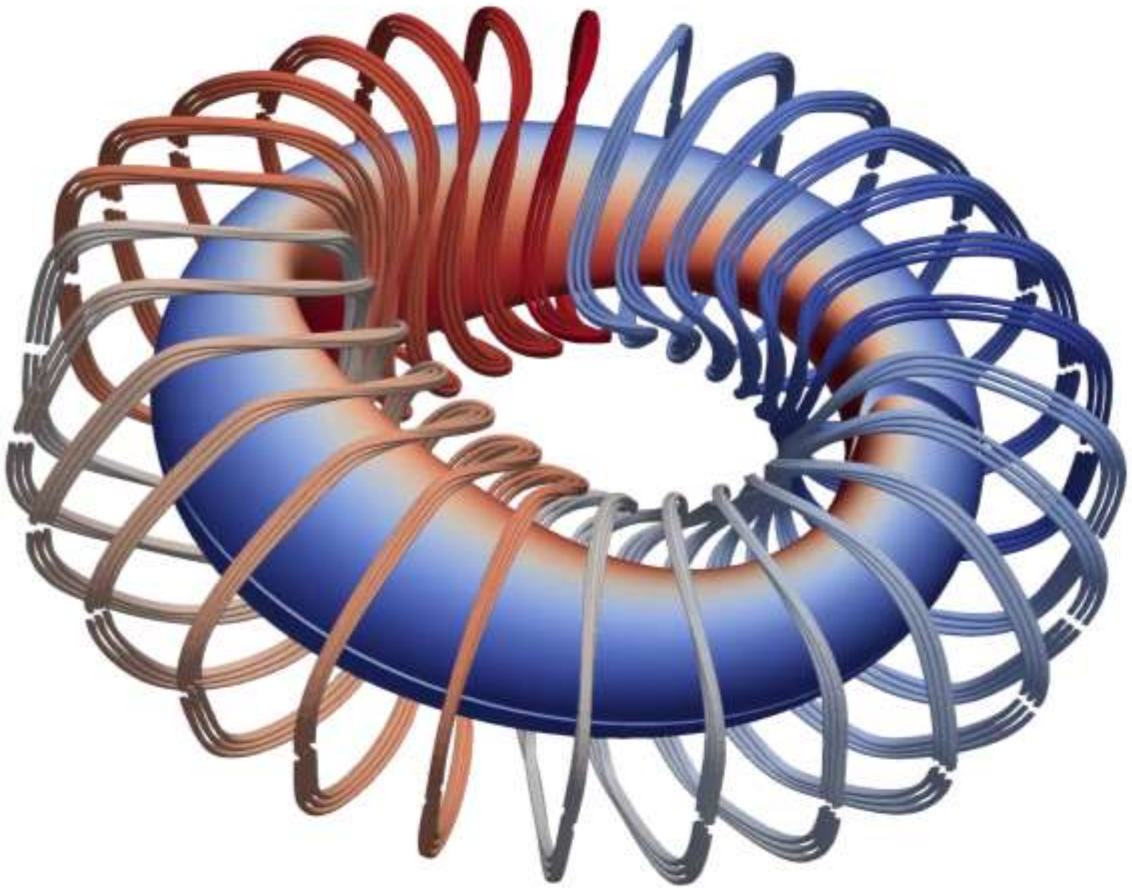


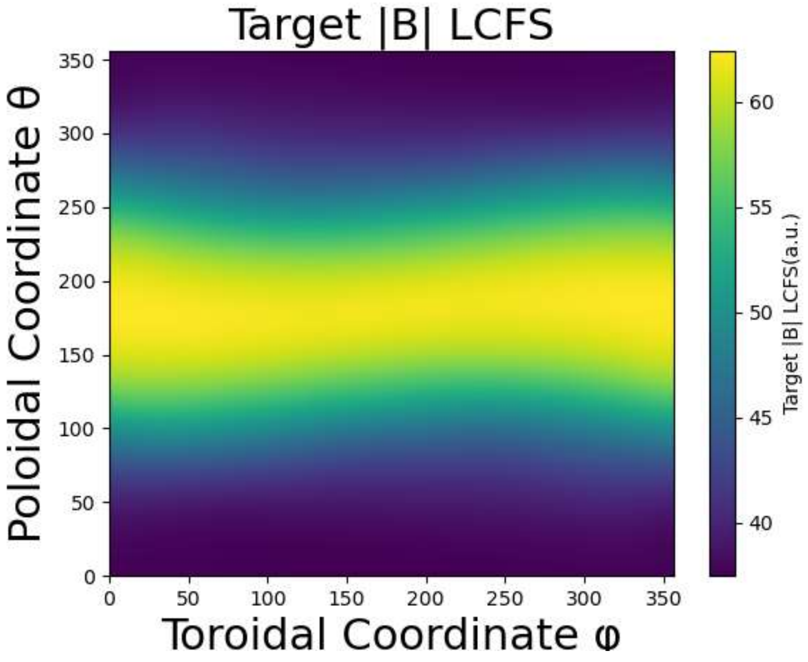
EPOS OPTIMIZATION

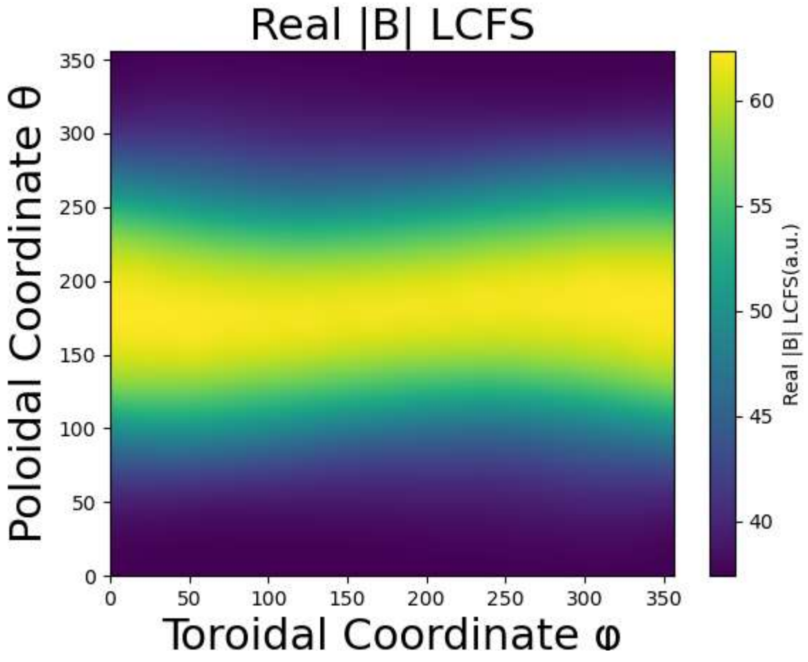


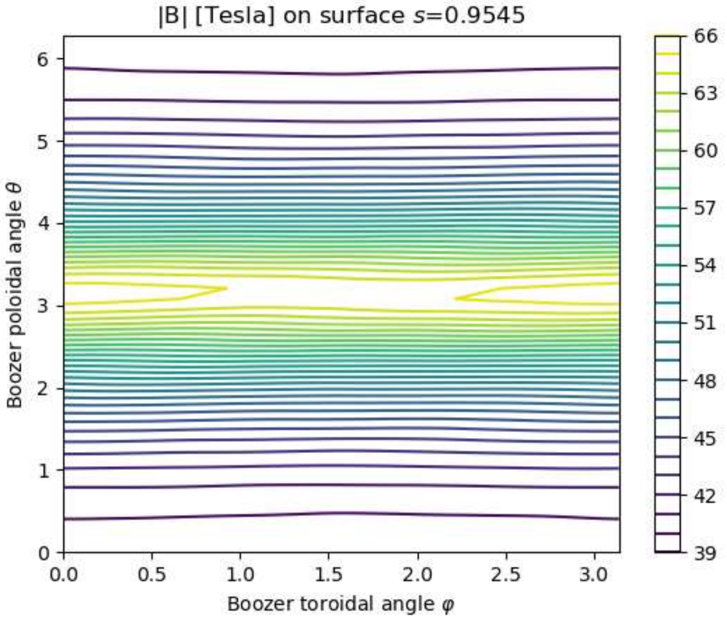
HALF-FIELD PERIOD SYMMETRIC STELLARATOR

- 28 Coils (independent currents per half field period)
- Max Curvature Strain (Hard Bending):
  - 1,3e-3 < 2e-3
- Squared Flux: 1,01e-6
- Max Torsional Strain
  - 9,4e-4 < 2e-3

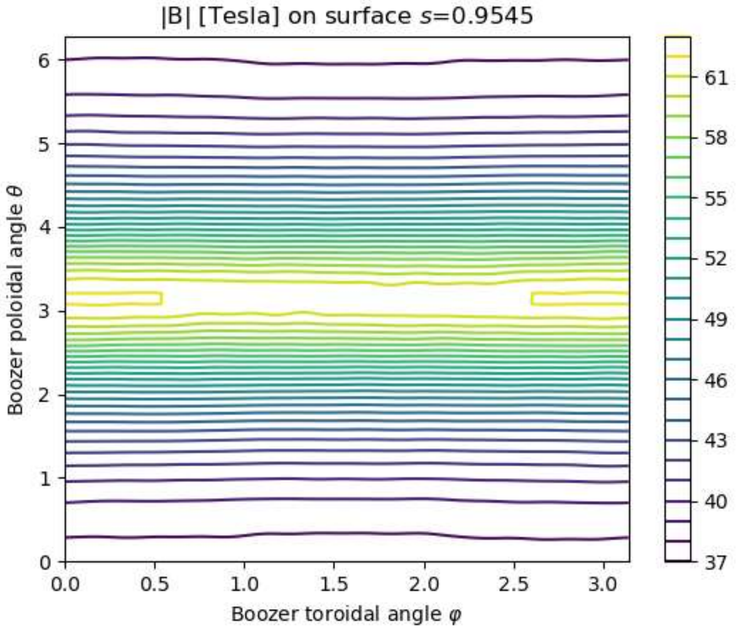






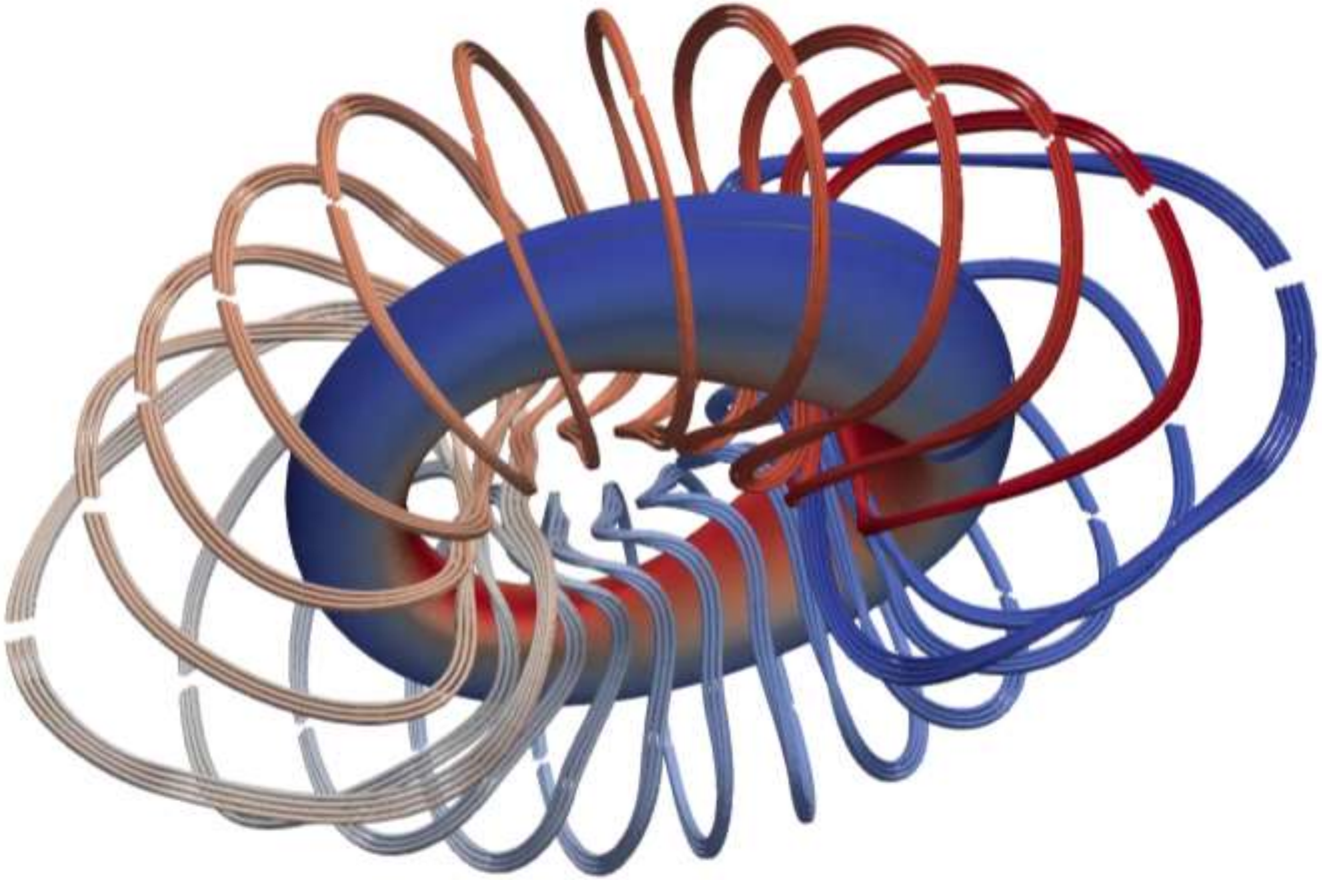


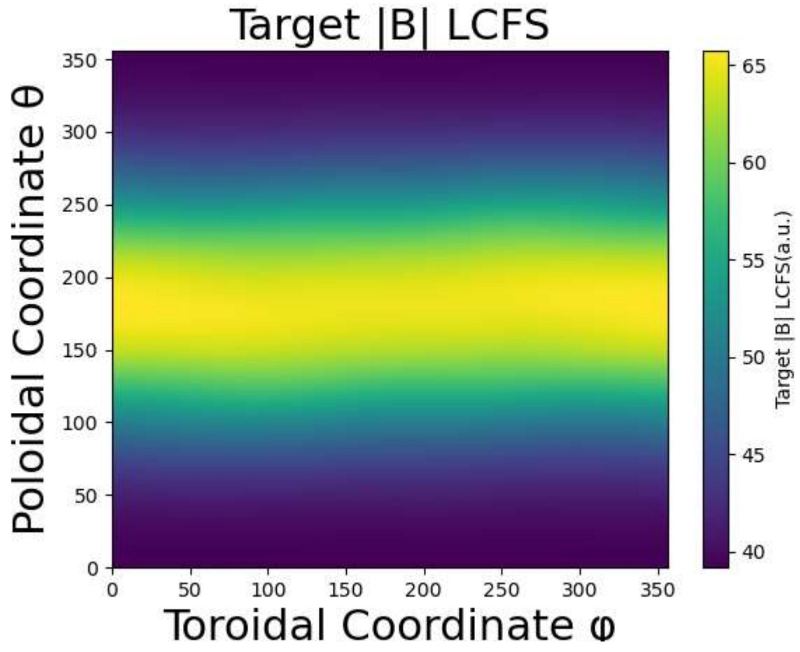
- Target QS: **8,2e-5**
- Reproduced QS: 1,1e-4

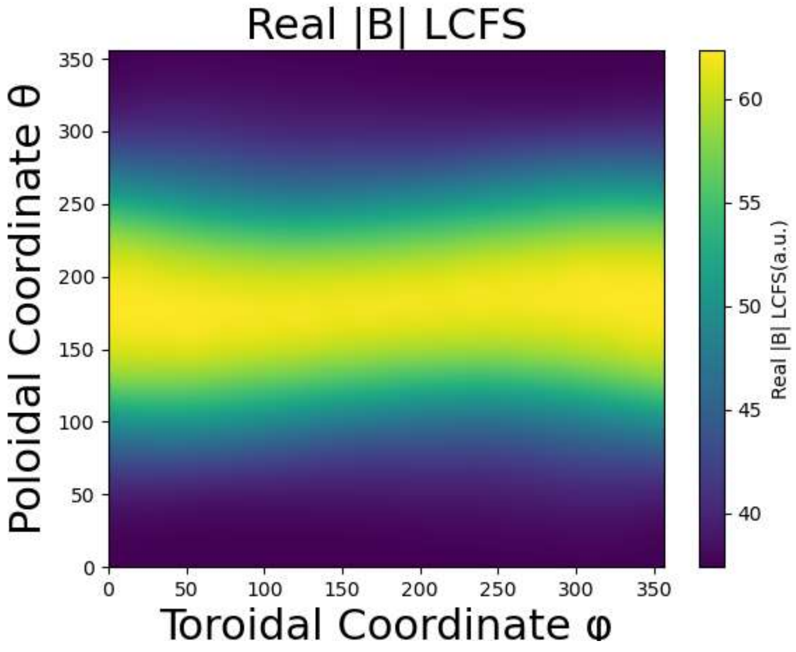


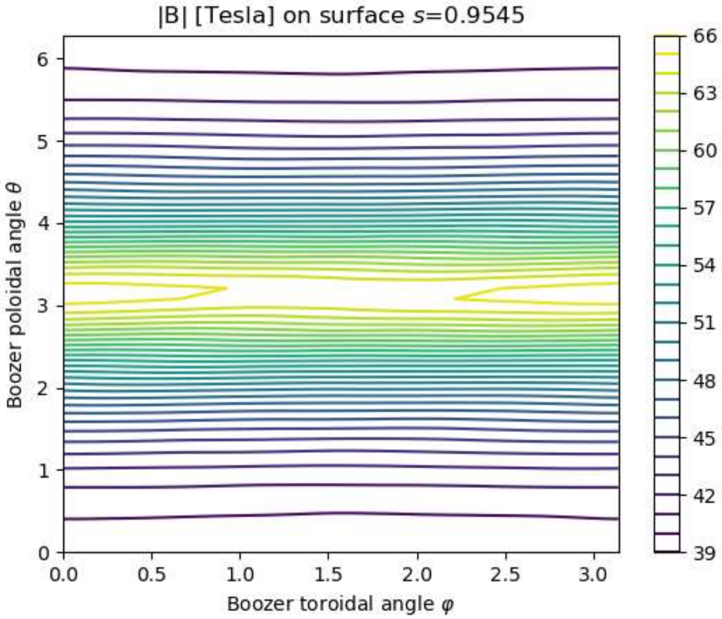
WEAVE-LANE COIL STELLARATOR

- 22 Coils (independent currents)
- Max Curvature Strain (Hard Bending):
  - 1,1e-3 < 2e-3
- Squared Flux: 1,29e-6
- Weave-Lane Gap ~ 7cm
- Max Torsional Strain:
  - 7,8e-4 < 2e-3

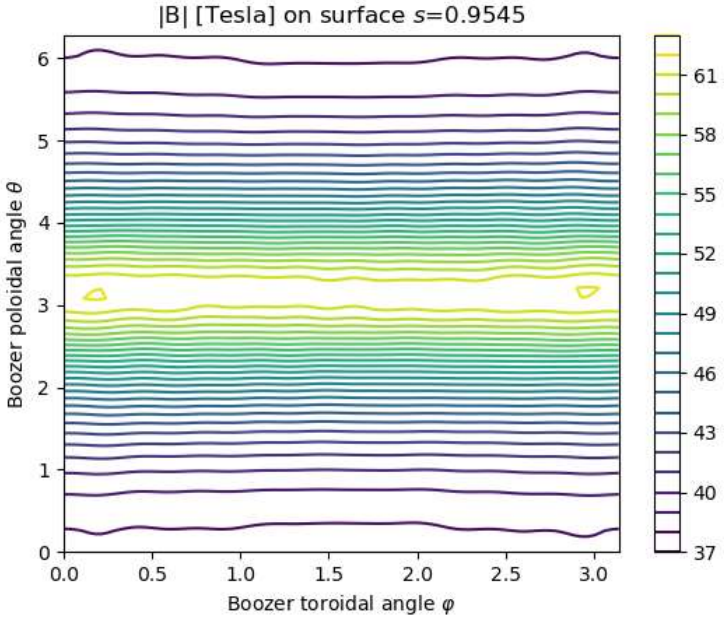






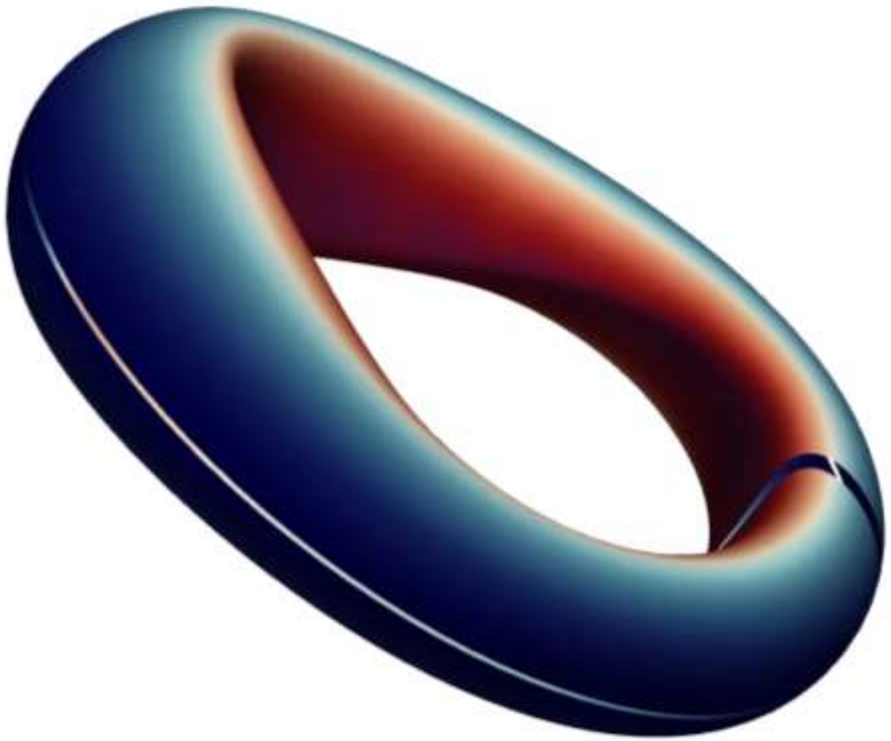


- Target QS: **8,2e-5**
- Reproduced QS: 4,6e-4



EQUILIBRIUM DATA:

- Iota on axis: **0,108**
- Iota on edge: **0,1039**
- Mean Iota: **0,111**
- Equilibrium Volume: **10,21 L**
- Aspect Ratio: **3,7**
- QS on LCFS / Cumulated : **1,47e-5 / 8,21e-5**
- Minor Radius a: **4,07 cm**
- Major Radius (Longuest): **19,4 cm**
- Positrons at 1eV to achieve a/lambda=10 : **1,63e10**

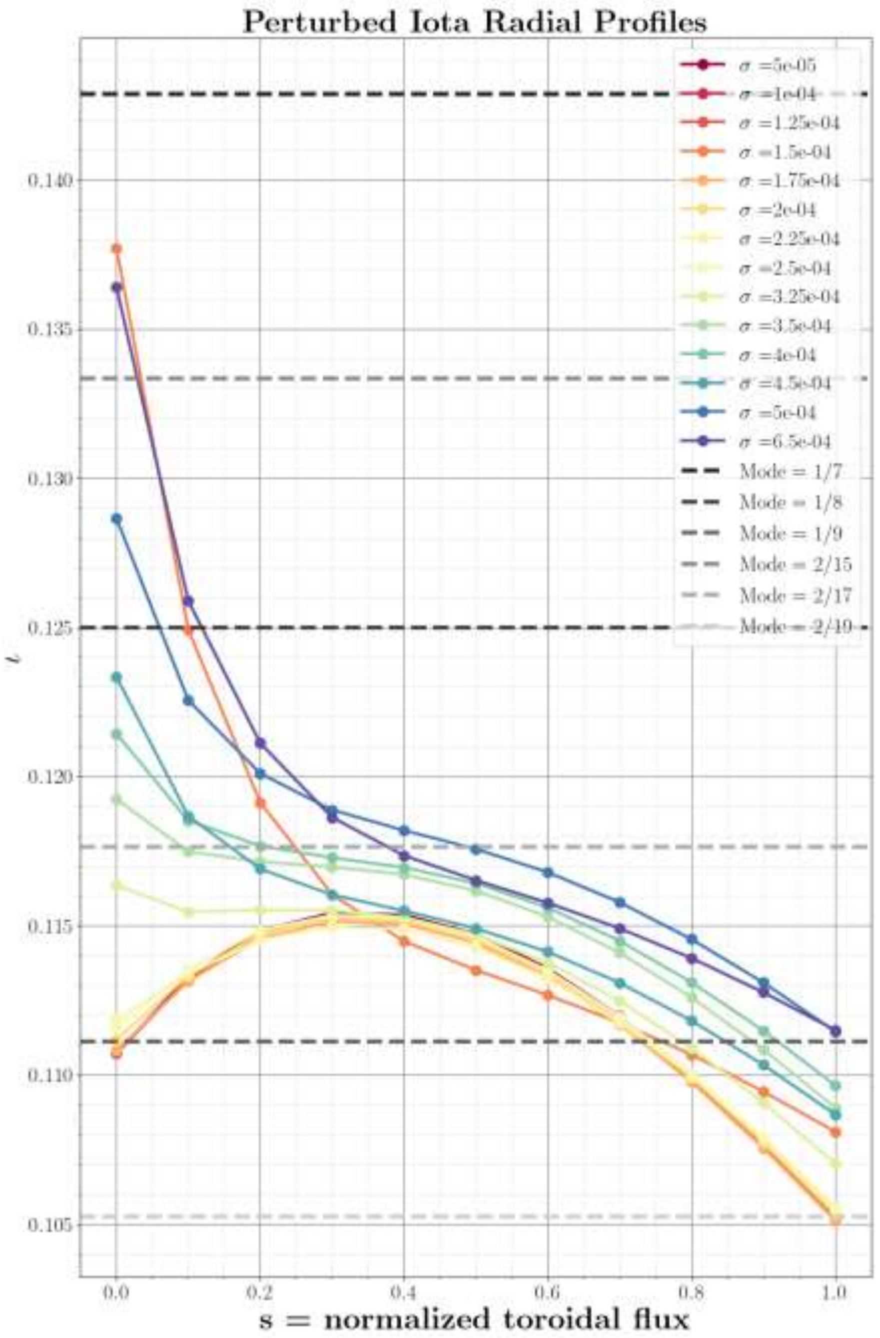


A POSTERIORI PERTURBATIONS

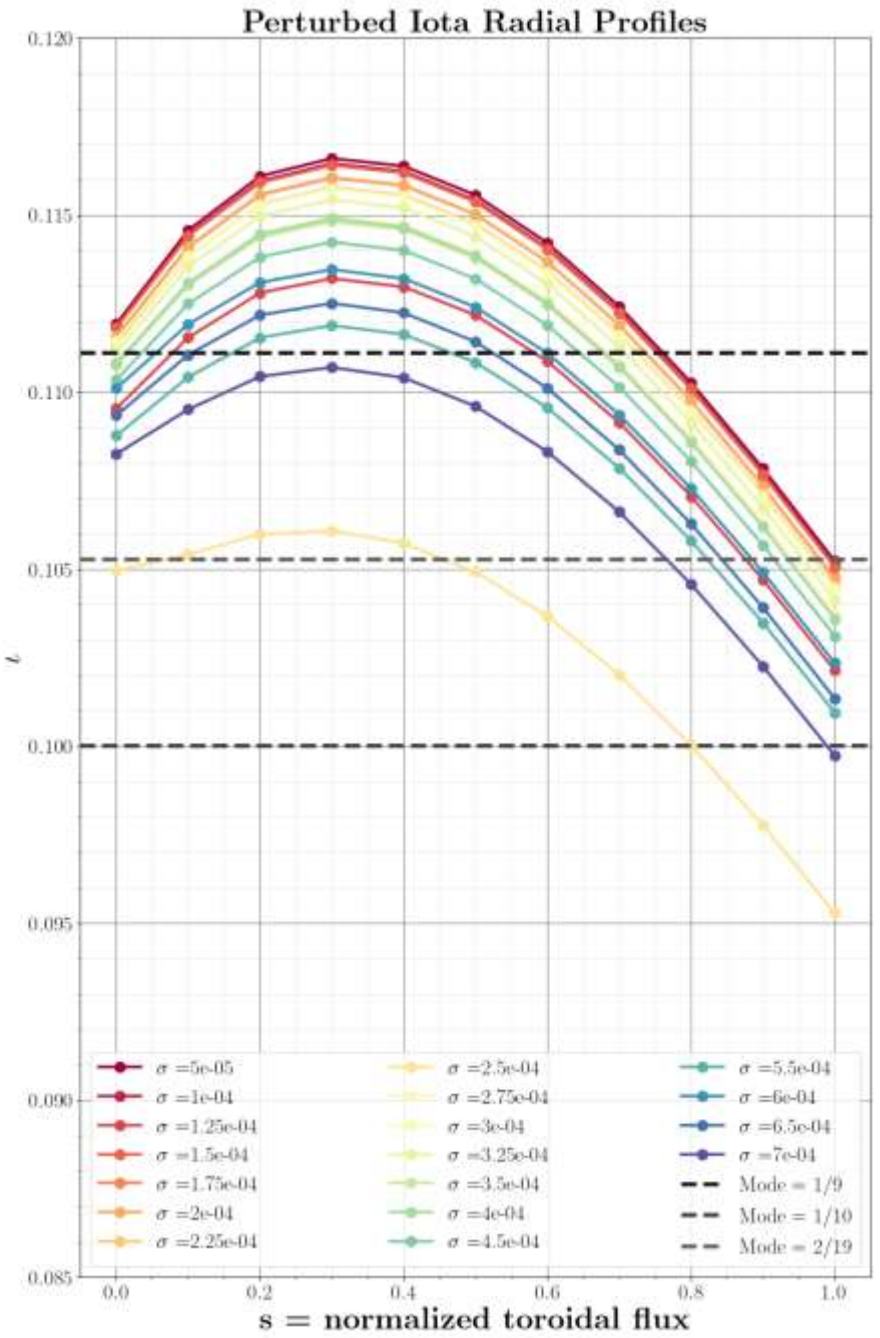
MAIN DATA COMPARISON, INITIAL A-POSTERIORI COIL PERTURBATION

- **8 SAMPLES, Gaussian process along the coil.**
- **Perturbations STD ranging from 5e-5 m to 7e-4 m. (Weave-lane coils undergo double the perturbation amplitude)**
- **Characteristic length kept constant at 0,2 m.**
- **Dimensions:**
  - Standard stellarator: minor radius coils: ~ **9 cm** ↔ perimeter of around 56 cm
  - Weave-lane stellarator: minor radius normal coils : **~ 12 cm** ↔ perimeter of around 75 cm
    minor radius WL : **~ 19 cm** ↔ perimeter of around 119 cm

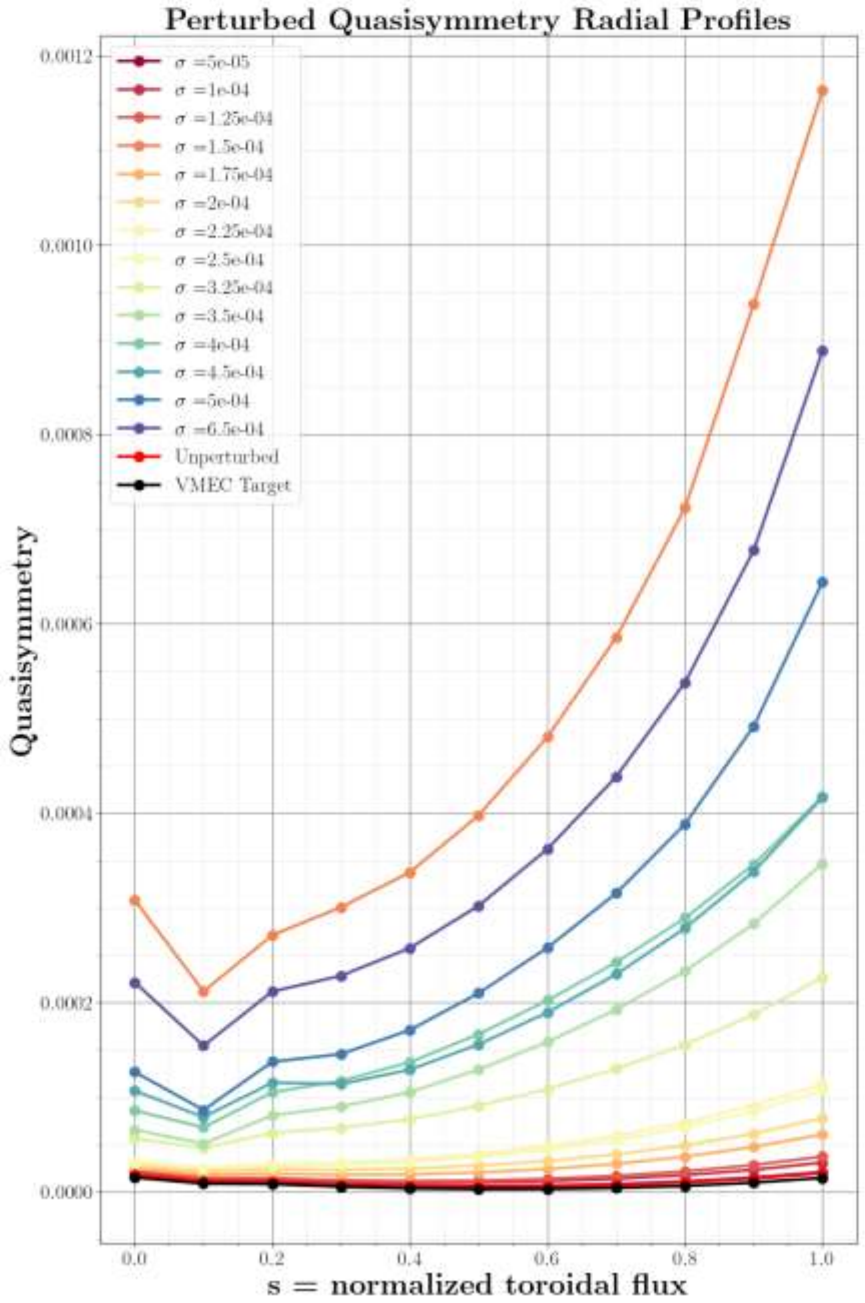
- **Normal**



- **Weave-Lane**



- **Normal**



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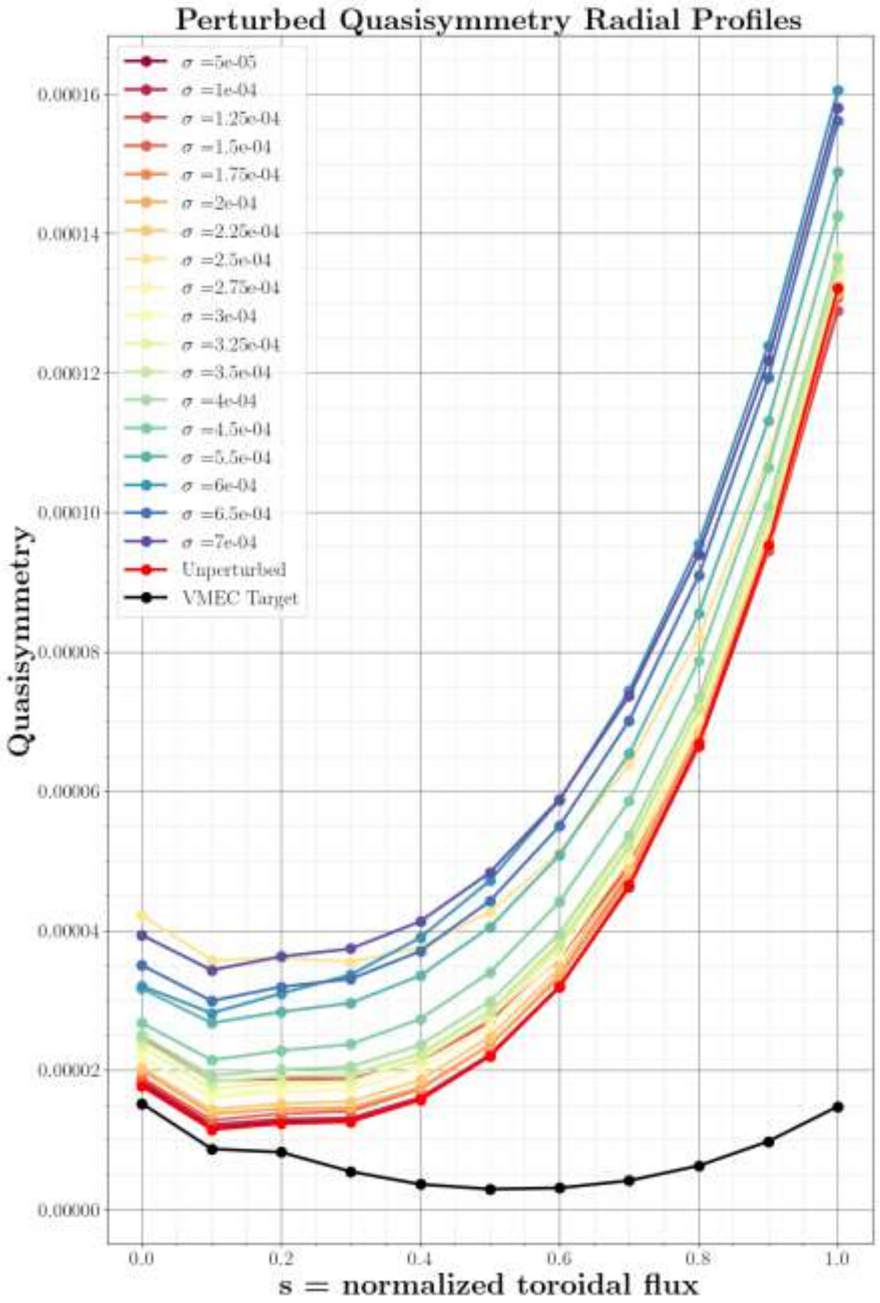
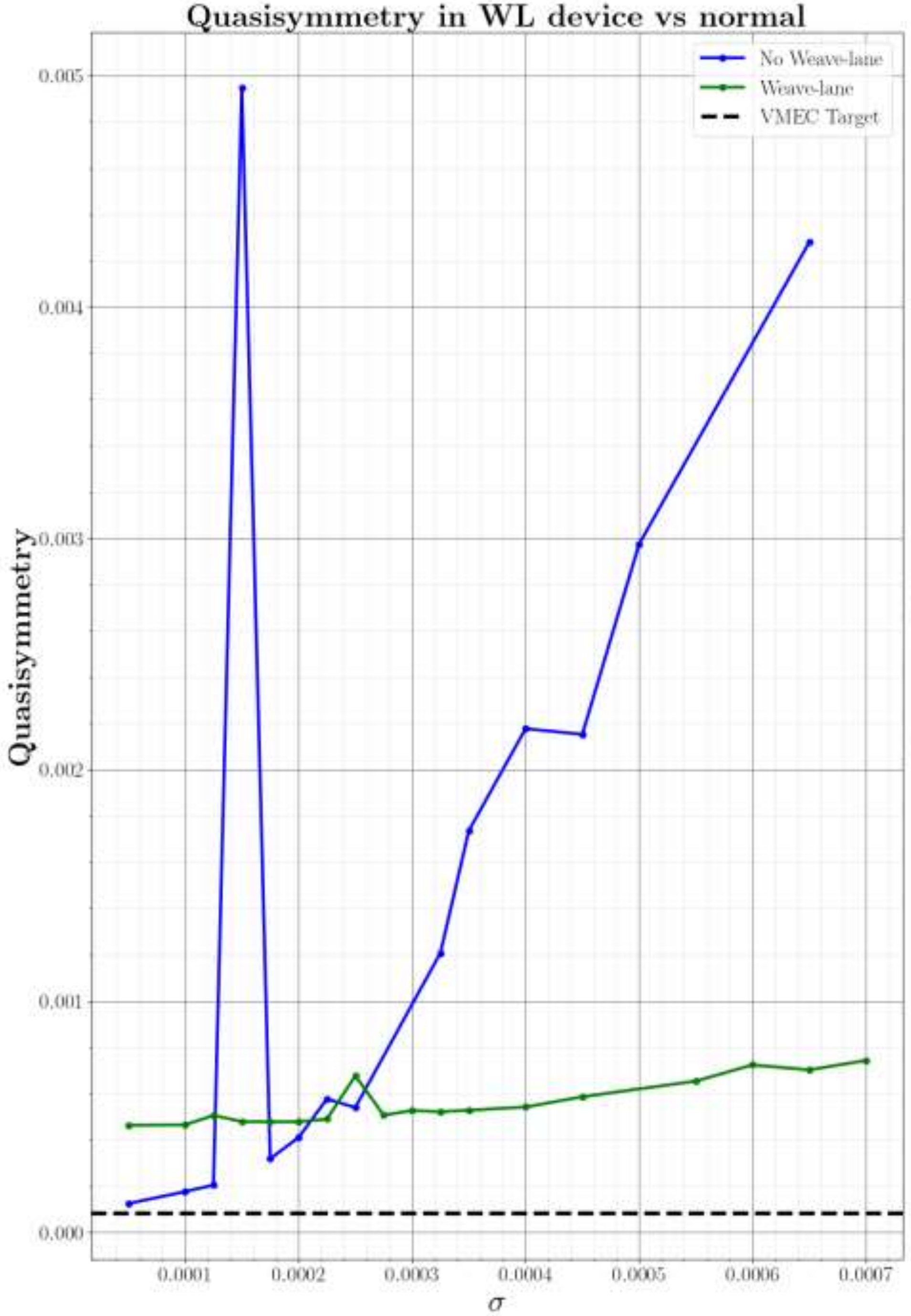
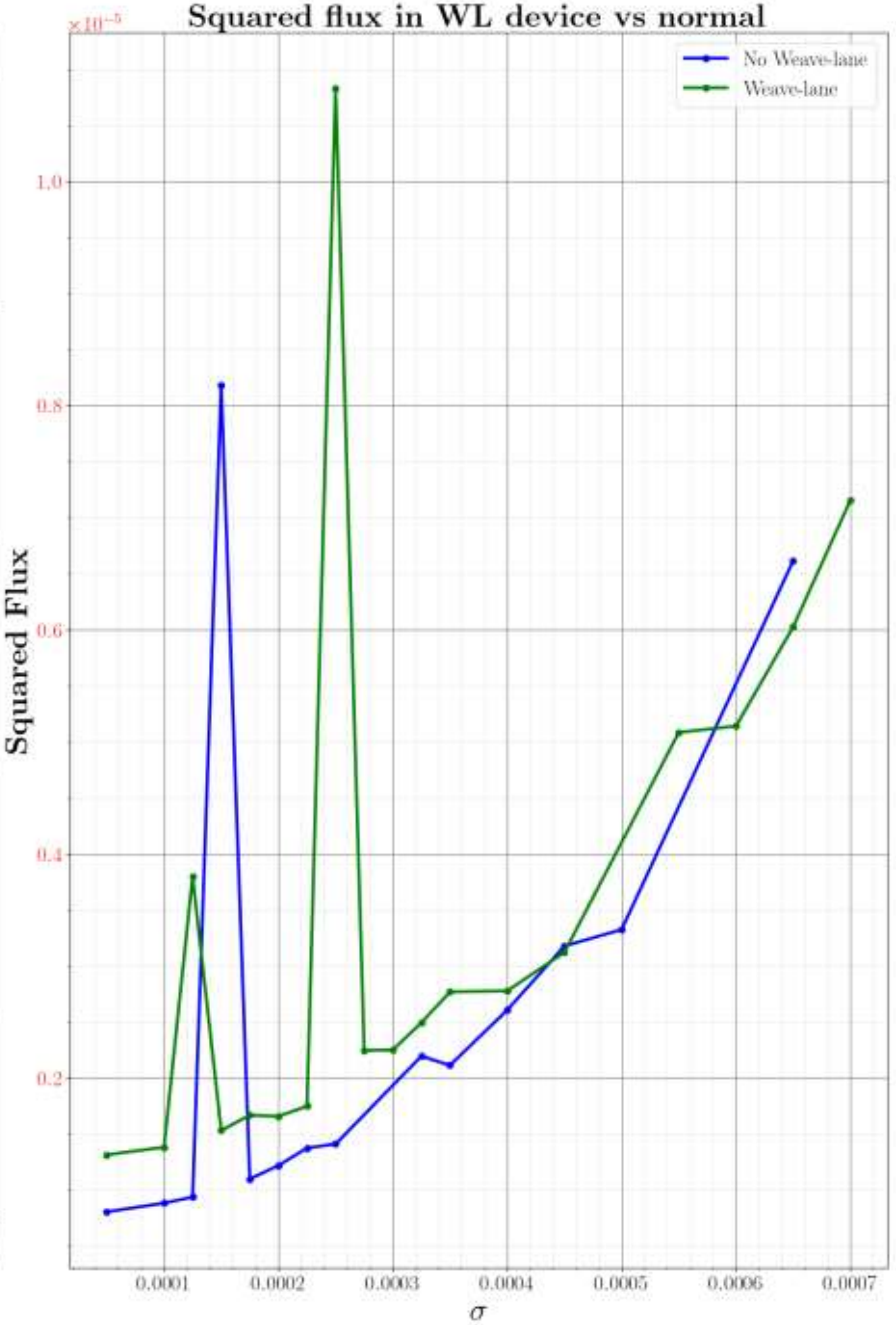



Quasisymmetry in WL device vs normal
No Weave-lane
Weave-lane
VMEC Target
Quasisymmetry
0.005
0.004
0.003
0.002
0.001
0.000
0.0001
0.0002
0.0003
0.0004
0.0005
0.0006
0.0007
σ


Squared flux in WL device vs normal
×10⁻⁵
No Weave-lane
Weave-lane
Squared Flux
1.0
0.8
0.6
0.4
0.2
0.0001
0.0002
0.0003
0.0004
0.0005
0.0006
0.0007
σ

PERTURBATION MODELLING

- **Motivation: For coils that are 1m in minor radius, a tilt angle +-0,1 degrees (W7X limit) <=> +-1mm in the sampling, PDF is modified.**

"The sources of these [magnetic field] errors are differences between the designed and fabricated coil shapes at the **manufacturing** stage or misalignments of the coils at the **assembly** stage."

- T. Andreeva et al. (2004) *Analysis of the Magnetic Field Perturbations during the Assembly of Wendelstein 7-X*, Fusion Science and Technology

- **Stochastic errors impact the most stellarator symmetry as opposed to systematic fabrication errors.**
- **Vertical axis rotations had a bigger impact than toroidal axis rotations.**
- **Distributions to be assymmetrical.**

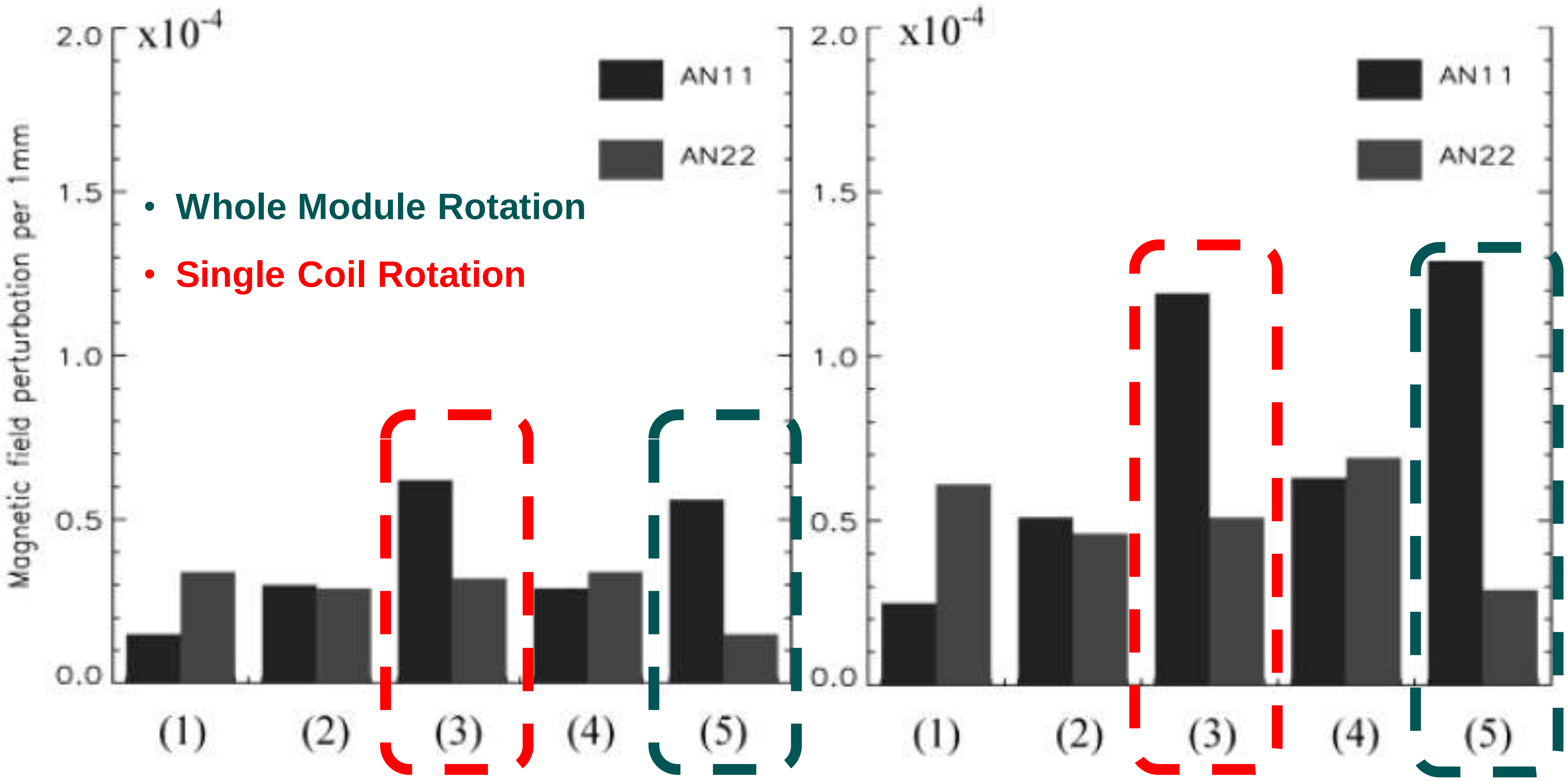
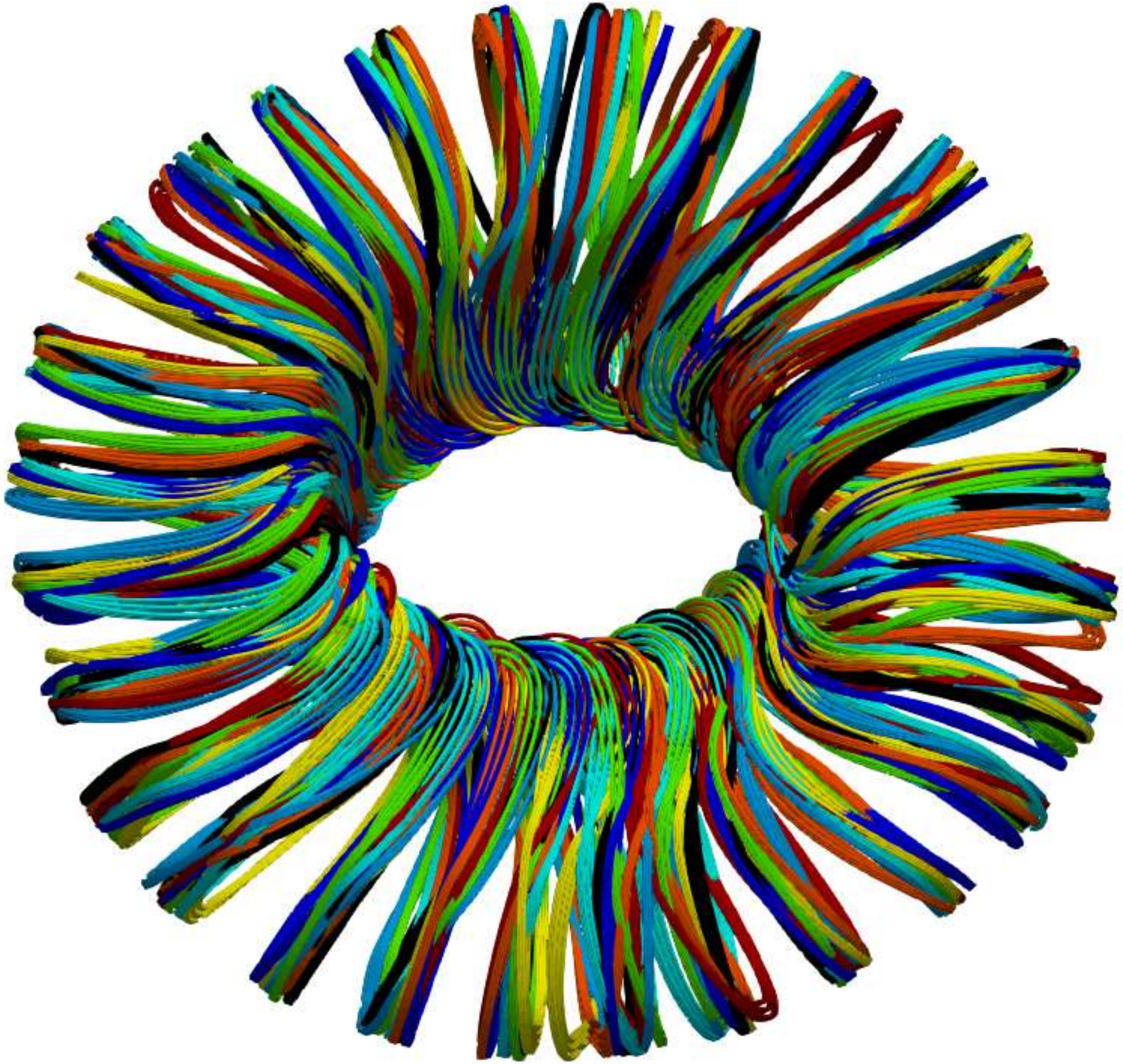
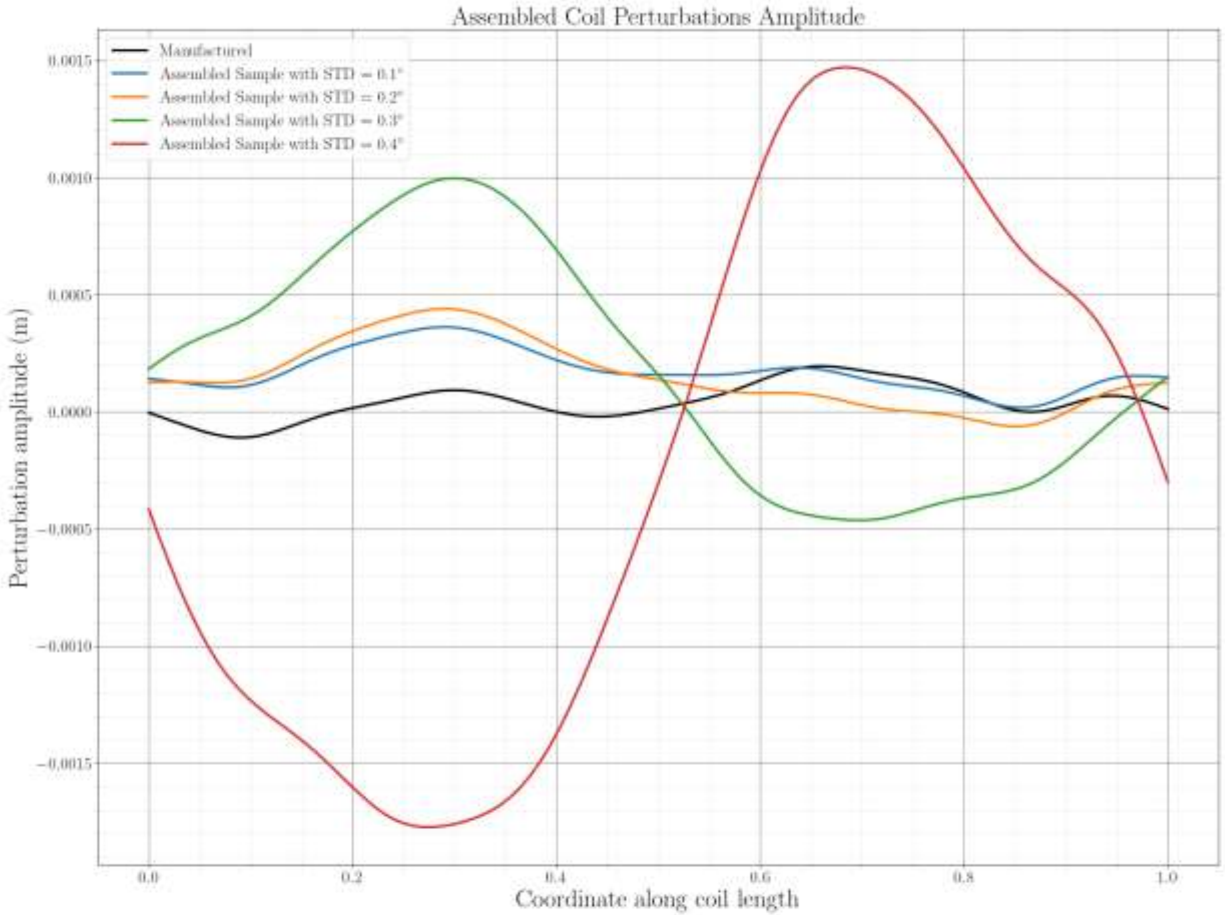


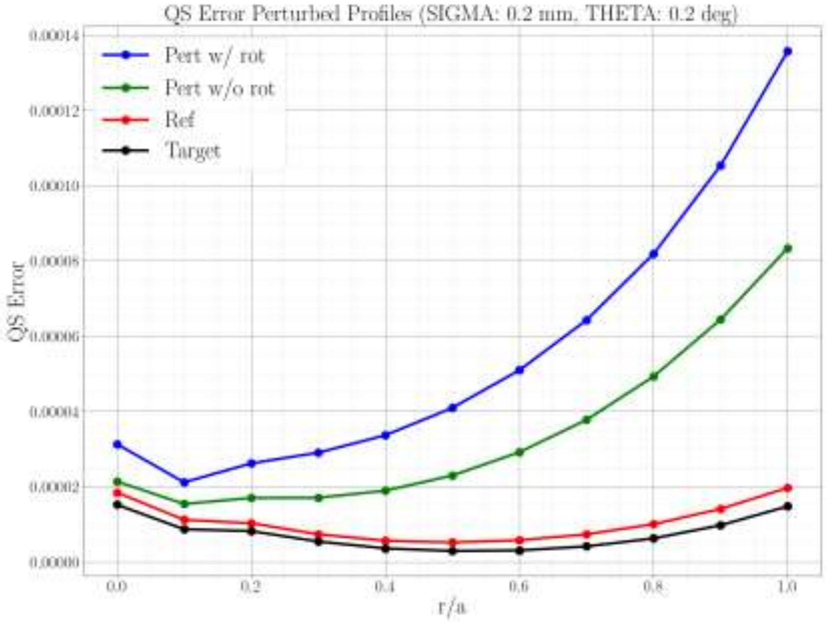
Fig. 9. Relative magnetic field perturbations for an average deviation of 1 mm: (left) average over 10 runs, (right) maximum of 10 runs; in each frame from left to right: (1) shifts of individual coils, (2) manufacturing errors, (3) rotations of individual coils, (4) shifts of whole modules, and (5) rotations of whole modules.

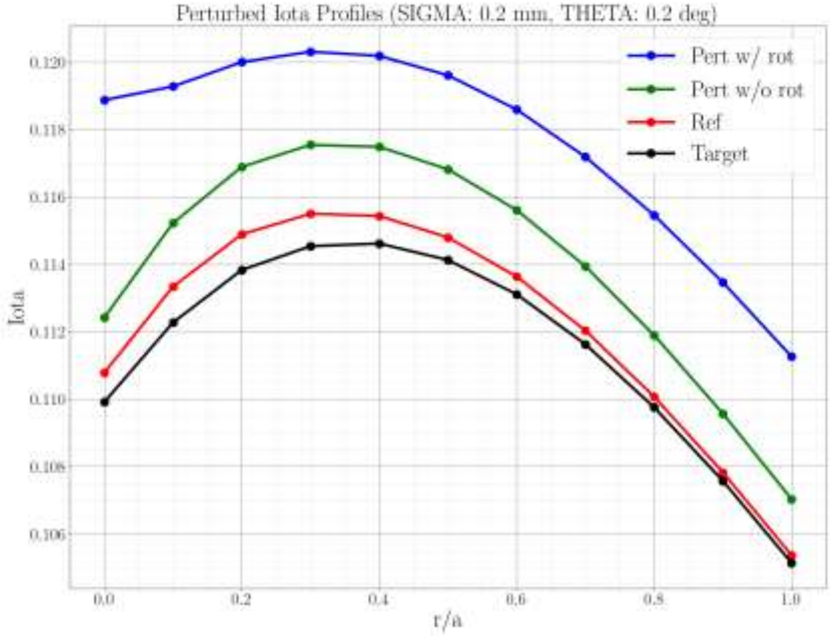
- T. Andreeva et al. (2004) *Analysis of the Magnetic Field Perturbations during the As* *of Wendelstein 7-X*, Fusion Science and Tech

ROTATION DISTRIBUTION: INDEPENDENT MULTIVARIATE GAUSSIAN OF ANGLES.




Assembled Coil Perturbations Amplitude
Manufactured
Assembled Sample with STD = 0.1°
Assembled Sample with STD = 0.2°
Assembled Sample with STD = 0.3°
Assembled Sample with STD = 0.4°
Perturbation amplitude (m)
Coordinate along coil length


QS Error Perturbed Profiles (SIGMA: 0.2 mm, THETA: 0.2 deg)
Pert w/ rot
Pert w/o rot
Ref
Target
QS Error
r/a


Perturbed Iota Profiles (SIGMA: 0.2 mm, THETA: 0.2 deg)
Pert w/ rot
Pert w/o rot
Ref
Target
Iota
r/a

CONCLUSION AND FUTURE WORK

- PROMISING OPTIMIZATION ROUTINE.
- GOOD FLUX SURFACES HAVE BEEN FOUND.
- GOOD QUASISYMMETRY IS POSSIBLE.
- ROBUSTNESS OF THE STELLARATOR IS STILL TO BE DETERMINED.
- EXPANSION OF STOCHASTIC OPTIMIZATION TO SIMULATE REAL LIFE PERTURBATIONS.
- UNDERSTANDING LINK BETWEEN QUASISYMMETRY AND STOCHASTIC PERTURBATIONS

GENERALIZATION OF STOCHASTIC PERTURBATIONS

"The sources of these [magnetic field] errors are differences between the designed and fabricated coil shapes at the manufacturing stage or misalignments of the coils at the assembly stage."

- T. Andreeva et al. (2004) *Analysis of the Magnetic Field Perturbations during the Assembly of Wendelstein 7-X*, Fusion Science and Technology

- **CURRENT TECHNIQUES ONLY SIMULATE MANUFACTURING ERRORS**
- **INTEGRATE ROTATIONS AND TRANSLATIONS (AND TILTS OF WP ?)**
- **ROTATIONS APPEAR TO CONTRIBUTE THE MOST TO MAGNETIC FIELD DEGRADATION**