Parallel length scale and nonlinear heat flux from gyrokinetic simulations

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Motivation

- Want to optimize stellarator geometry to minimize turbulent heat fluxes.
- Can we use purely geometric quantity to predict turbulence?
- Critical balance prediction: $Q_i \propto L_{||}$ (for slabs[1] and tokamaks[2])
- In tokamaks: $L_{||} \propto qR \propto 1/\iota$
- What about QA stellarators?

[1] T. Adkins+, JPP **89**(4) 905890406 (2023)
[2] T. Adkins+, in preparation

Critical balance

- Parallel dissipation rate and non-linear dissipation rate are assumed to balance [M. Barnes+ 2011, T. Adkins+ 2022]
- Assumes far above marginality. (Q_i "large", meaning $L_{||}$ large if $Q_i \propto L_{||}$)
- Assumes FLR effects are negligible ($k_\perp \rho_s \ll 1$)

Scale invariance

- Electrostatic, collisionless, no FLR ($k_\perp \rho_s \ll 1$) gyrokinetics:

$$0 = \frac{\partial}{\partial t}\left(h_s - \frac{q_s\phi}{T_{0s}} f_{0s}\right) + \left(v_\parallel \vec{b}_0 + \vec{v}_{\mathrm{m}s}\right)\cdot\vec{\nabla} h_s + \frac{1}{B_0^2}\vec{B}_0\cdot\left[\vec{\nabla}\phi\times\vec{\nabla}\left(h_s + f_{0s}\right)\right]$$

$$0 = \sum_s q_s\left[-\frac{q_s\phi}{T_{0s}} n_{0s} + \int \mathrm{d}^3\vec{v}\, h_s\right].$$

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- Symmetry. For any λ:

$$\tilde{h}_s = \lambda\, h_s(x/\lambda, y/\lambda, z/\lambda, t/\lambda),$$
$$\tilde{\phi} = \lambda\, \phi(x/\lambda, y/\lambda, z/\lambda, t/\lambda),$$

Implications for transport

- Suppose h_s is periodic in x, y and z with domain sizes L_x, L_y, $L_\parallel$.
  $\implies$ $\tilde{h}_s$ is periodic with domain sizes λL_x, λL_y and $\lambda L_\parallel$.

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- Electrostatic heat flux

$$Q_s[h_s](L_x, L_y, L_\parallel, t) = \int_{Lx,Ly,L\parallel} \mathrm{d}^3\vec{r}\, \left(\vec{v}_E \cdot \vec{\nabla} x\right) \int \mathrm{d}^3 v\, \frac{mv^2}{2} h_s(t) \Bigg/ \int_{Lx,Ly,L\parallel} \mathrm{d}^3\vec{r}\,,$$

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- Stationarity: Q_s has been able to reach a statistical steady-state.

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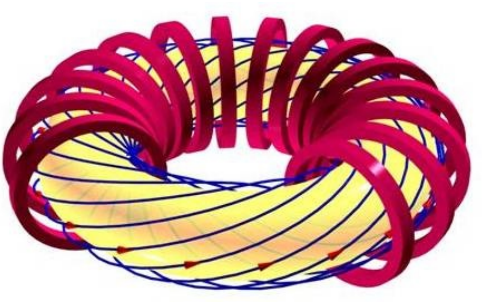
- Locality: Q_s is independent of perpendicular domain size.
- Stationarity: Q_s has been able to reach a statistical steady-state.
- Given that λ can be chosen arbitrarily, it follows that:

$$\boxed{Q_s \propto L_\parallel}$$

Scale invariance in a tokamak



- What is $L_{||}$ in toroidal geometry?

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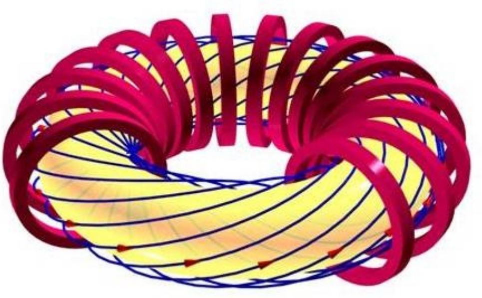

- What is $L_{\parallel}$ in toroidal geometry?
- Parallel scale invariance broken due to variation in geometry. For a tokamak:

$$L_{\parallel} \sim qR, \quad \text{where} \quad q = \frac{\#\text{ of toroidal turns}}{\#\text{ of poloidal turns}} \quad \Rightarrow \quad \boxed{Q_s \propto q}$$

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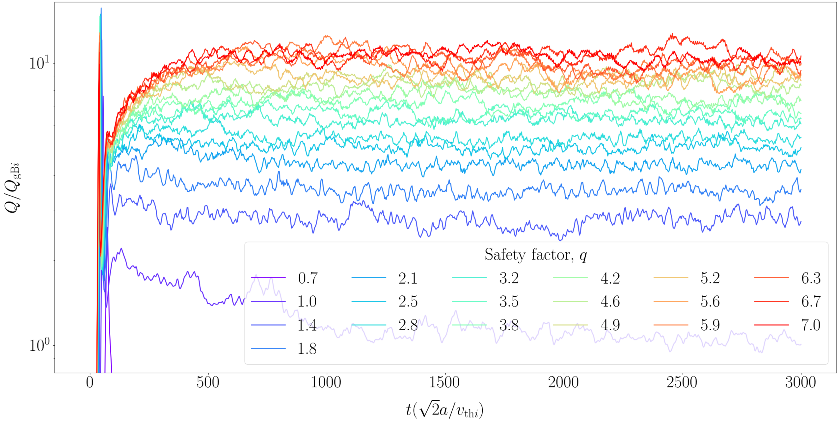

- ion-scale (adiabatic electron) Cyclone Base Case `GX` simulations.
- Miller geometry:
  $r/a = 0.5$, $R/a = 2.8$, $\hat{s} = 0.8$, $a/L_{T_i} = 2.5$, $a/L_n = 0.8$, $\nu_{ii}/(v_{\mathrm{th}i}/a) = 1.2 \times 10^{-4}$.

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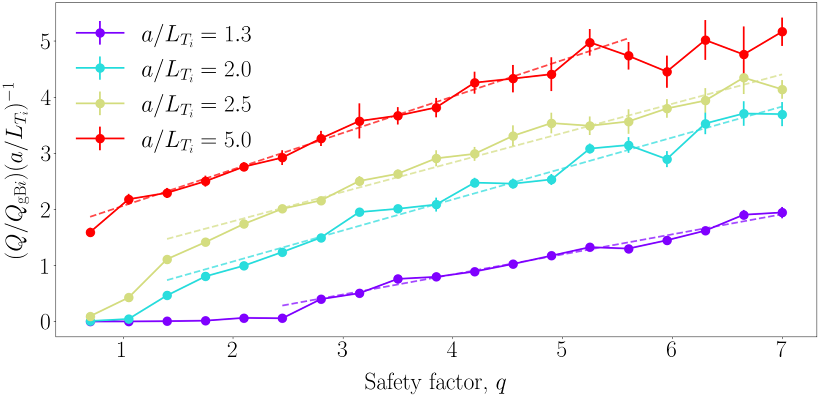

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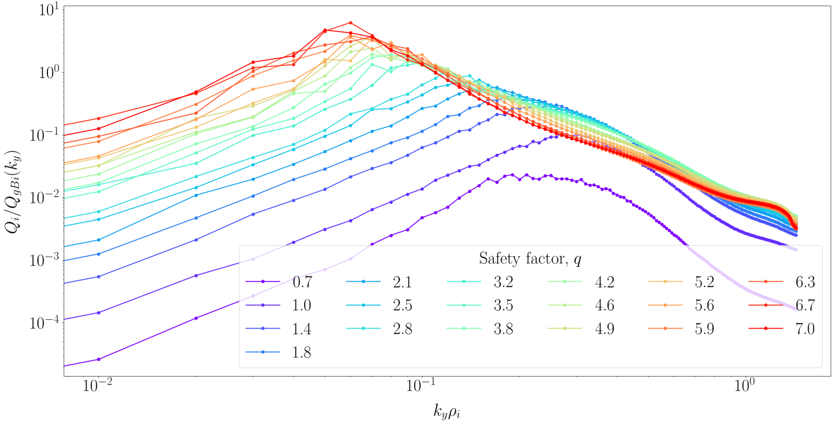

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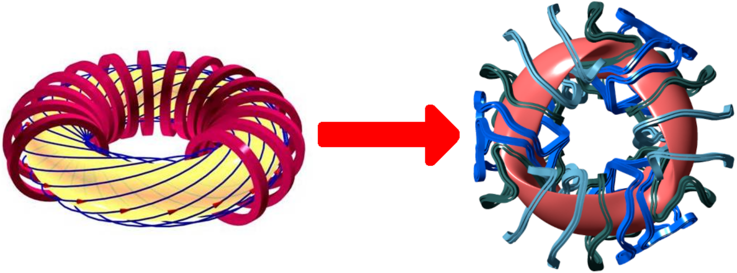
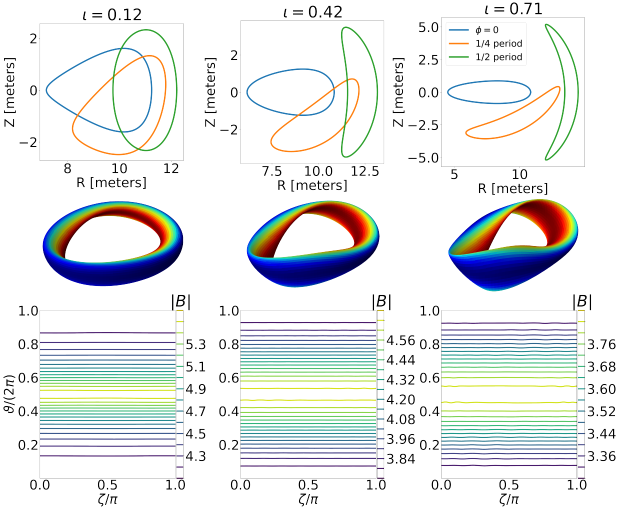
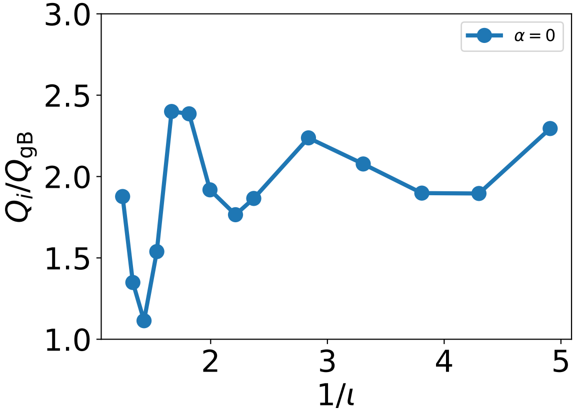
From tokamak to QA stellarator



Series of precise QA with varying ι



Q_i in QA ι scan



- GX simulations: $r/a = 0.5$, $\alpha = 0.0$, $a/L_T = 3.0$, $a/L_n = 1.0$

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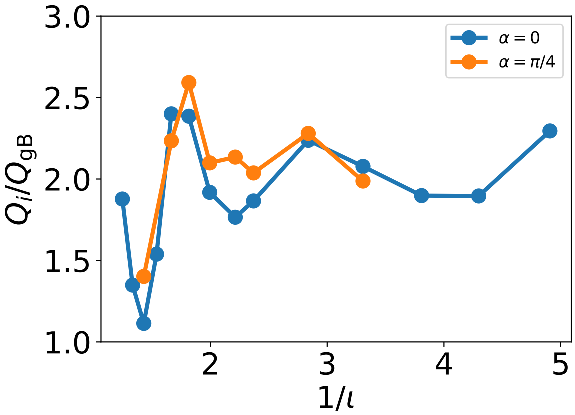

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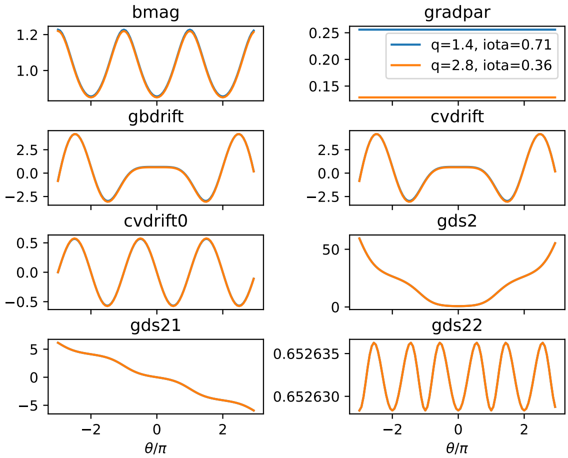
Changes in safety factor for Miller tokamak – and for QA stellarator

- Changing q in Miller geometry only changes $\nabla\theta$



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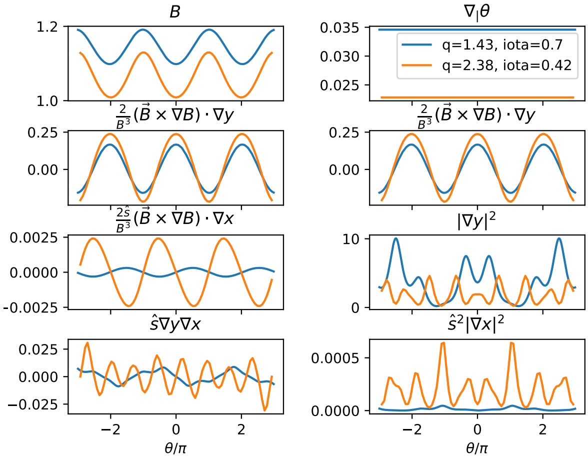

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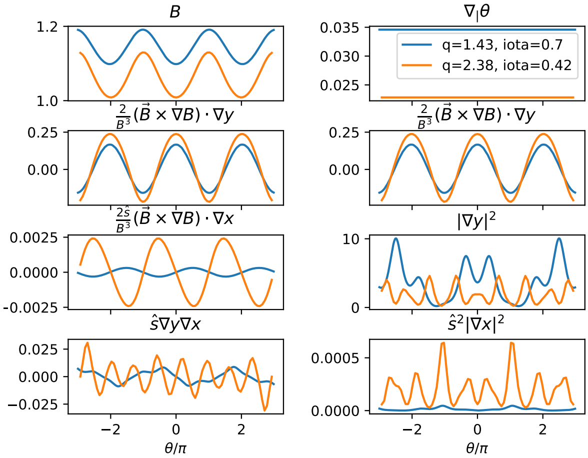
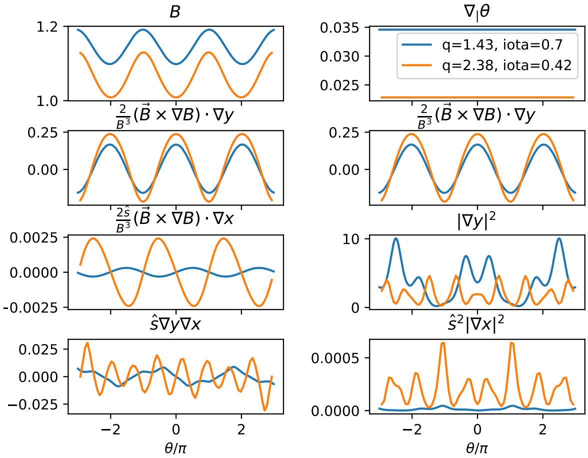
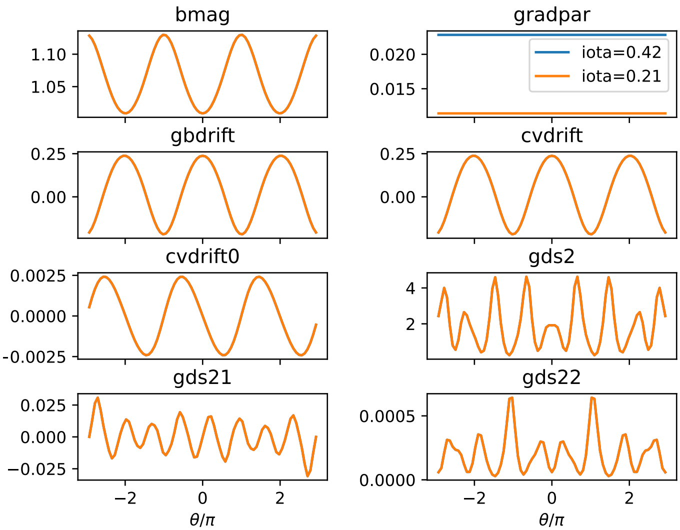


Changes in safety factor for Miller tokamak – and for QA stellarator

- Changing q in Miller geometry only changes $\nabla\theta$
- Changing ι in stellarator equilibrium is more complicated
- Stellarator has more than 1 lenght scale for parallel variation in geometry.
- If $Q_i \propto L_{||}$ for any lenght scale, changing $\nabla\theta$ should change it.



Artificial flux-tubes: Miller-like fake ι scan



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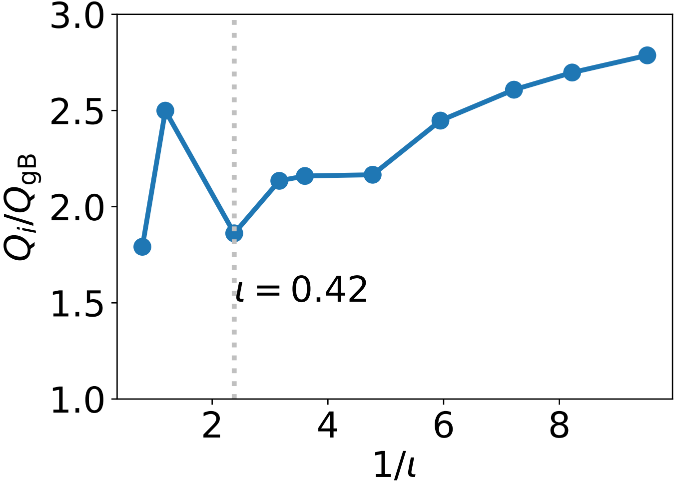

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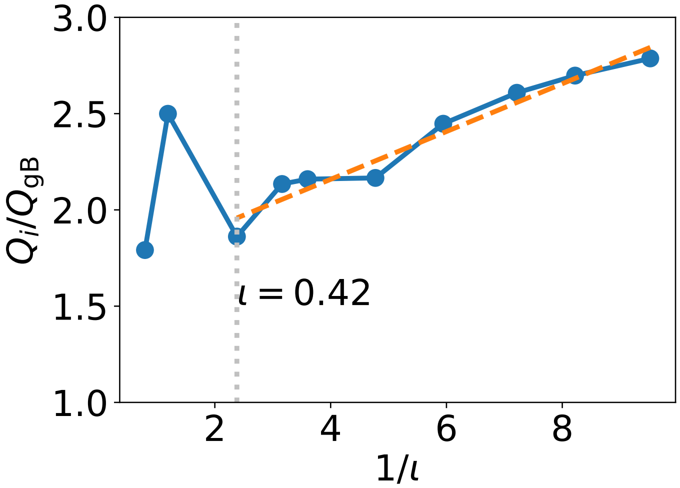


Artificial flux-tubes: Miller-like fake ι scan

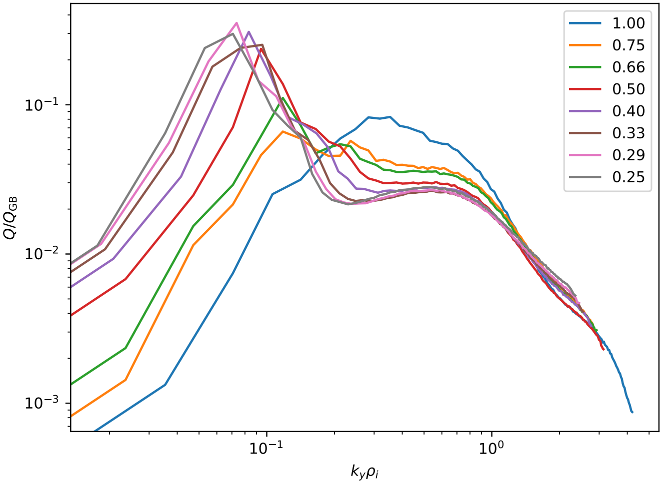
- Lower $1/\iota$ has heat flux at higher k_y

$\Longrightarrow$ FLR effects become important.

- Lower $1/\iota$ has smaller Q_i

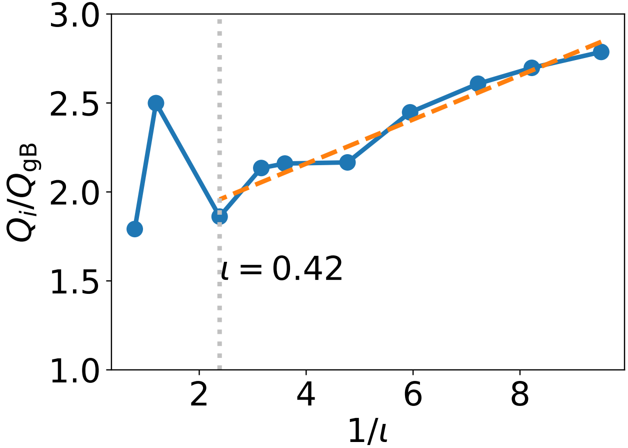
$\Longrightarrow$ Closer to marginality. (Critical balance does not apply)

- Thus, we don't expect scaling to apply for low $1/\iota$



Summary - Relevance to turbulence optimization?

- Critical balance scaling may apply to low ι (here, $\iota \lesssim 0.4$)



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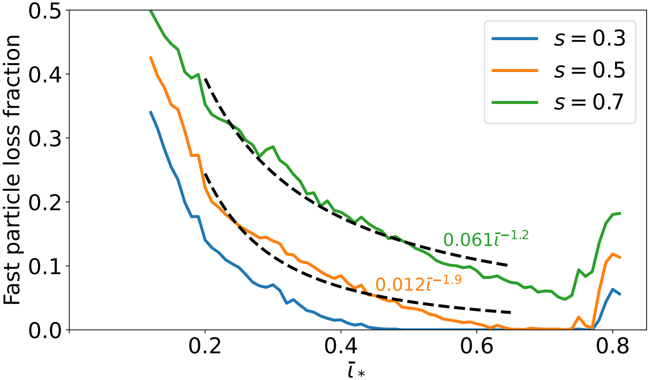

SIMPLE: C. G. Albert+ (2020)

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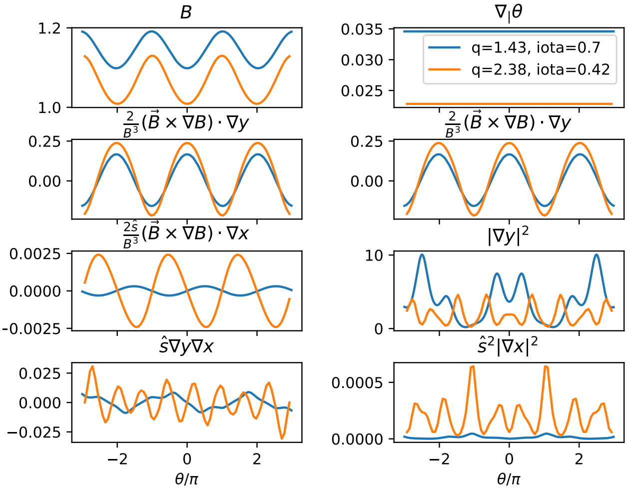
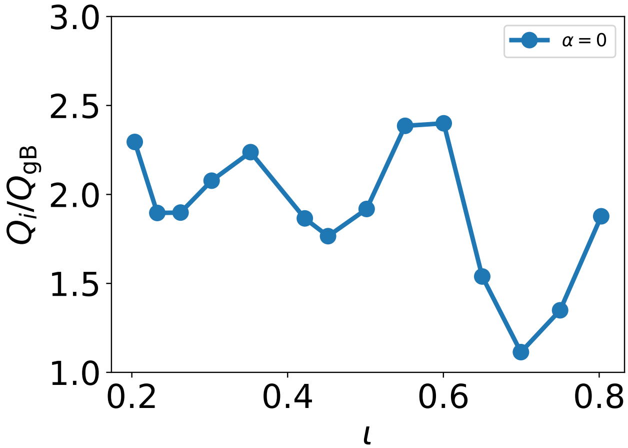
- Typically, do not want ι too low
- Scaling is complicated by several parallel lenght scales



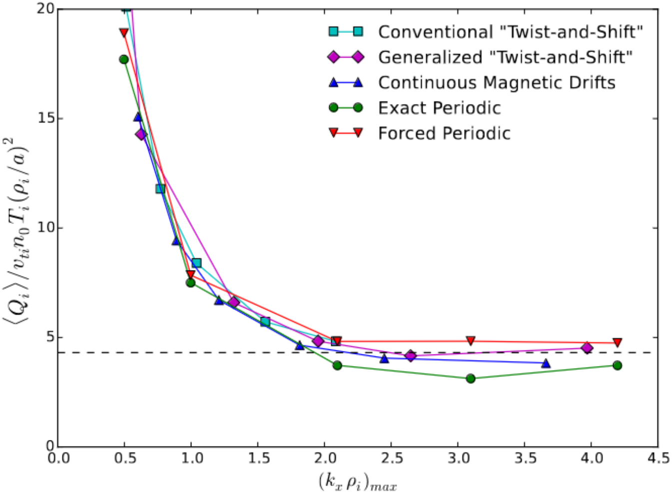
Summary - Relevance to turbulence optimization?

- Critical balance scaling may apply to low ι (here, $\iota \lesssim 0.4$)
- Typically, do not want ι too low
- Scaling is complicated by several parallel lenght scales
- Simple ι scaling not observed

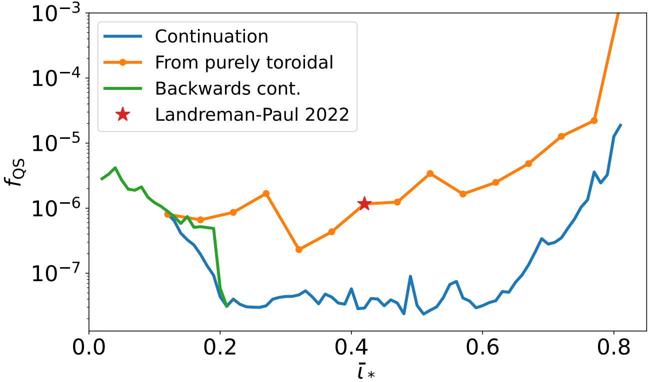


BONUS SLIDES

The Parallel Boundary Condition for Turbulence Simulations in Low Magnetic Shear Devices. M. Martin+ (2018)



QA iota scan



Continuation
From purely toroidal
Backwards cont.
Landreman-Paul 2022
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
f_{QS}
0.0
0.2
0.4
0.6
0.8
$\bar{\iota}_*$