2012 ANU Maths Day Relay Contest

Please remember NO Calculators allowed in the relay contest

Relay Question 1 (10 marks) (190 marks remain)

Points on a circle:

For how many points lying on the circle $x^2+y^2 = 18$ are both the x-coordinate and the y-coordinate integers?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 2 (10 marks) (180 marks remain)

Minimum value:

If

$$f(x) = (\log x)^4 + \frac{2}{(\log x)^2}, \quad x > 1,$$

find x at which f takes the minimum value. (log is logarithm base e)

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 3 (10 marks) (170 marks remain)

Equation of a line:

Write the equation of the line L defined by the condition that the distances from every point of L to the points $A(-1,-2)$ and $B(2,3)$ are equal.

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 4 (10 marks) (160 marks remain)

Points on a circle:

For how many points lying on the circle $x^2+y^2 = 20$ are both the x-coordinate and the y-coordinate integers?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 5 (10 marks) (150 marks remain)

Minimum value:

If

$$f(x) = 3(\log x)^2 + \frac{1}{(\log x)^6}, \quad x > 1,$$

find x at which f takes the minimum value. (log is logarithm base e)

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 6 (10 marks) (140 marks remain)

Inequalities:

Find all pairs of real numbers x, y satisfying the system of inequalities

$$\begin{cases} y \geq x^2 + 1 \\ y \leq 2x \\ x \geq 1 \end{cases}$$

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 7 (10 marks) (130 marks remain)

Triangle:

Suppose for a triangle ABC we have $|AC| = 3.8$, $|AB| = 0.6$, and $|BC|$ is an integer. Find $|BC|$.

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 8 (10 marks) (120 marks remain)

Ferris wheel:

A ferris wheel has 24 equally spaced passenger cars and rotates at a constant speed. How many passenger cars does one have to add to reduce the waiting time between two passenger cars by 25%?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 9 (10 marks) (110 marks remain)

Inequalities:

Find all pairs of real numbers x, y satisfying the system of inequalities

$$\begin{cases} y \geq x^2 \\ y \leq 2x - 1 \\ x \geq 1 \end{cases}$$

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 10 (10 marks) (100 marks remain)

Equation of a line:

Write the equation of the line L defined by the condition that the distances from every point of L to the points $A(-2,-4)$ and $B(4,6)$ are equal.

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 11 (10 marks) (90 marks remain)

Ferris wheel:

A ferris wheel has 24 equally spaced passenger cars and rotates at a constant speed. How many passenger cars does one have to add to reduce the waiting time between two passenger cars by 20%?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 12 (10 marks) (80 marks remain)

Triangle:

Suppose for a triangle ABC we have $|AC| = 3.9$, $|AB| = 0.5$, and $|BC|$ is an integer. Find $|BC|$.

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 13 (10 marks) (70 marks remain)

Decimal fractions:

Find all pairs of digits x, y satisfying the identity

$$\frac{x+y}{5} = x.y$$

Here, a digit is an element of the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and by $x.y$ we mean decimal point notation, e.g. if $x = y = 2$, then $x.y = 2.2$.

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 14 (10 marks) (60 marks remain)

Scales:

Two watermelons are weighed with a scale that has a constant error. One watermelon weighs 3 kg, the other one 4 kg, and both of them 8 kg. What is the true weight of the watermelons?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 15 (10 marks) (50 marks remain)

Three glasses:

Suppose we have three glasses of water. Pour one-third of the water contained in the first glass into the second glass. Next, pour one-third of the water contained in the second glass into the third glass. Finally, pour one-third of the water contained in the third glass into the first glass. Assume that after this process each glass contains 100 ml of water. How much water did we have in each glass initially?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 16 (10 marks) (40 marks remain)

Cube:

Suppose that the faces of a cube are labelled with positive integers in the following way: the top face is labelled with 25, the right face with 7, the front face with 66, and the other faces are labelled with prime numbers. Assume that the sums of the numbers for each pair of opposite faces are all equal. Find the the number on the left face.

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 17 (10 marks) (30 marks remain)

Scales:

Two watermelons are weighed with a scale that has a constant error. One watermelon weighs 2 kg, the other one 3 kg, and both of them 6 kg. What is the true weight of the watermelons?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Please remember NO Calculators allowed in the relay contest

Relay Question 18 (10 marks) (20 marks remain)

Three glasses:

Suppose we have three glasses of water. Pour one-third of the water contained in the first glass into the second glass. Next, pour one-third of the water contained in the second glass into the third glass. Finally, pour one-third of the water contained in the third glass into the first glass. Assume that after this process each glass contains 200 ml of water. How much water did we have in each glass initially?

Attempt 1:	**Attempt 2:**	**Attempt 3:**

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Relay Question 19 (10 marks) (10 marks remain)

Cube:

Suppose that the faces of a cube are labelled with positive integers in the following way: the top face is labelled with 25, the right face with 7, the front face with 66, and the other faces are labelled with prime numbers. Assume that the sums of the numbers for each pair of opposite faces are all equal. Find the the number on the bottom face.

Attempt 1:	**Attempt 2:**	**Attempt 3:**

2012 ANU Maths Day Relay Contest

Please remember NO Calculators allowed in the relay contest

Relay Question 20 (10 marks) (0 marks remain)

Decimal fractions:

Find all pairs of digits x, y satisfying the identity

$$\frac{x+y}{4} = x.y$$

Here, a digit is an element of the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and by $x.y$ we mean decimal point notation, e.g. if $x = y = 2$, then $x.y = 2.2$.

Attempt 1:	**Attempt 2:**	**Attempt 3:**