Simplifying coil sets using efficiently-placed windowpane coils

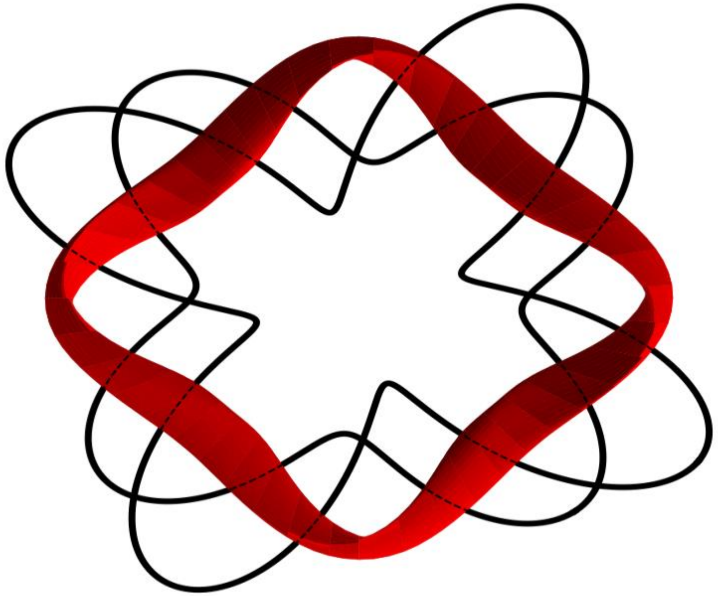
Todd Elder, Allen Boozer, Matt Landreman

Columbia University*, University of Maryland at College Park

*: Recently graduated

Stellarator coil sets suffer from poor access properties and difficulty of engineering

Idea: Use simple helical or modular coils to provide bulk shaping, localized groups of windowpane coils to produce the rest of the shaping



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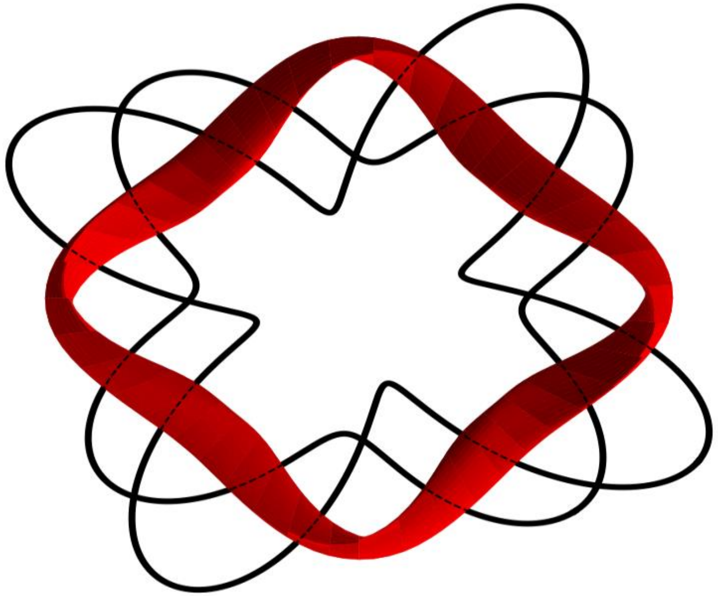

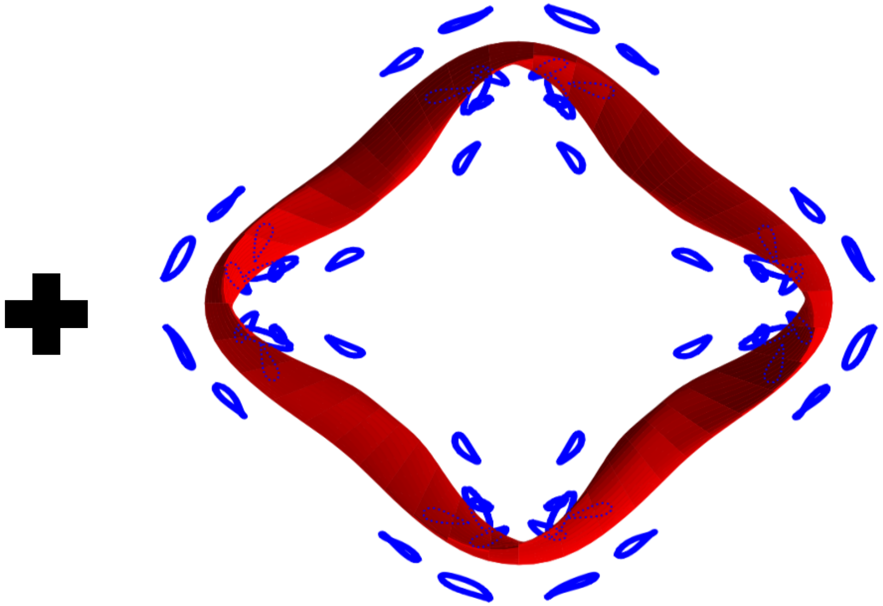

Simple helical coils

Well-placed windowpanes for shaping

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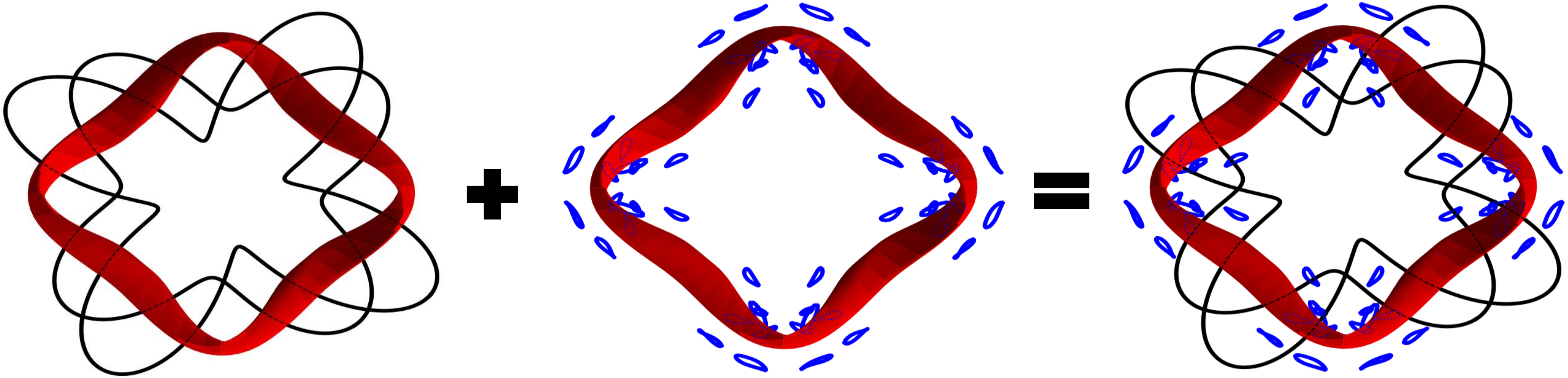

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Engineeringly feasible open-access coil set

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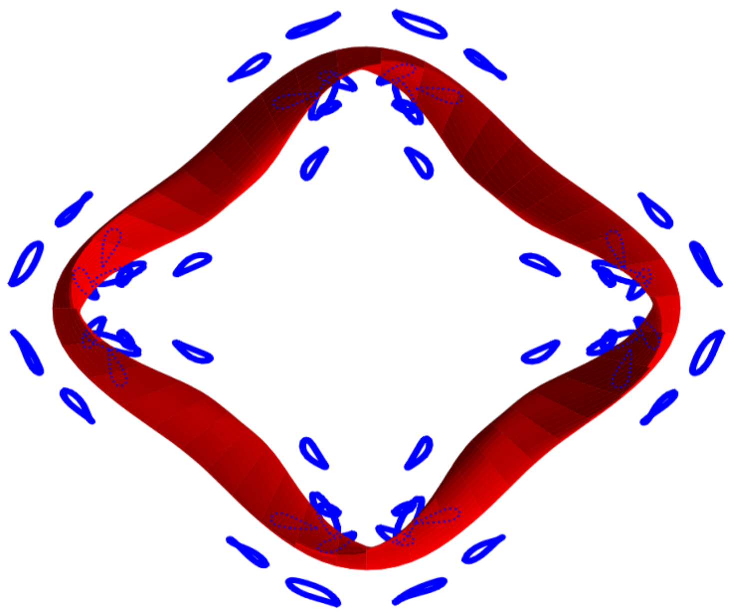


Well-placed windowpanes for shaping

How do we determine windowpane coil shape, location, and current?

Current potentials are magnetic-surface-generating stream functions

To generate a magnetic surface, find current sheet **I**

$$\mathbf{I} = \boldsymbol{\nabla}\Phi \times \hat{\mathbf{n}}$$

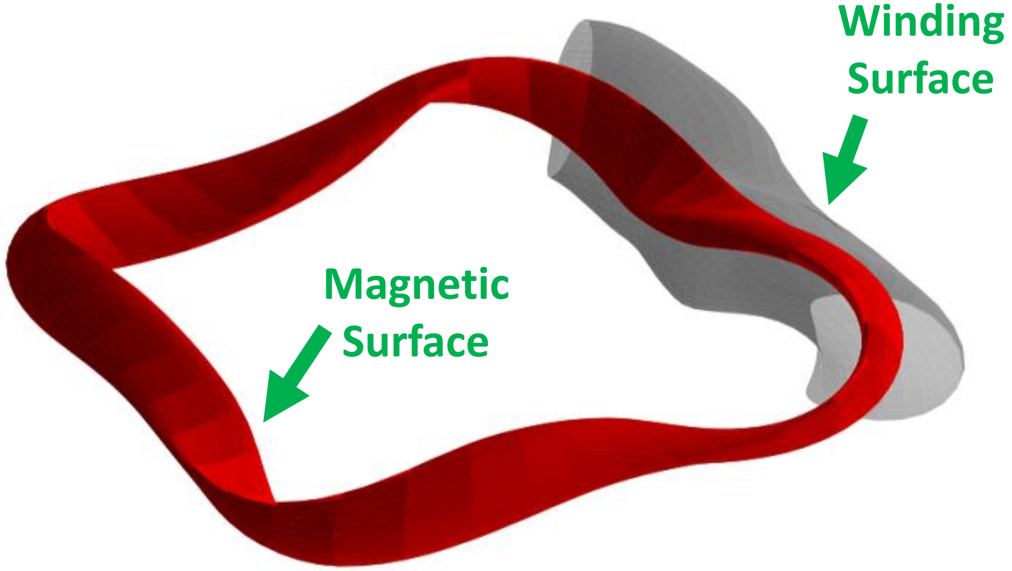
by solving underdetermined least-squares problem

$$\min\left(||\mathbf{A}\cdot\mathbf{x} - \mathbf{b}||_2\right)$$

A: Normal fluxes from non-secular Φ_j

b: Normal fluxes from the secular part of Φ

x: Vector Φ_j



Φ is known as current potential (CP)

$$\Phi = \Phi_{SV} + \frac{I\theta}{2\pi} + \frac{G\zeta}{2\pi}$$

Φ_{SV}: Degrees of freedom

I: Set by toroidal flux

G: modular/helical switch

Current potentials are typically in Fourier basis

$$\Phi_{SV} = \sum_{m,n} \Phi^{s}_{m,n}\sin(m\theta - n\varsigma) + \Phi^{c}_{m,n}\cos(m\theta - n\varsigma)$$

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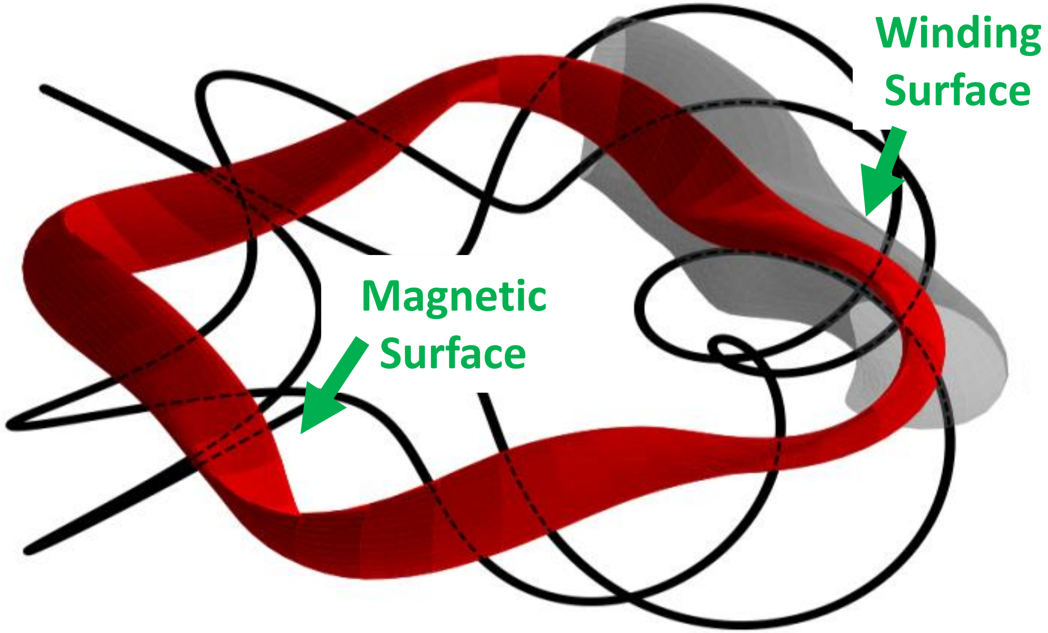
If the toroidal flux is supplied externally – say by modular or helical coils, the leftover field may be represented by a single-valued current potential (I=0,G=0)



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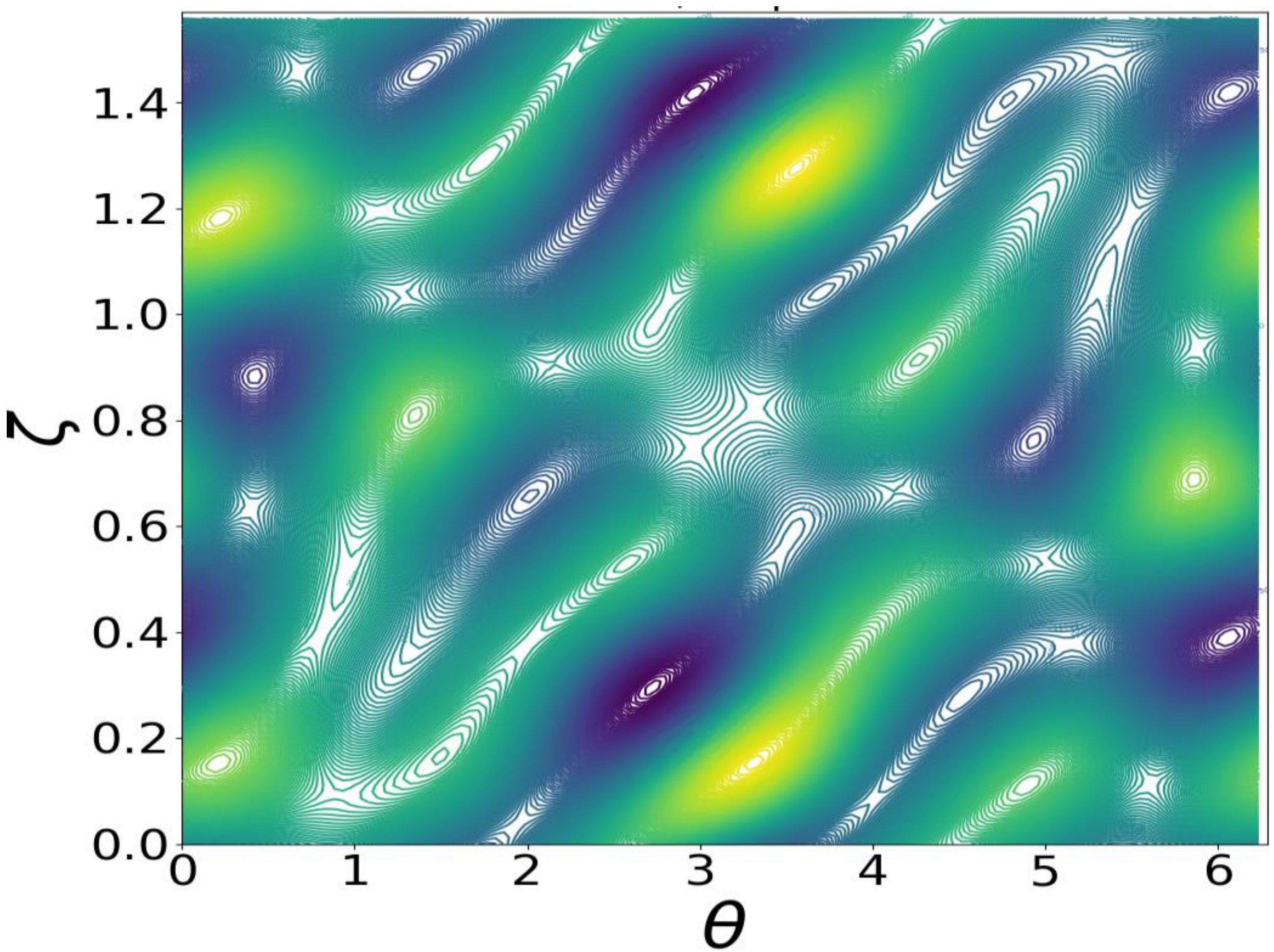
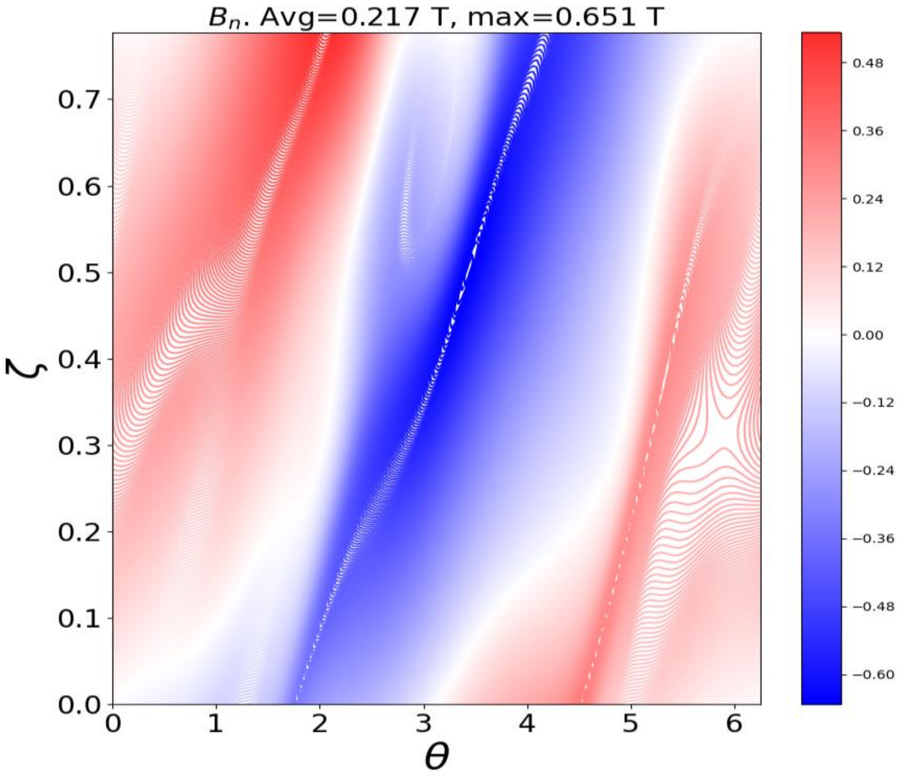
Contours of single-valued current potential provide initial windowpane coil guesses

Quick example showing windowpane coil generation from an HSX CP:





HSX Bn to cancel for one-half field period

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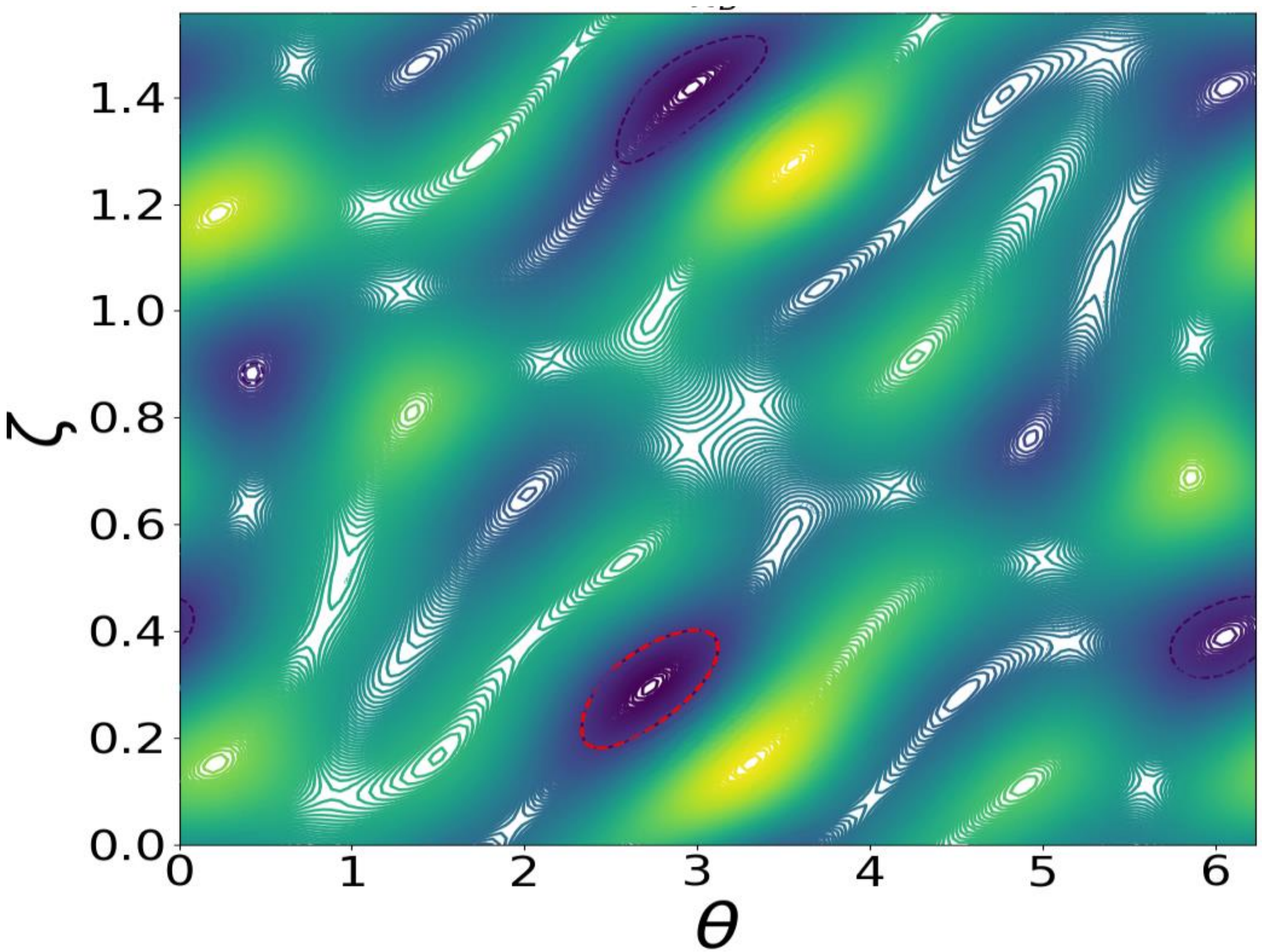

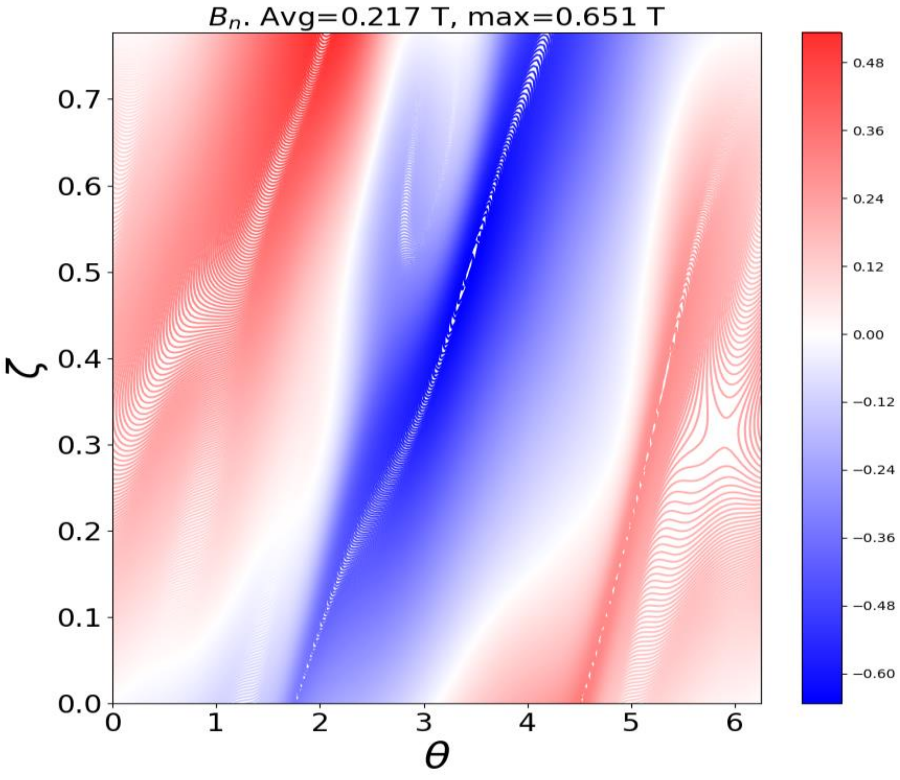

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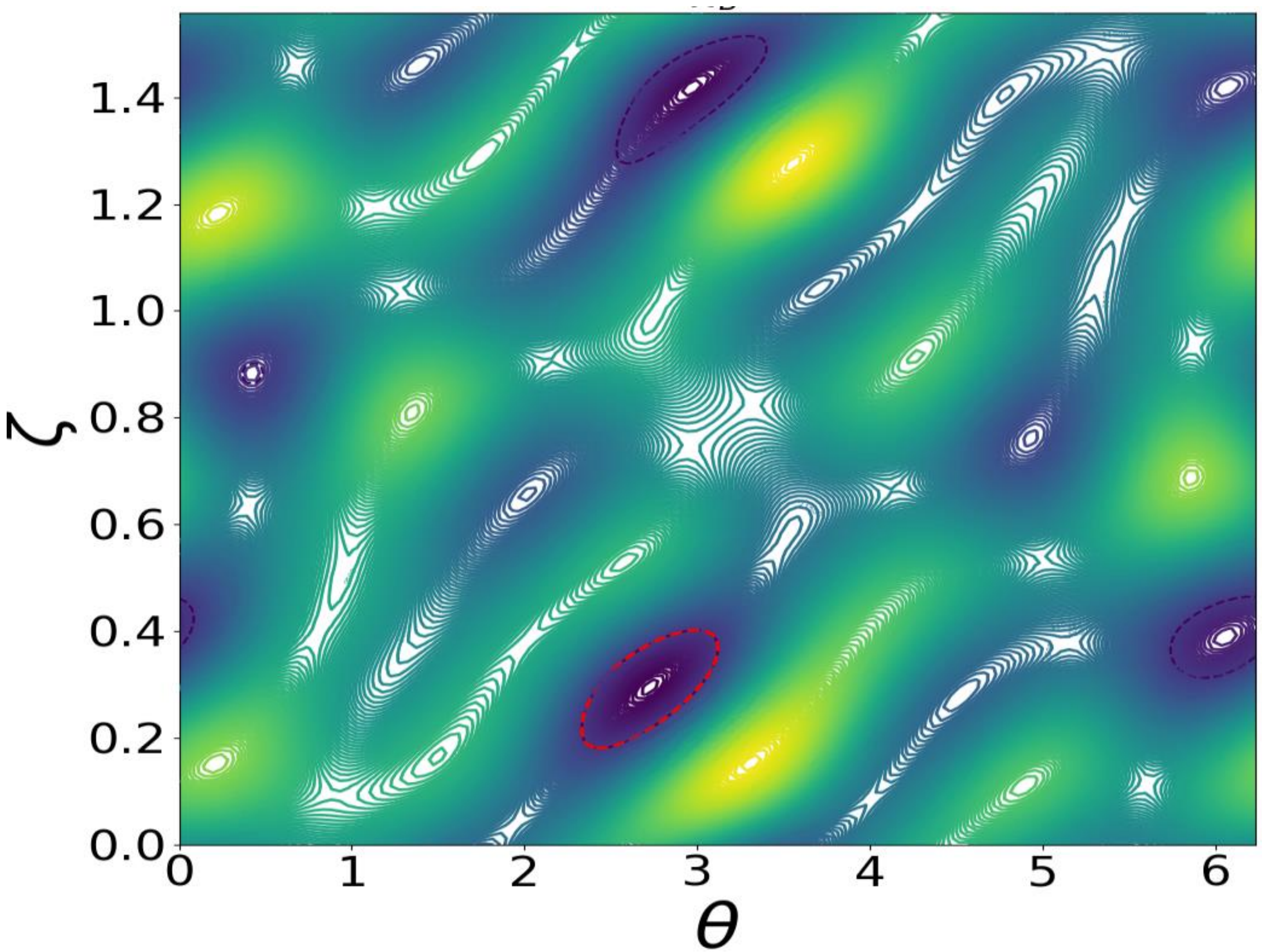

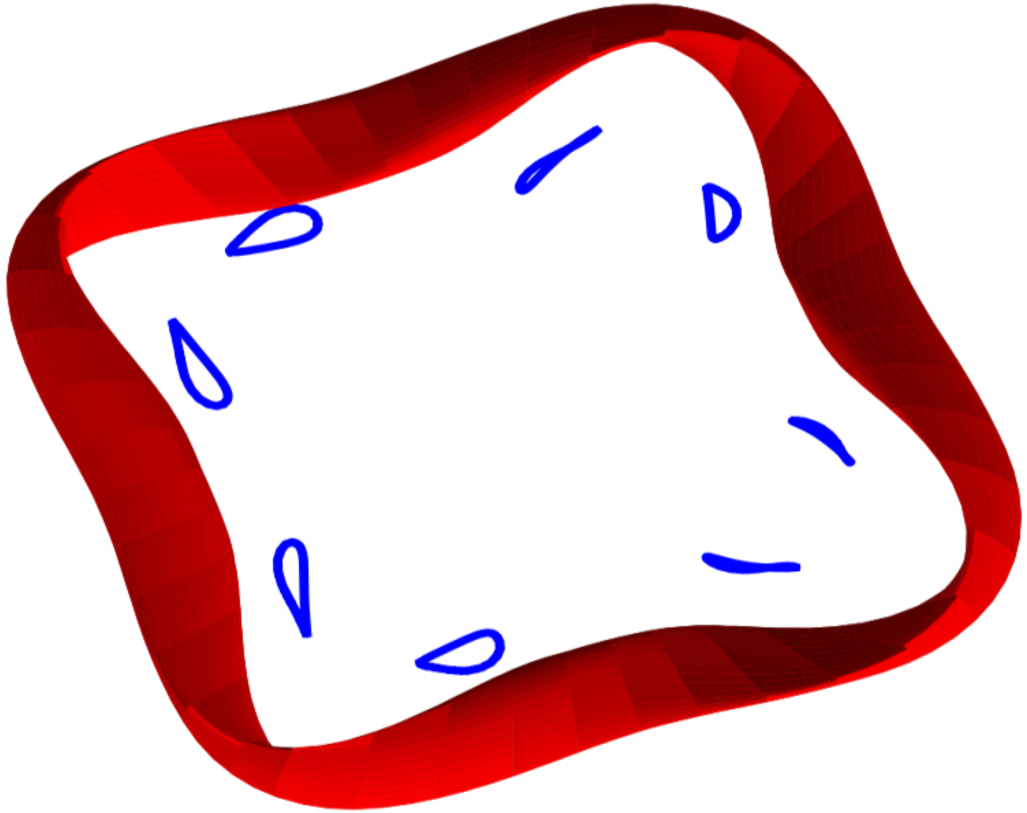

$$\mathbf{I} = \boldsymbol{\nabla}\Phi \times \hat{\mathbf{n}}$$

For good access properties, windowpane currents should be concentrated in crucial shaping locations

The underdetermined formulation of the least-squares problem allows for regularization techniques to influence current potential properties

$$\min(||\boldsymbol{A}\cdot\boldsymbol{x}-\boldsymbol{b}||_2+\lambda|\chi_K|_n)$$

A: Normal fluxes from non-secular Φ_j

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Current density **K**

$$\chi_K=\int|\boldsymbol{K}(\theta,\zeta)|_n\,d^2a$$

λ: Regularization parameter

n: Norm number

n=2: Tikhonov (or L2) regularization, produces smooth distributed currents (REGCOIL)

n=1: Sparse optimization (or L1 regularization), produces concentrated currents at crucial locations, low currents elsewhere

n=0: Sharp, sparse optimization (L0 regularization). Concentrated currents at certain locations, 0 elsewhere. Often NP-hard.

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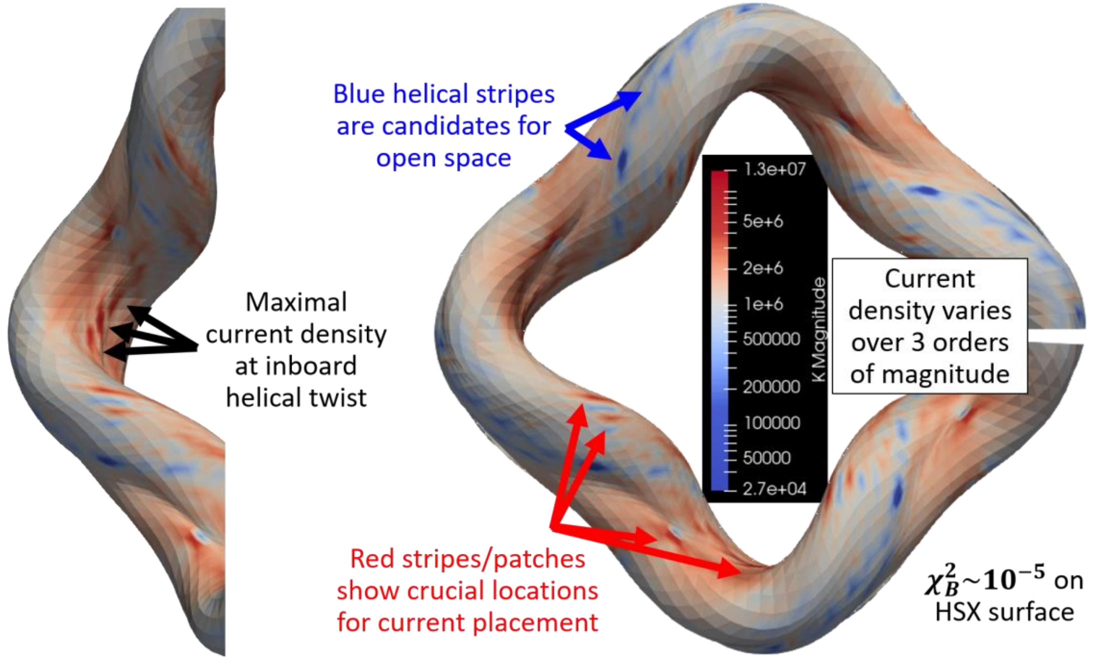
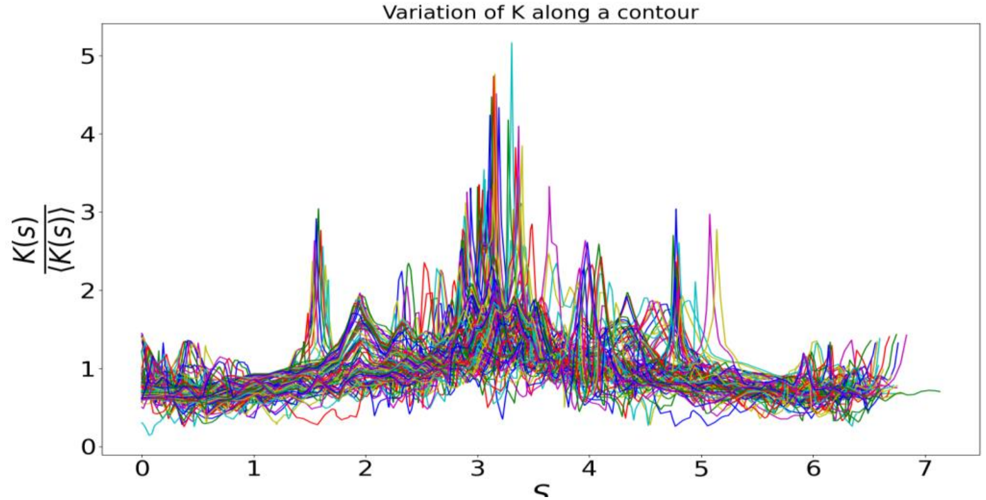
→ L1 and L0 regularization should be pursued to find open-access current configurations

Sparse optimization of current potentials was attempted, and failed due to Fourier basis

L1-regularized $|\mathbf{K}|_2$ on HSX winding surface



Patchiness due to Fourier terms “fighting” sparsity: Fourier basis likes diffuse, smooth functions – L1 regularization promotes sharp features



Along CP contours (candidates for coils), current density from L1-regularized **K** varies up to 500% (bad!)

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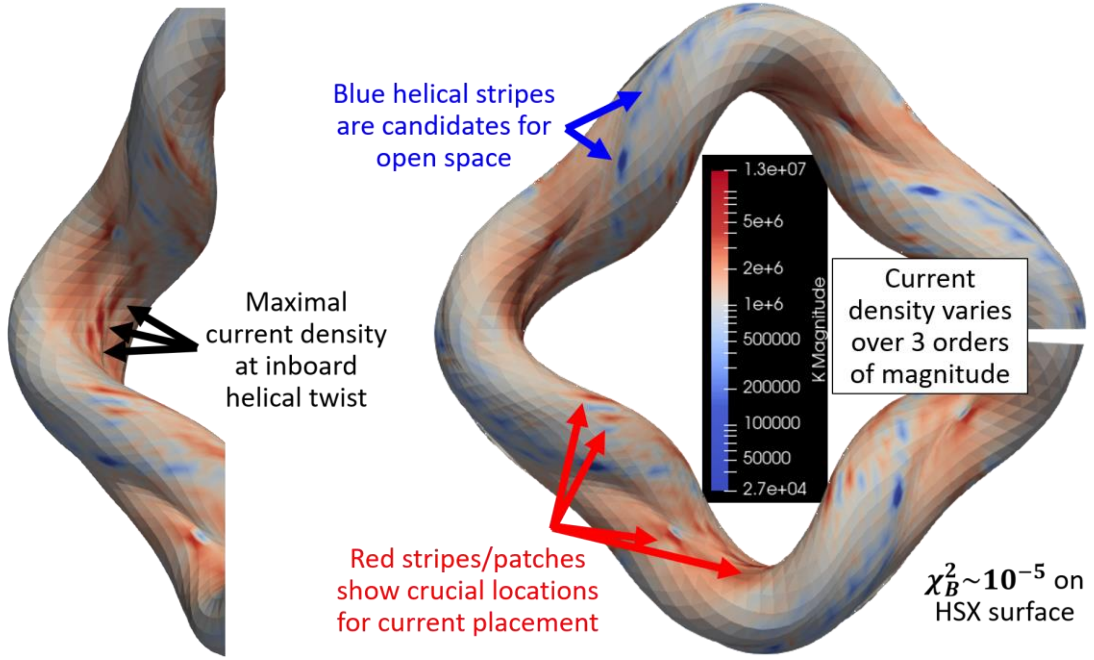

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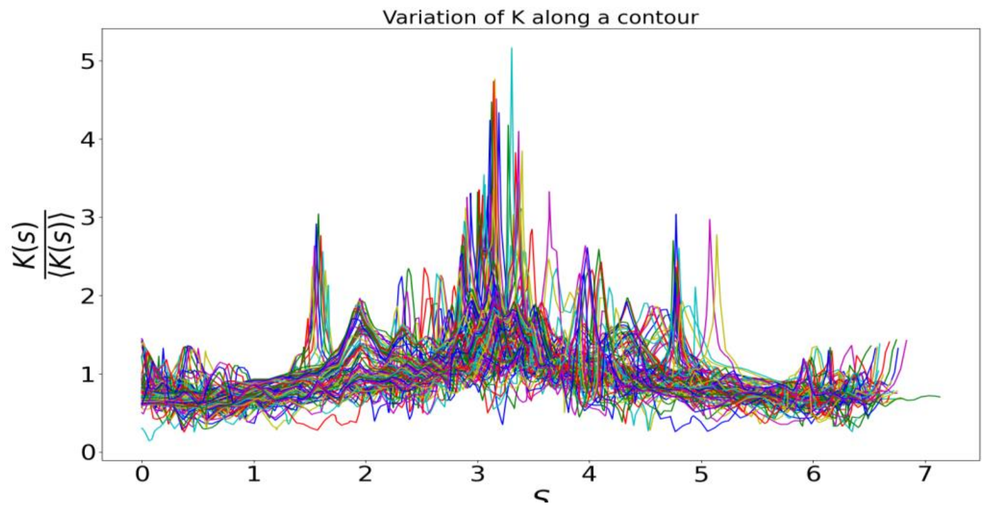

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→ **A finite element method basis may be best for current potential sparsity**

Magnetic dipoles provides quick finite-element-like basis

Current potentials are distributions of magnetic dipoles (Merkel 1988)

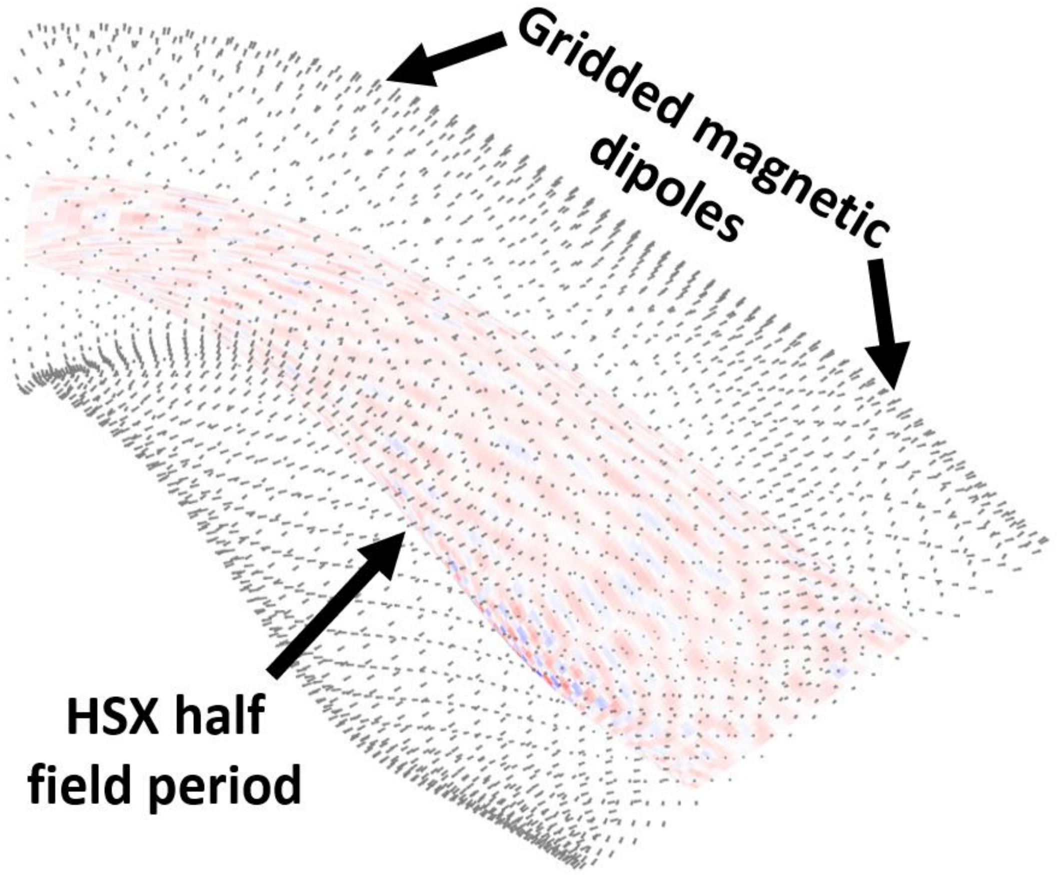
This provides a finite-element-like basis to work from

$$\boldsymbol{d} = \hat{\boldsymbol{n}} \iint \Phi_{SV}\, da$$

Dipole strengths found via least-squares solution

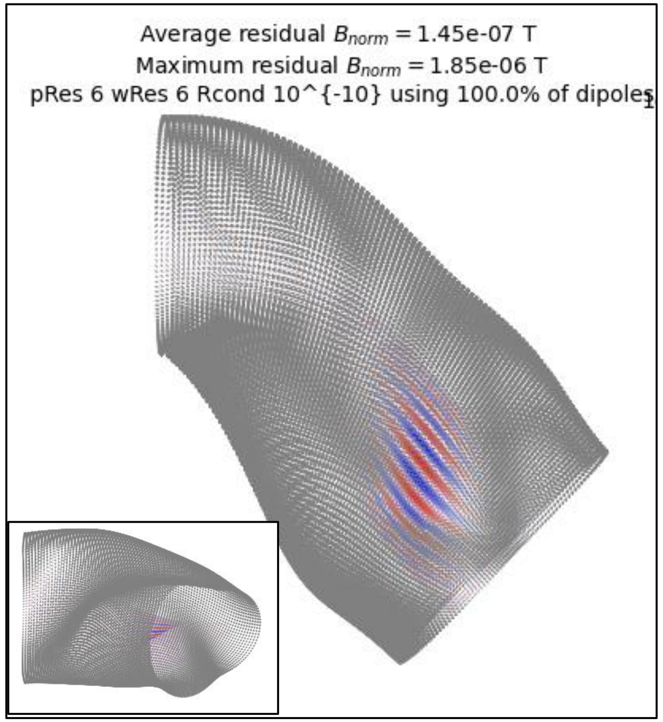
$$\boldsymbol{M} \cdot \boldsymbol{d} = B_n$$

Supports “sharp” current distributions



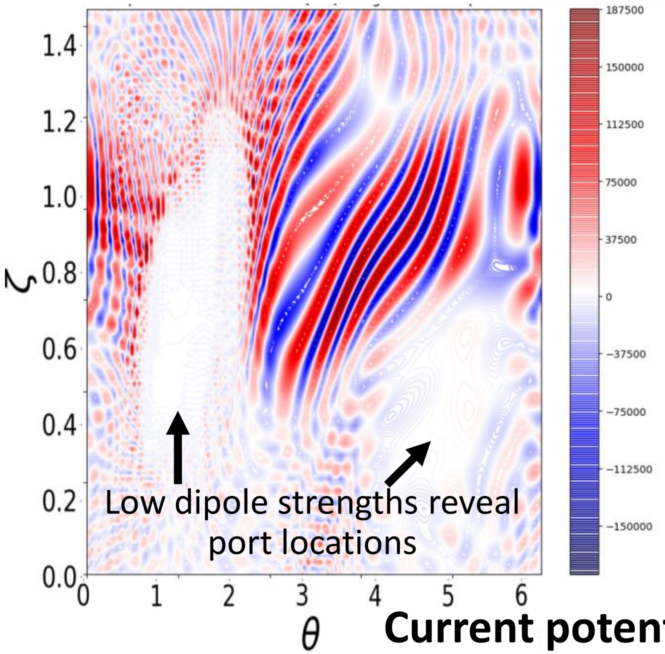
This approach differs from permanent magnet work in that the dipole strength is unlimited as the gradient of dipole strengths gives current densities

FEM CPs produce field shaping with open access properties



Method:

1. Compute dipole strengths using full winding surface
2. Eliminate dipoles with strength less than some magnitude
3. Re-compute new dipole strengths using fraction of winding surface area
4. Reduce fraction of surface used until 0.1% error achieved



Current potential analog (dip. strs)

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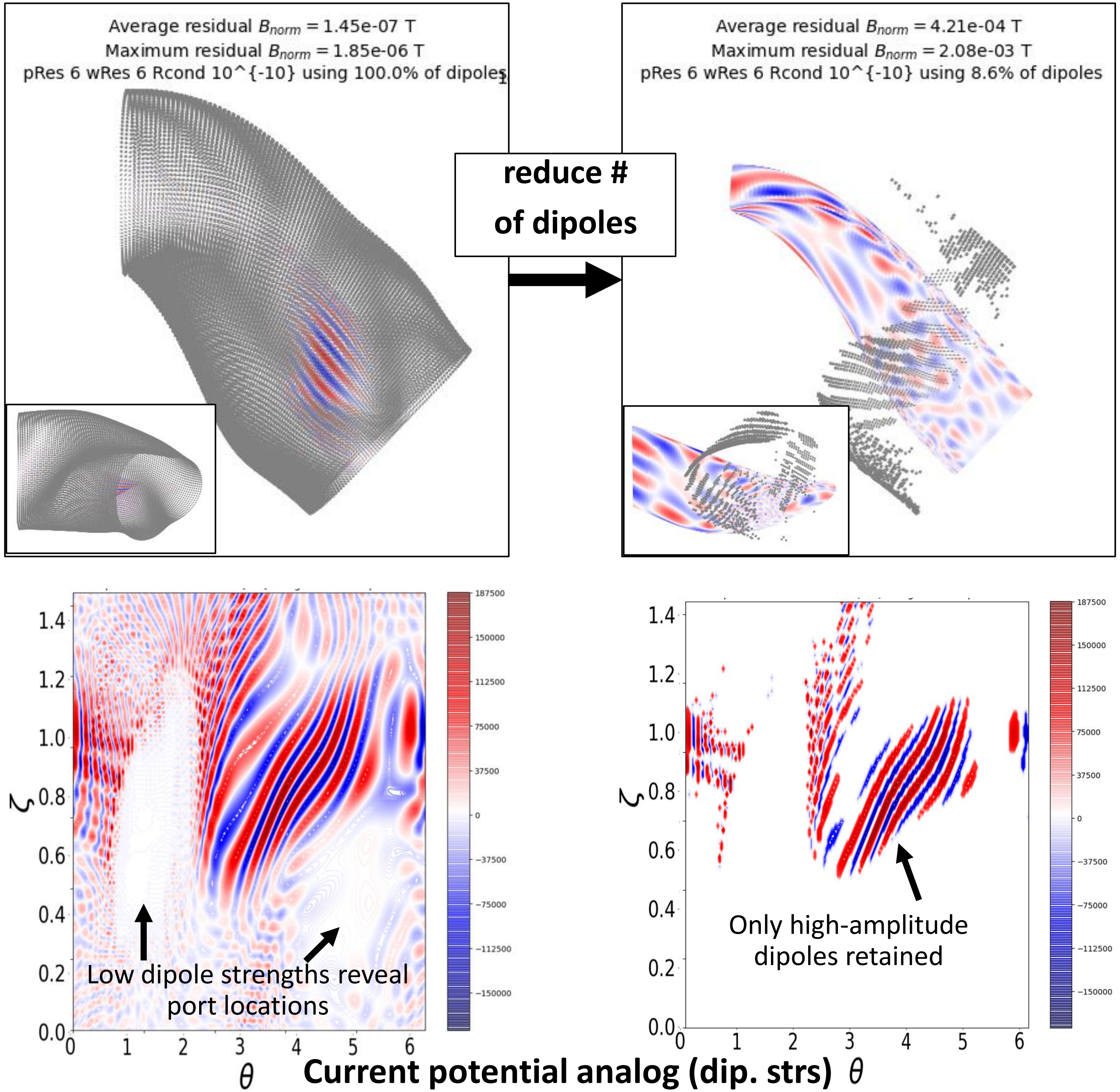

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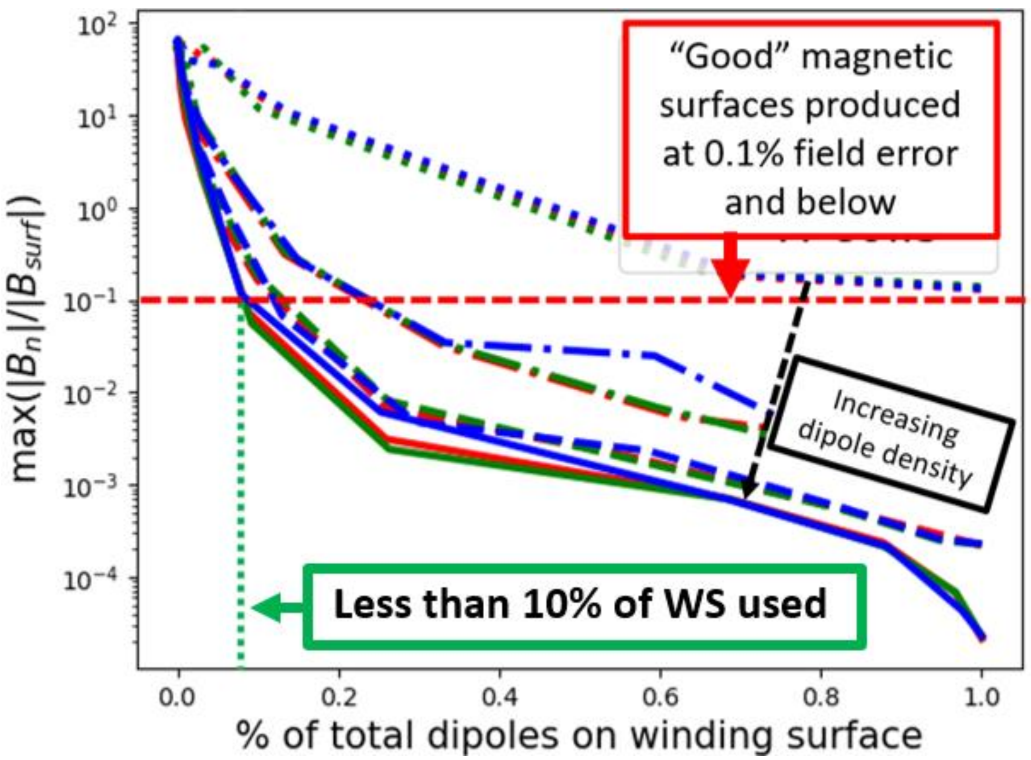
Finite-element-like basis produces encouraging results but suffers from short-wavelength features in the current potential



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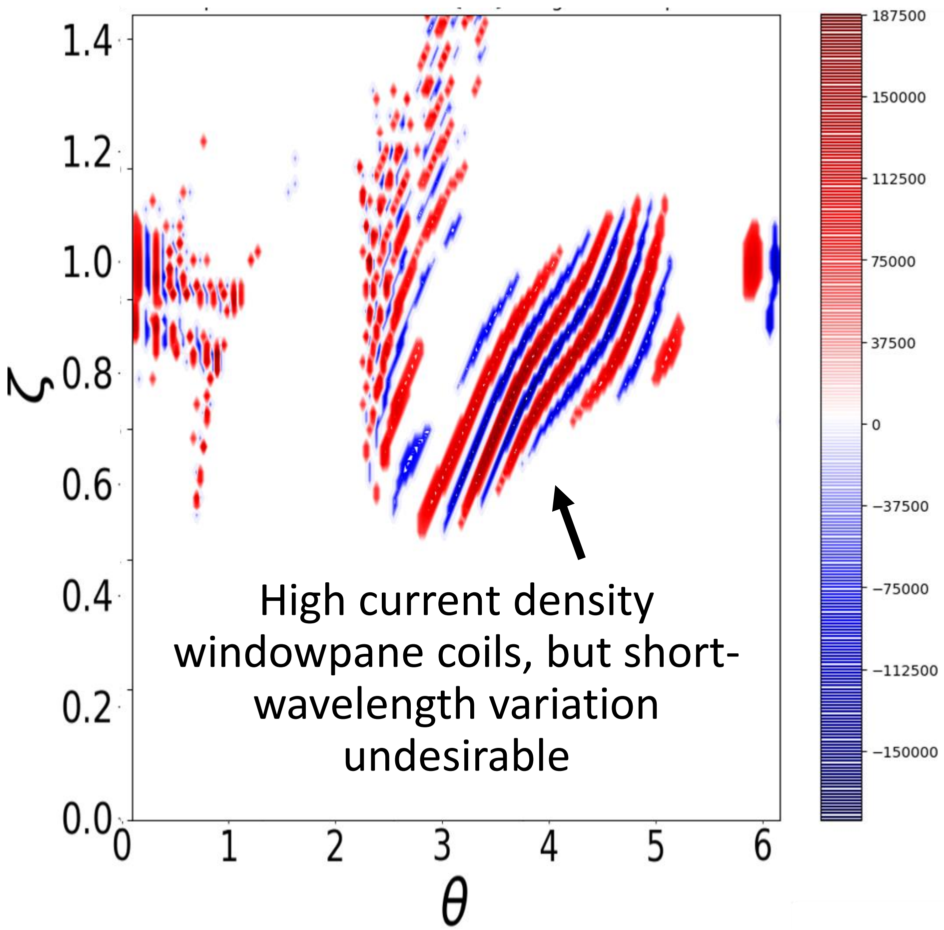

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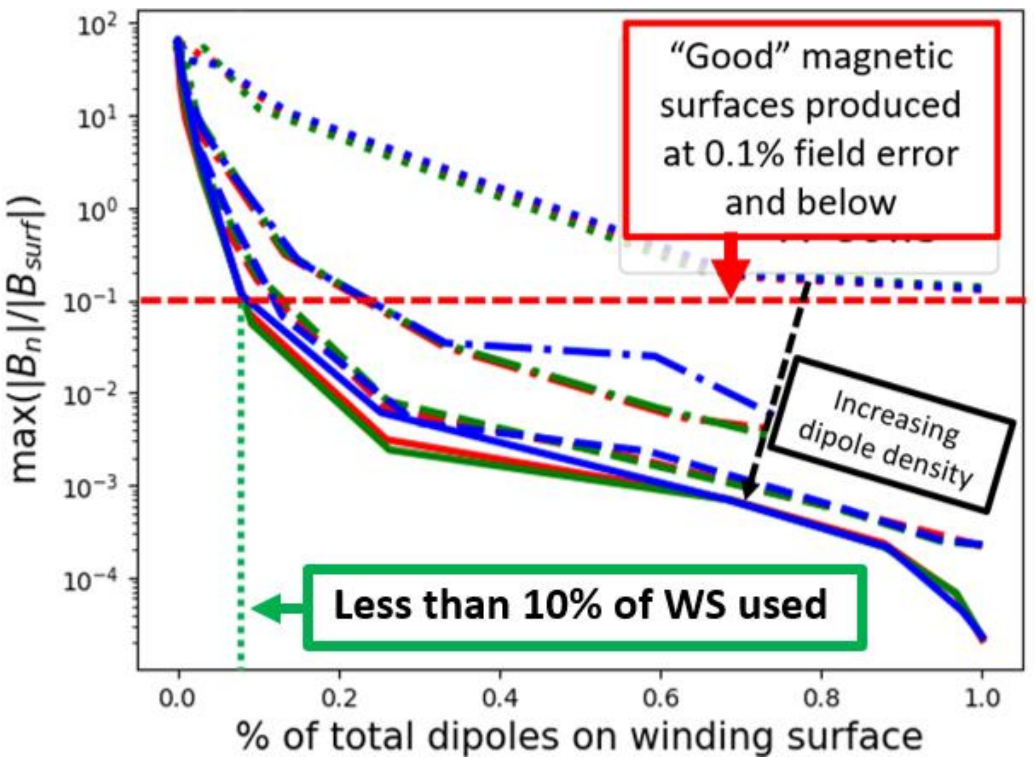

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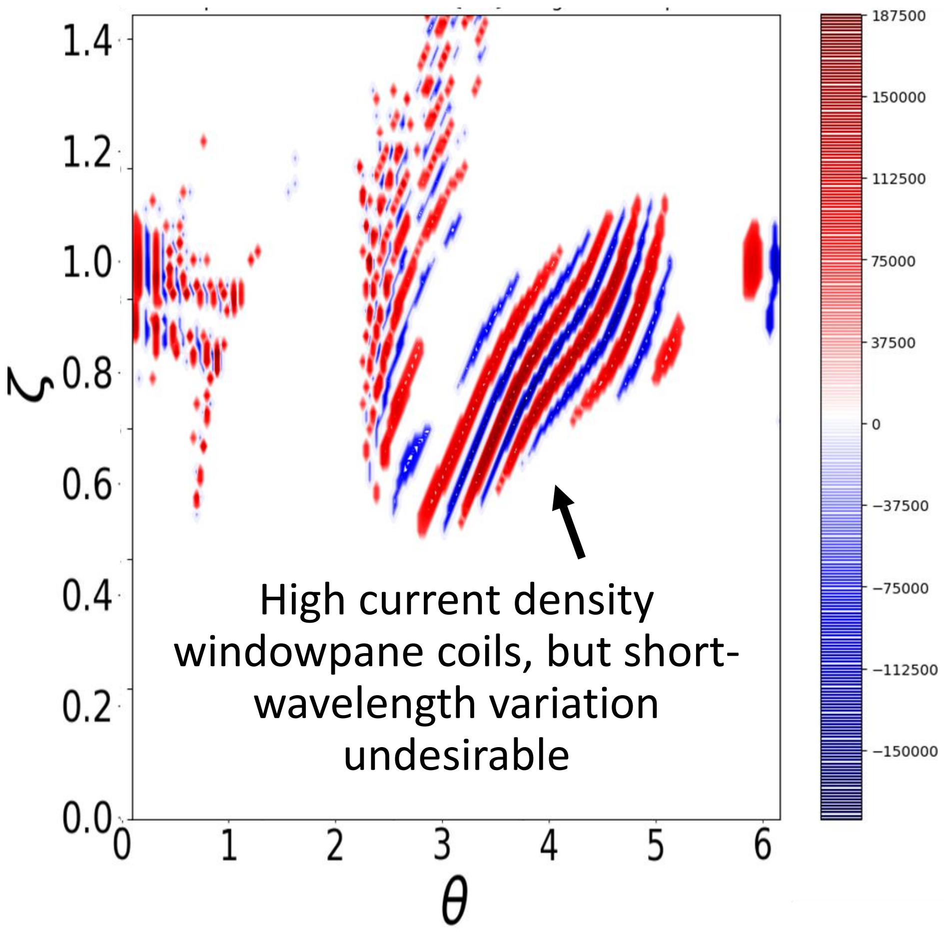


→ Truncated SVD retaining only most efficient singular values may eliminate short-wavelength features

Truncated SVD analysis may yield simple, sparse FEM current potentials

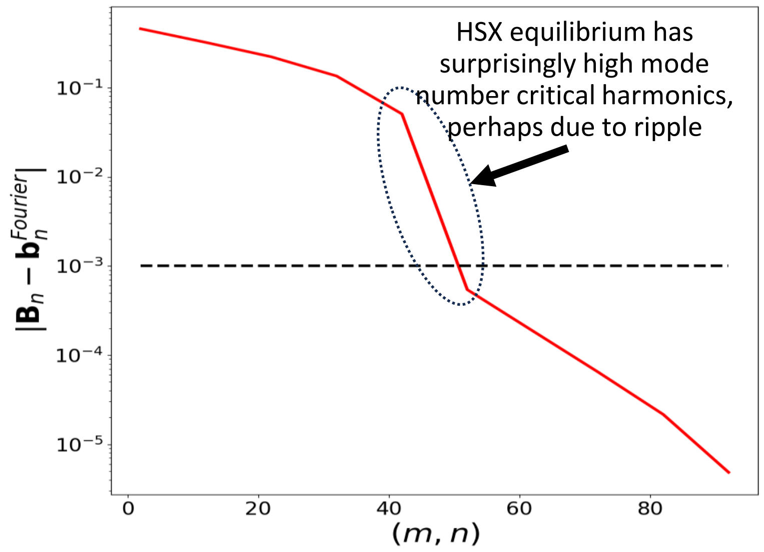
Dipole strengths found via LSS

$$\boldsymbol{M} \cdot \boldsymbol{d} = B_n$$

M: Dipole transfer matrix

d: Dipole strengths

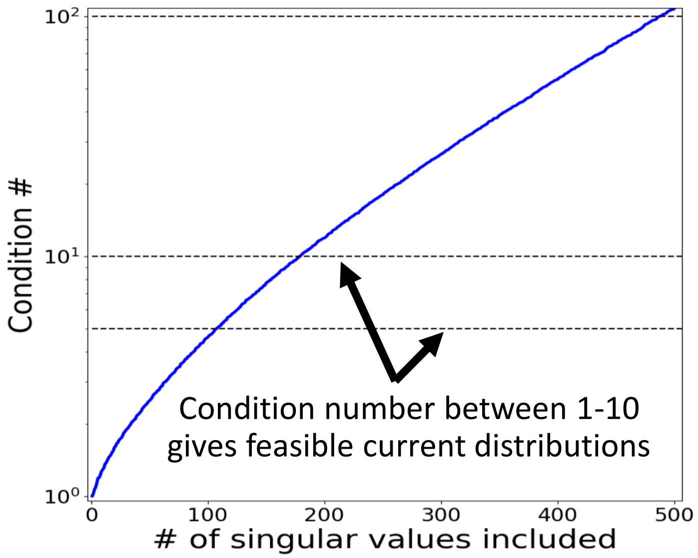
B_n: Error field



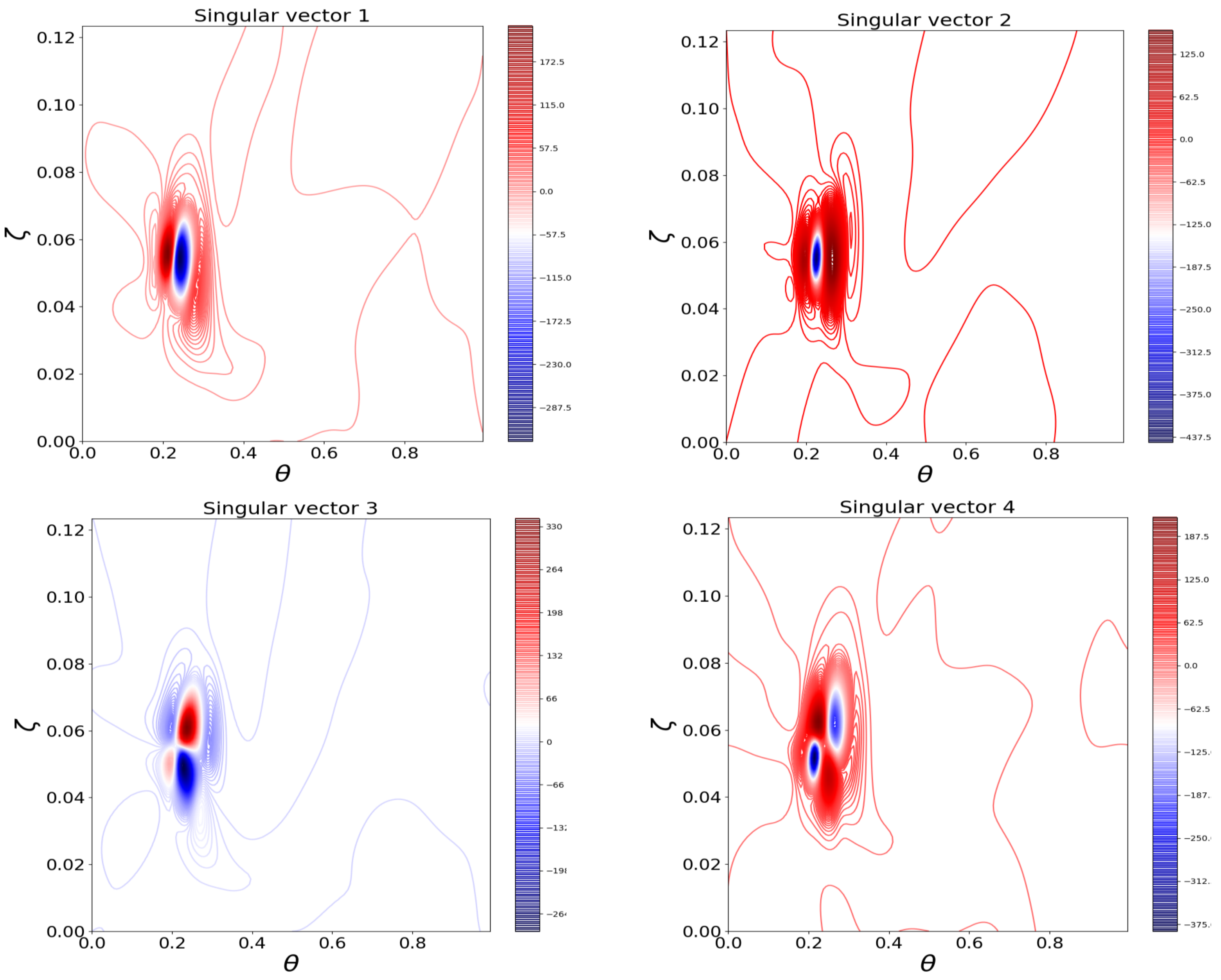
Method:

1. Fourier decompose $\boldsymbol{B} \cdot \hat{n}$, **M** matrix to relate current potential patches to global error fields
2. Compute SVD of $\boldsymbol{M} = \boldsymbol{U}\boldsymbol{S}\boldsymbol{V}^{\boldsymbol{h}}$**,** retain singular values only up to a condition number between 1-10 for feasible current distributions
3. Proceed with fractional solves:
   1. Eliminate dipoles with strengths less than some magnitude
   2. Re-compute new dipole strengths using fraction of winding surface area
   3. Reduce fraction of surface used until 0.1% error achieved

Condition number of **M** increases exponentially w/ SVs



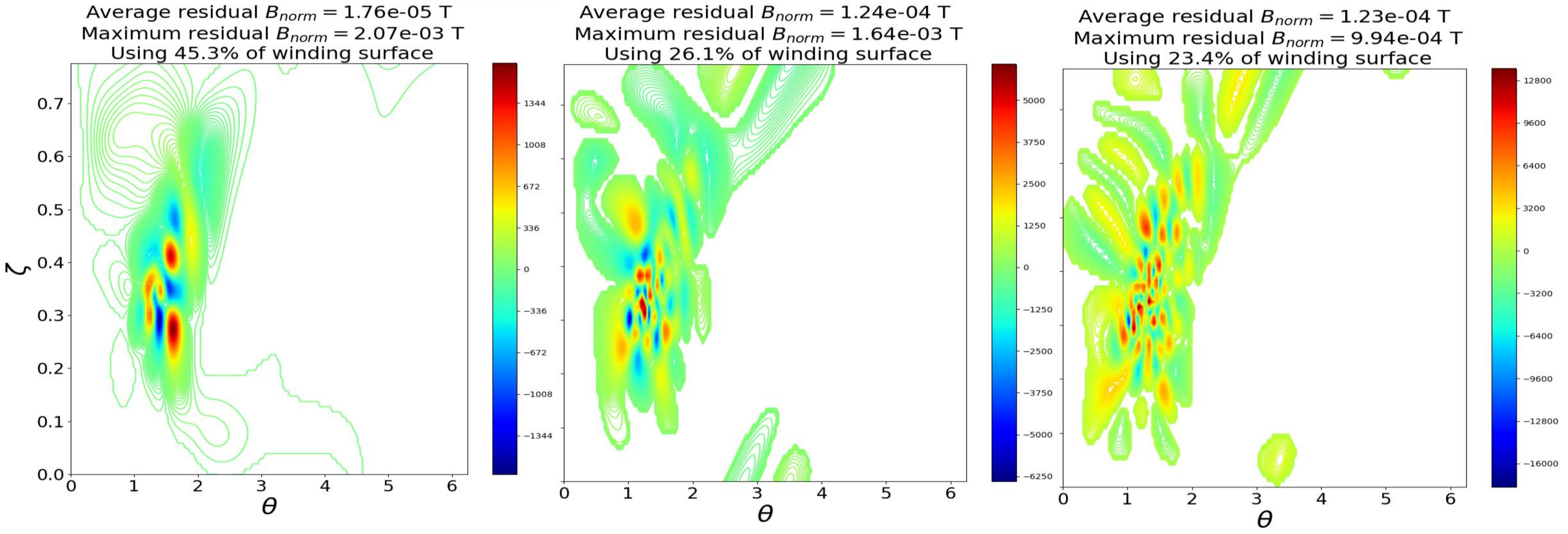
First few singular vectors surprisingly localized, not smooth



First few singular values typically have simple features, though this FEM-like basis yields singular vectors with some complexity

Fractional area required to produce field to 0.1% accuracy decreases with increasing condition number

Increasing condition number

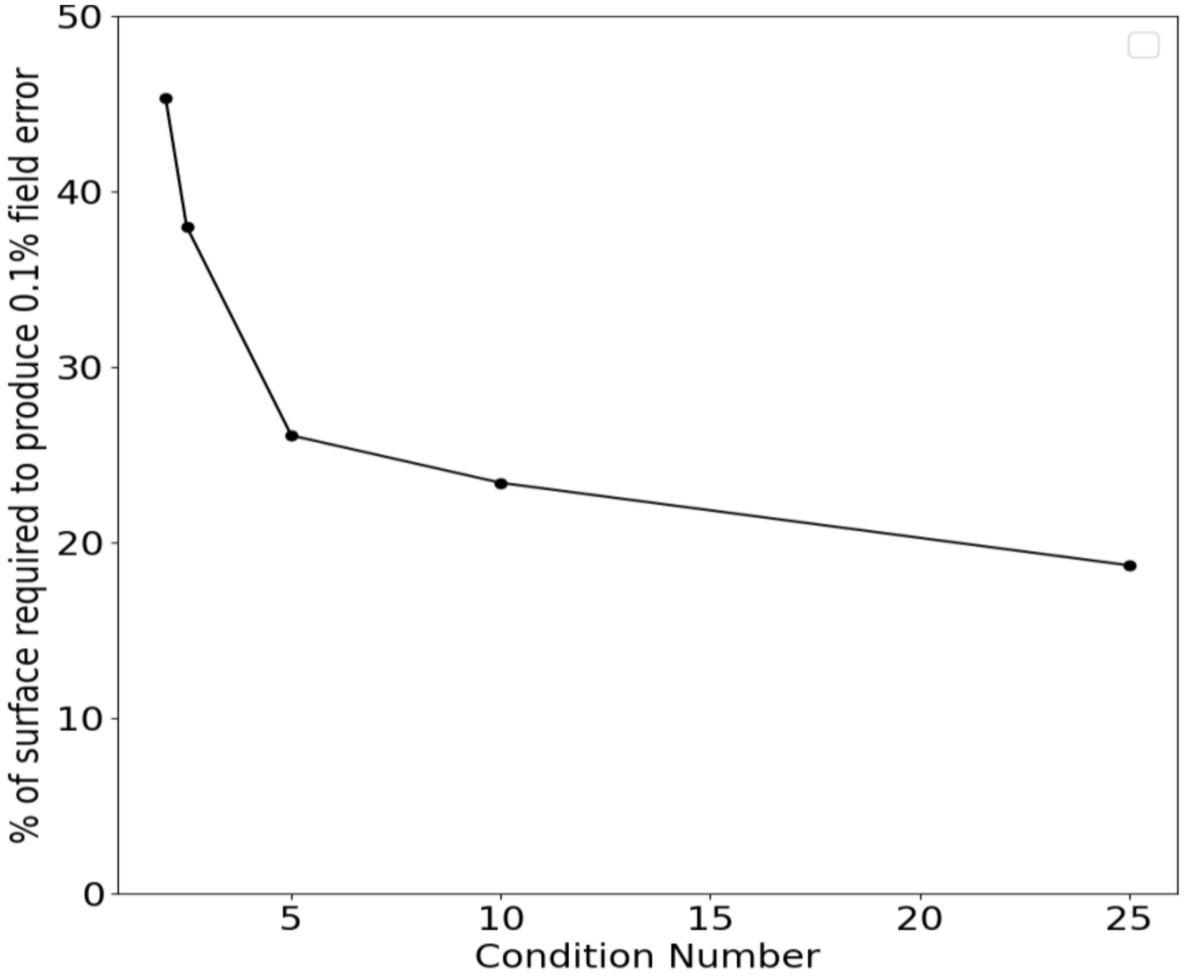


Surface plots of dipole strengths analogous to current potentials. Contours are potential windowpane coils.

Amplitudes of larger singular eigenvectors larger, still have short-wavelength features which are undesirable for coil production

Fractional area required to produce field to 0.1% accuracy decreases with increasing condition number

Fractional area required to produce field to specified accuracy saturates quickly w.r.t. condition number



Summary

Attempting to generate sparse current potentials using a finite-element-method-like basis and L0,L1 regularization. This could be used for efficient windowpane coil placement, yielding simple open-access stellarator coil configurations. Method can be improved by:

- Minimizing the appearance of short-wavelength features, perhaps by regularization w.r.t dipole amplitude gradient
- Moving to an actual, not approximate FEM basis and enforcing smoothness via regularization
- Any suggestions?